Other Modulation Techniques
- CAP, QAM, DMT

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Complex Signals

- Concept useful for describing a pair of real signals
- Let $j = \sqrt{-1}$

Two Important Properties of Real Signals

- Amplitude is symmetric ($|A(j\omega)| = |A(-j\omega)|$)
- Phase is anti-symmetric ($\angle A(j\omega) = -1 \times \angle A(-j\omega)$)

Two Important Complex Relationships

- Continuous-time

$$e^{j\omega t} = \cos(\omega t) + j\sin(\omega t) \qquad (1)$$

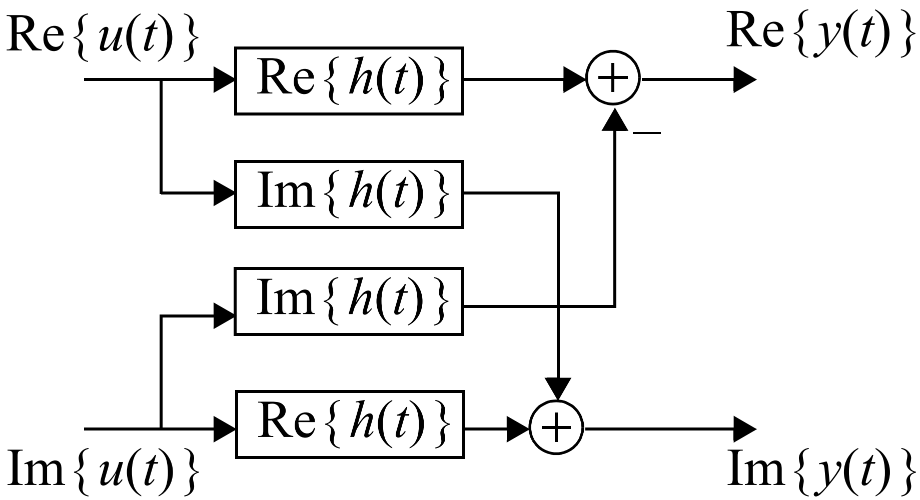
- Discrete-time

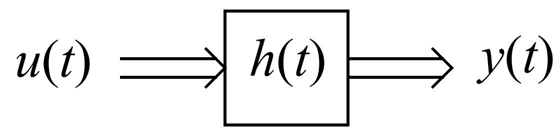
$$e^{j\omega nT} = \cos(\omega nT) + j\sin(\omega nT) \qquad (2)$$

Complex Transfer Function

- Let $h(t)$ be a complex impulse response

$$h(t) = \mathrm{Re}\{h(t)\} + j\mathrm{Im}\{h(t)\} \tag{3}$$





- 4 systems needed if both $h(t)$ and $u(t)$ complex
- 1 system needed if both $h(t)$ and $u(t)$ real
- 2 systems needed if one is complex and other real

Hilbert Transform

- Often need a complex signal with all negative frequency components zero — use Hilbert transform
- Hilbert transform is a ***real*** filter with response

$$h_{bt}(t) = \frac{1}{\pi t} \tag{4}$$

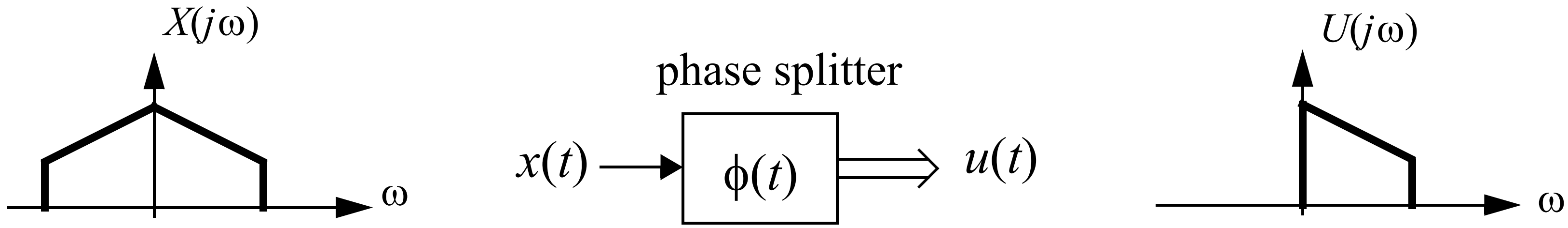
$$H_{bt}(j\omega) = -j\,\mathrm{sgn}(\omega) \tag{5}$$

- The Hilbert transform of a signal $x(t)$ is denoted as $\hat{x}(t)$ and can be found using filter in (5)

$$\hat{X}(j\omega) = -j\,\mathrm{sgn}(\omega)X(j\omega) \tag{6}$$

- Shift phase of signal by -90 degrees at all frequencies — allpass filter with phase shift
- Recall $j = e^{-j(\pi/2)}$

Phase Splitter



- A complex system, $\phi(t)$, that removes negative frequency components referred to as a ***phase splitter***.

$$\Phi(j\omega) = \begin{cases} 1, & \omega \ge 0 \\ 0, & \omega < 0 \end{cases} \tag{7}$$

- A phase splitter is built using a Hilbert transform (hence the name phase splitter)

Phase Splitter

- To form a signal, $u(t)$, having only positive freq components from real signal, $x(t)$

$$u(t) = 0.5(x(t) + j\hat{x}(t)) \tag{8}$$

- $u(t)$ is two real signals where we think of signals as

$$x(t) = \mathrm{Re}\{2u(t)\} \tag{9}$$

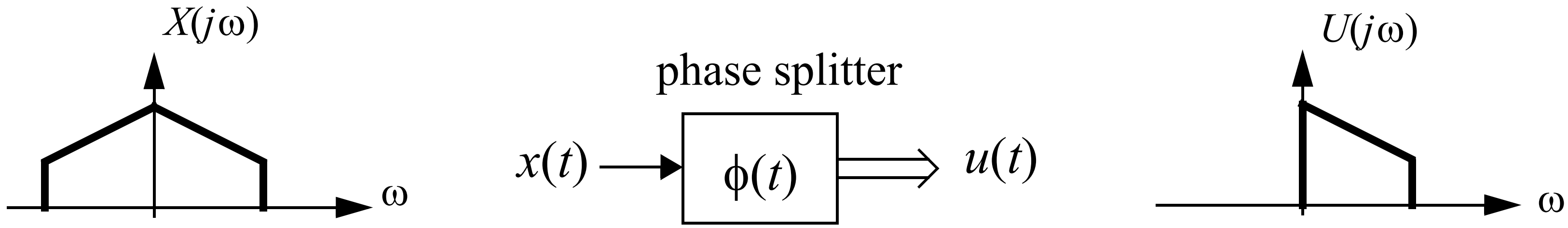
$$\hat{x}(t) = \mathrm{Im}\{2u(t)\} \tag{10}$$

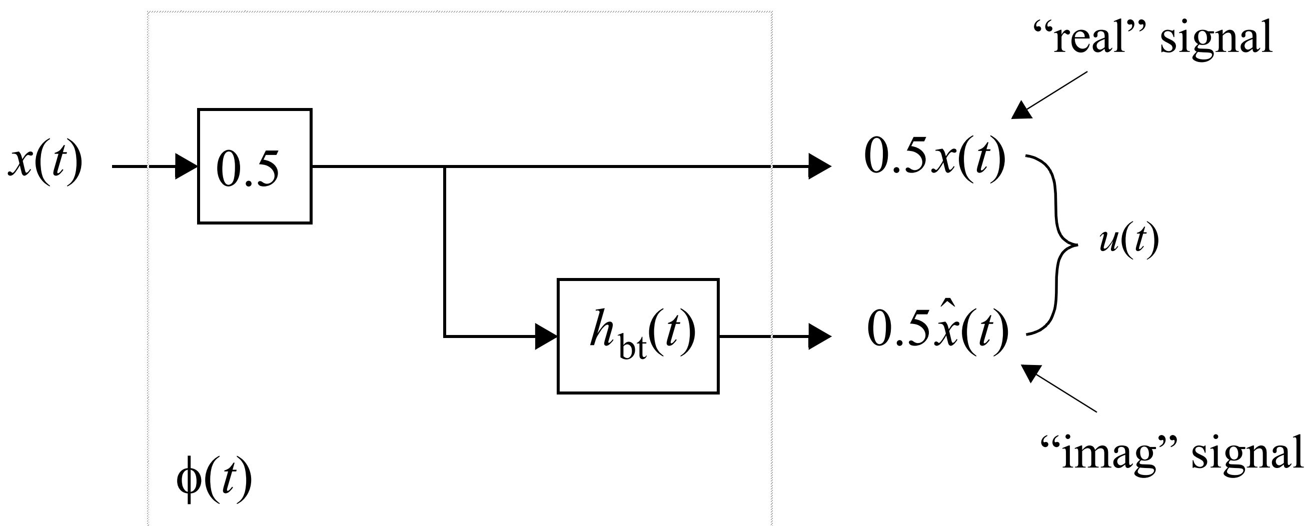
- To see that only positive frequency components remain — use (6) and (8)

$$U(j\omega) = 0.5(X(j\omega) + j \times (-j\,\mathrm{sgn}(\omega)X(j\omega))) \tag{11}$$

$$U(j\omega) = 0.5(X(j\omega) + \mathrm{sgn}(\omega)X(j\omega)) \tag{12}$$

Phase Splitter

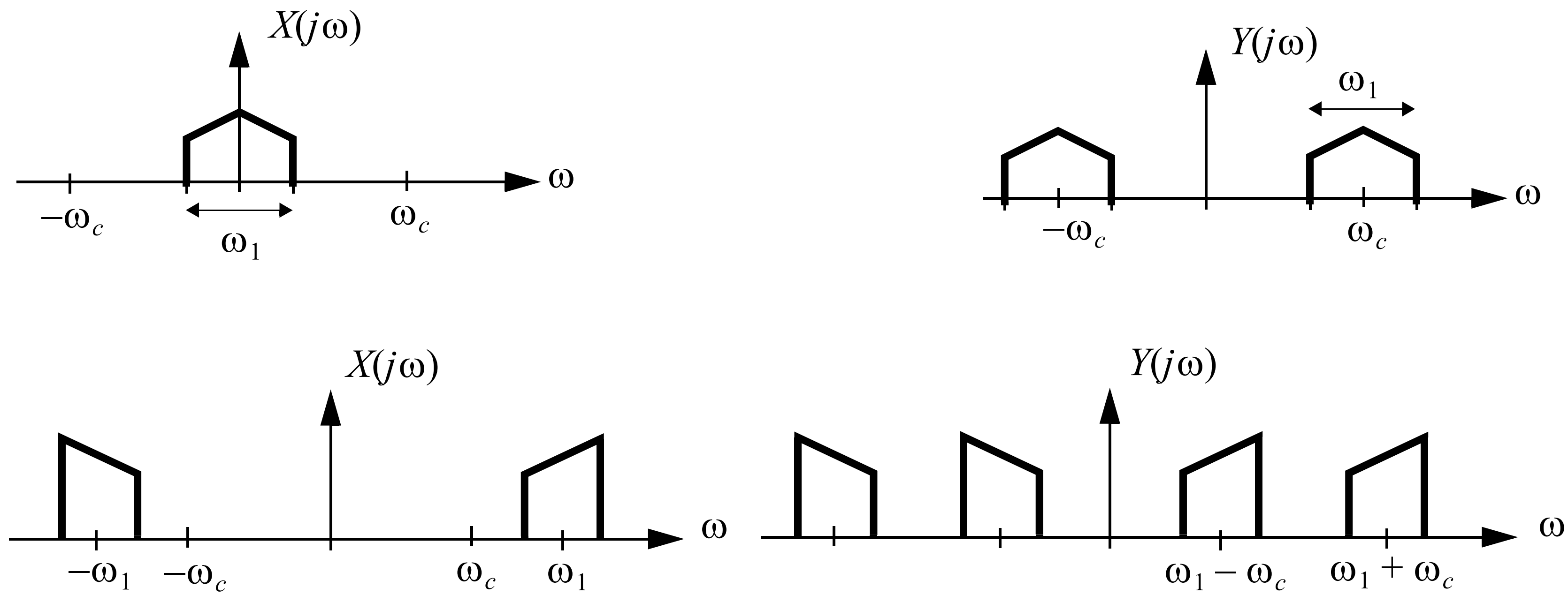




Real-Valued Modulation

$$y(t) = x(t)\cos(\omega_c t) \tag{13}$$

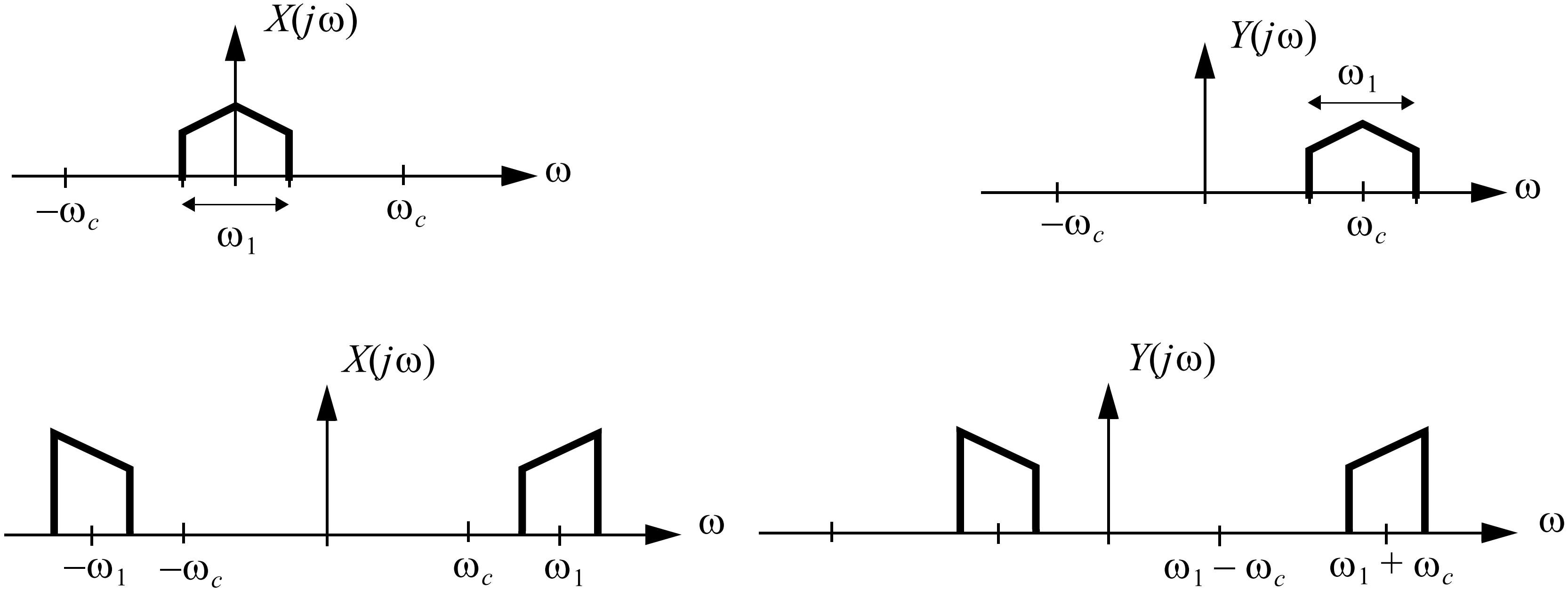
- Multiplication by $\cos(\omega_c t)$ results in convolution of frequency spectrum with two impulses at $+\omega_c$ and $-\omega_c$



Complex Modulation

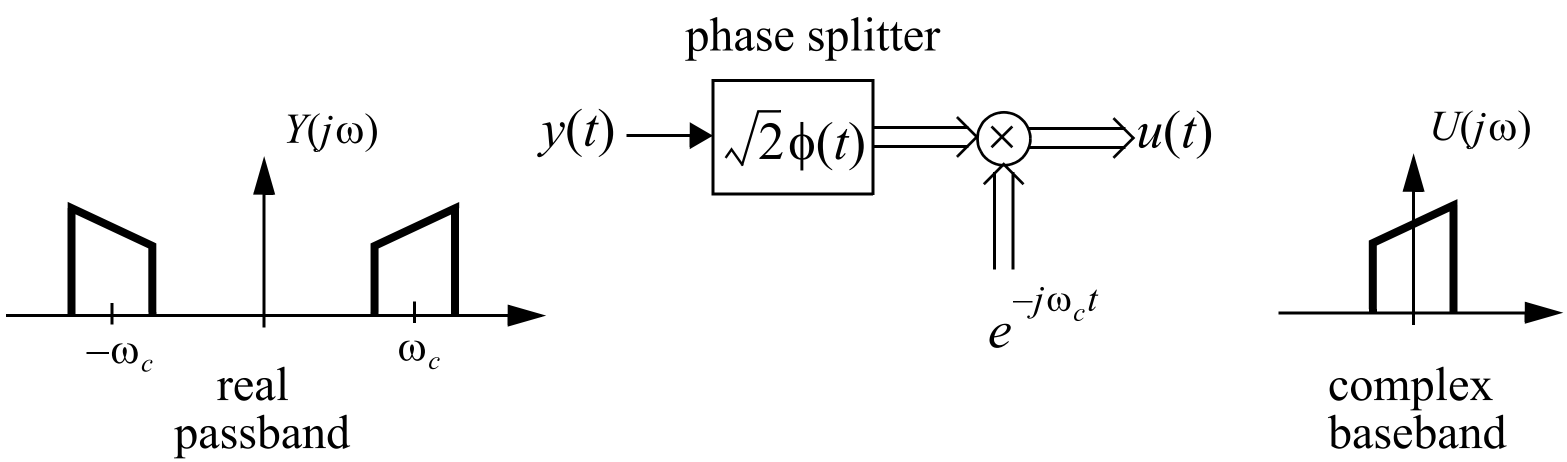
$$y(t) = e^{j\omega_c t} x(t) \quad (14)$$

- Mult a signal by $e^{j\omega_c t}$ shifts spectrum by $+\omega_c$,



Passband and Complex Baseband Signals

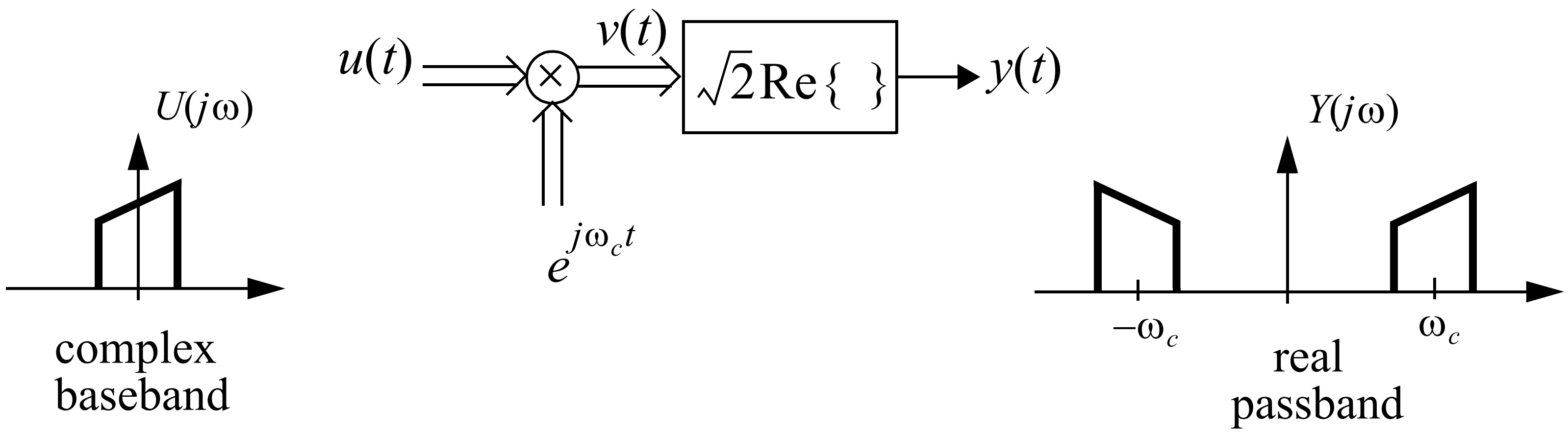
- Can represent a passband signal as a complex baseband signal.
- Need complex because passband signal may not be symmetric around ω_c



- $\sqrt{2}$ factor needed to keep the same signal power.

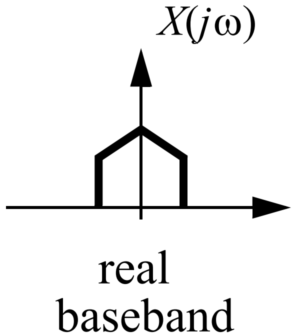
Modulation of Complex Baseband

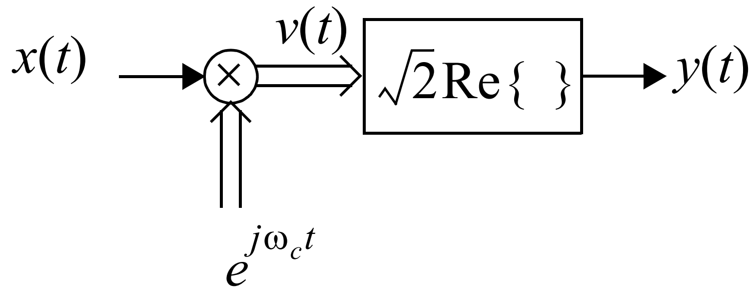
- It is only possible to send ***real*** signals along channel
- Can obtain passband modulation from a complex baseband signal by complex modulation then taking real part.

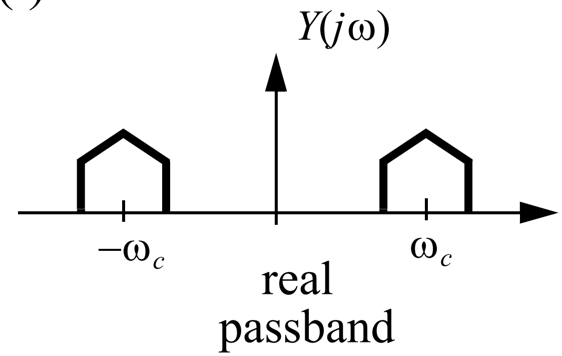


- Works because $v(t)$ has only positive freq. therefore its imag part is its Hilbert transform and taking real part restores negative frequencies.

Double Sideband





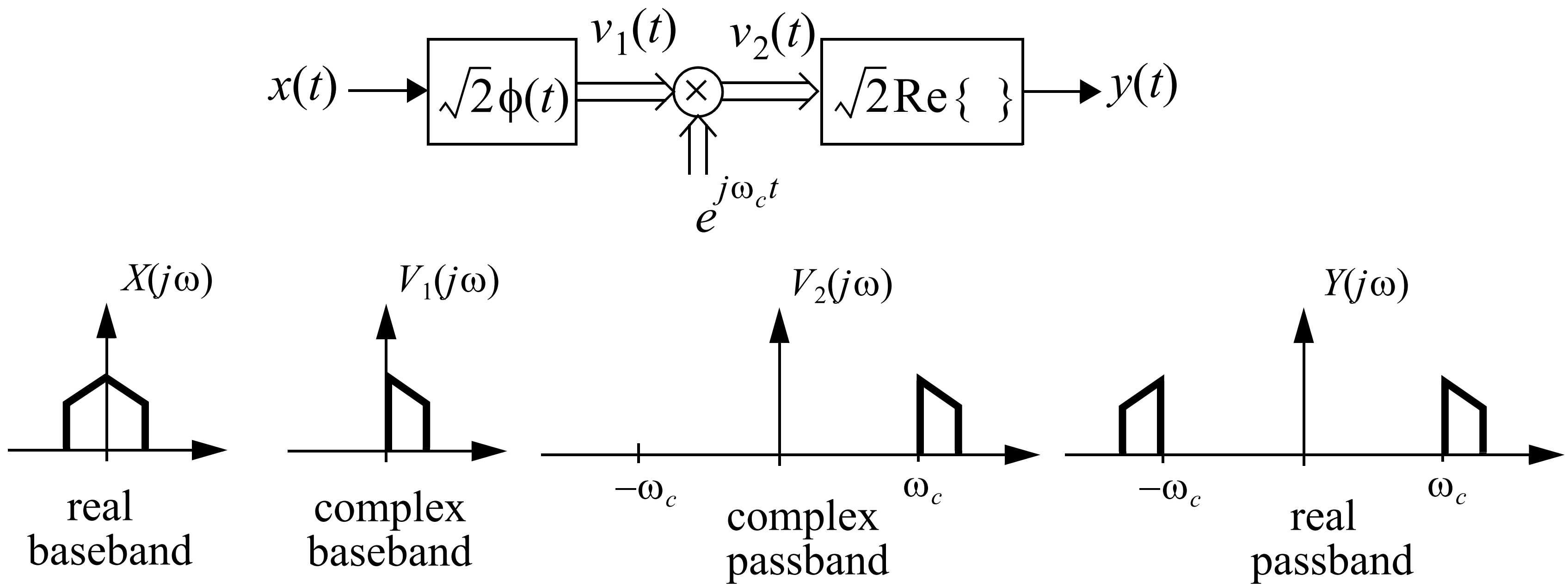


$$v(t) = x(t) \times (\cos(\omega_c t) + j\sin(\omega_c t)) \tag{15}$$

$$y(t) = \sqrt{2}x(t)\cos(\omega_c t) \tag{16}$$

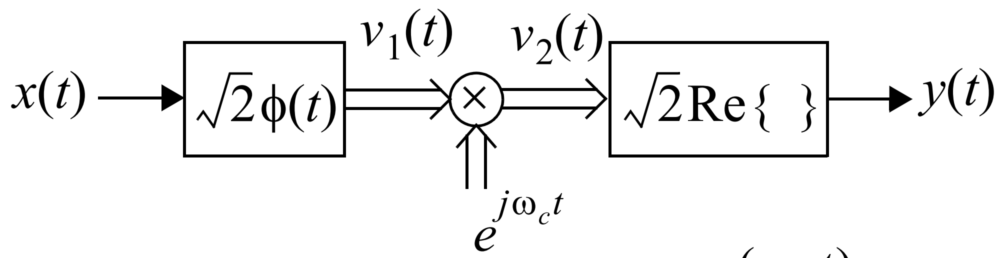
- $x(t)$ is a real signal so positive and negative frequencies symmetric
- Modulated signal, $y(t)$, has symmetry above and below carrier freq, ω_c — using twice minimum bandwidth necessary to send baseband signal.

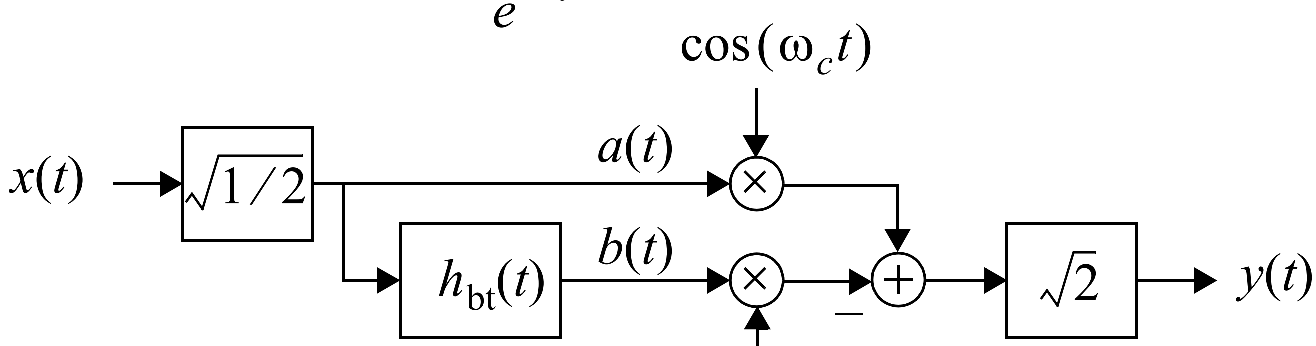
Single Sideband



- Twice as efficient as double sideband
- Disadvantage — requires a phase-splitter good to near dc (difficult since a phase discontinuity at dc)

Single Sideband





$v_1(t) = a(t) + jb(t)$

- If $v_1(t) = a(t) + jb(t)$, then $y(t) = \mathrm{Re}\{e^{j\omega_c t} v_1(t)\}$ becomes

$$y(t) = \sqrt{2}\mathrm{Re}\{(\cos(\omega_c t) + j\sin(j\omega_c t)) \times (a(t) + jb(t))\} \tag{17}$$

$$y(t) = \sqrt{2}a(t)\cos(\omega_c t) - \sqrt{2}b(t)\sin(\omega_c t) \tag{18}$$

Quadrature Amplitude Modulation (QAM)

- Start with two independent real signals

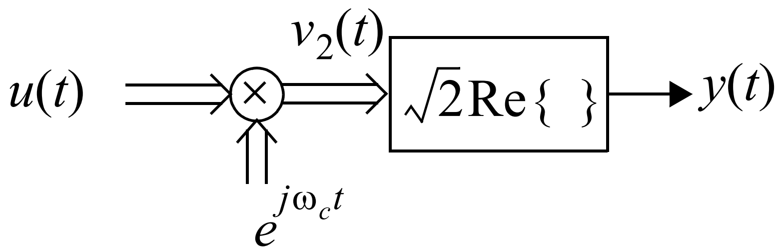
$$u(t) = a(t) + jb(t) \tag{19}$$

- In general, they will form a complex baseband signal
- Modulate as in single-sideband case

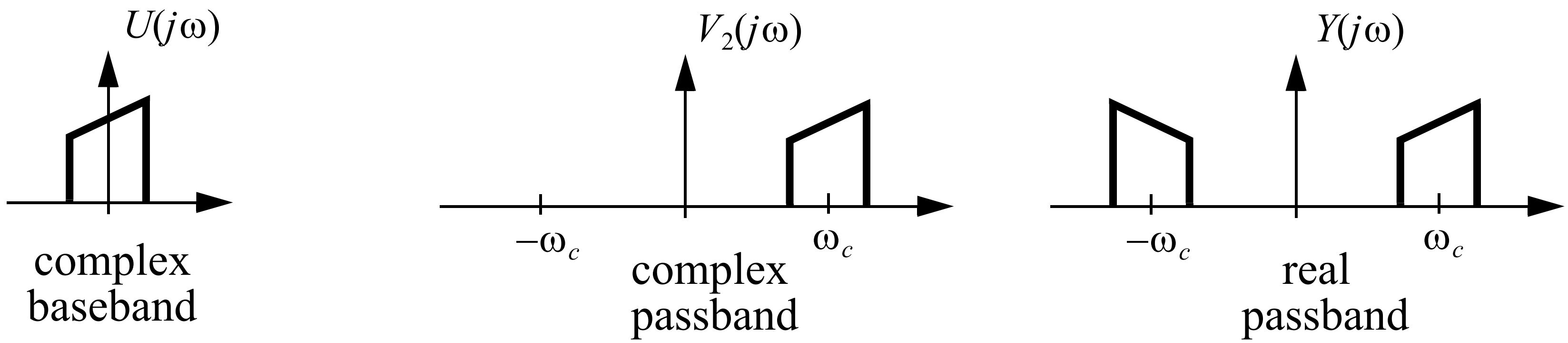
$$y(t) = \sqrt{2}a(t)\cos(\omega_c t) - \sqrt{2}b(t)\sin(\omega_c t) \tag{20}$$

- Data communications: $a(t)$ and $b(t)$ are outputs of two pulse shaping filters with multilevel inputs, A_k and B_k
- While QAM and single sideband have same spectrum efficiency, QAM does not need a phase splitter
- Typically, spectrum is symmetrical around carrier but information is twice that of double-side band.

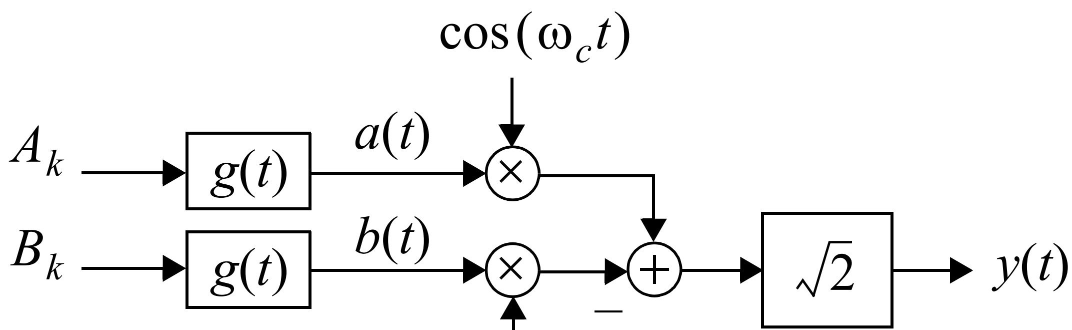
QAM



$v_2(t)$
$u(t)$
$\sqrt{2}\mathrm{Re}\{\ \}$
$y(t)$
$e^{j\omega_c t}$



$U(j\omega)$
complex
baseband
$V_2(j\omega)$
$-\omega_c$
ω_c
complex
passband
$Y(j\omega)$
$-\omega_c$
ω_c
real
passband

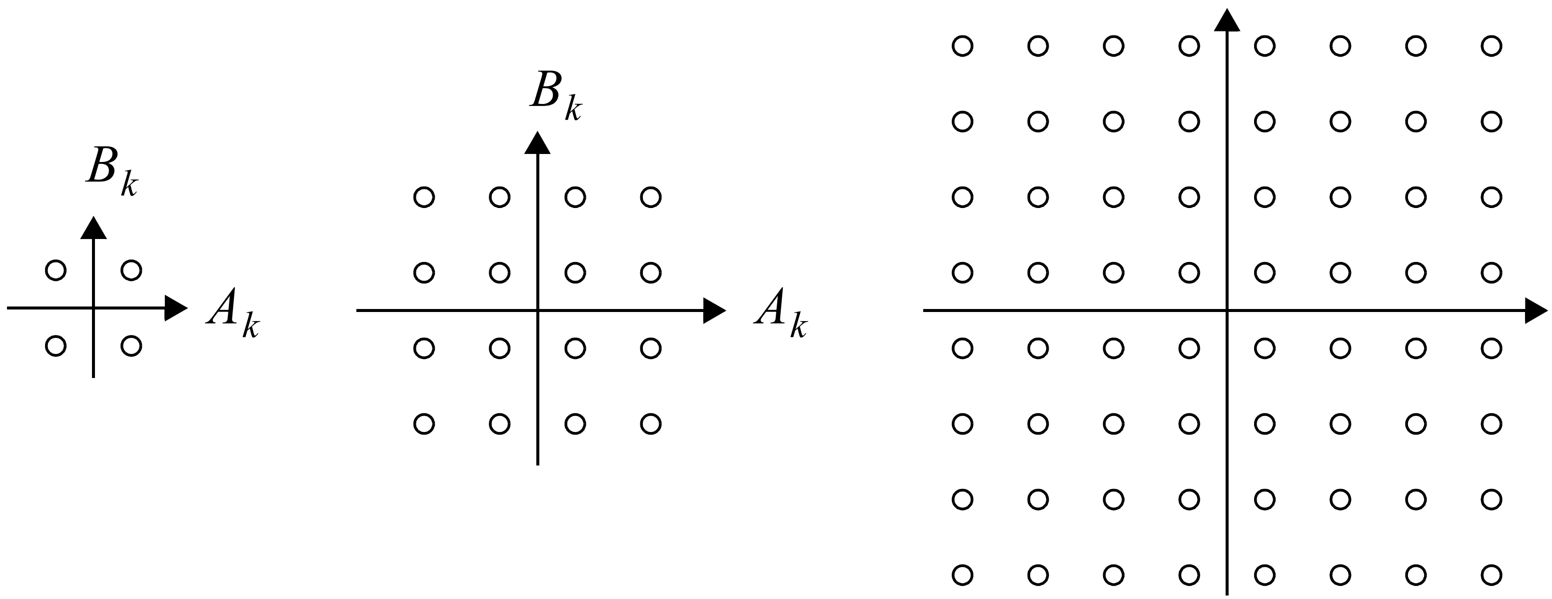


$\cos(\omega_c t)$
A_k
$g(t)$
$a(t)$
B_k
$g(t)$
$b(t)$
$-$
$\sqrt{2}$
$y(t)$
$\sin(\omega_c t)$

$$u(t) = a(t) + jb(t)$$

QAM

- Can draw signal constellations

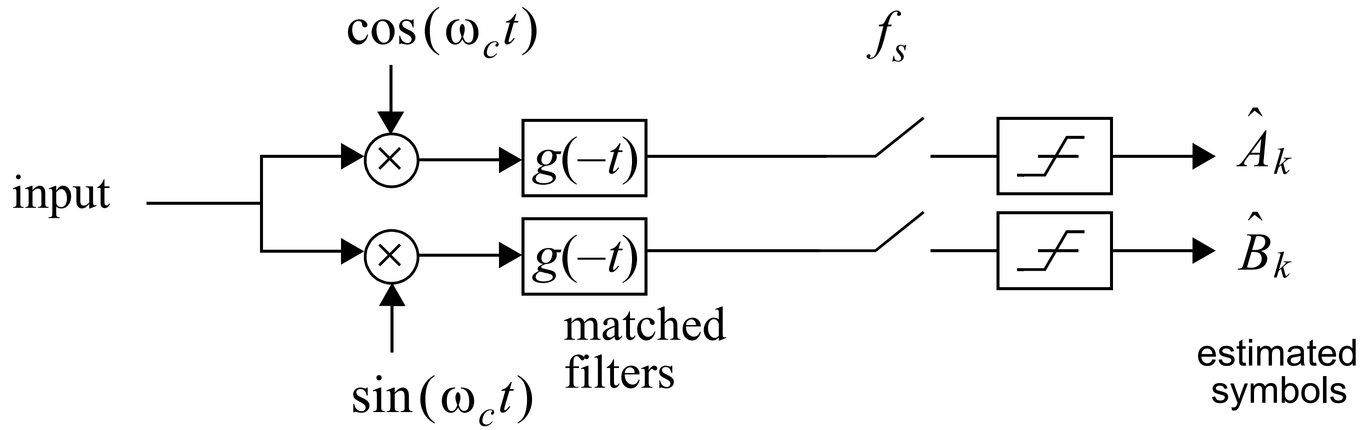


QAM 4 QAM 16 QAM 64

- Can Gray encode so that if closest neighbor to correct symbol chosen, only 1 bit error occurs

QAM

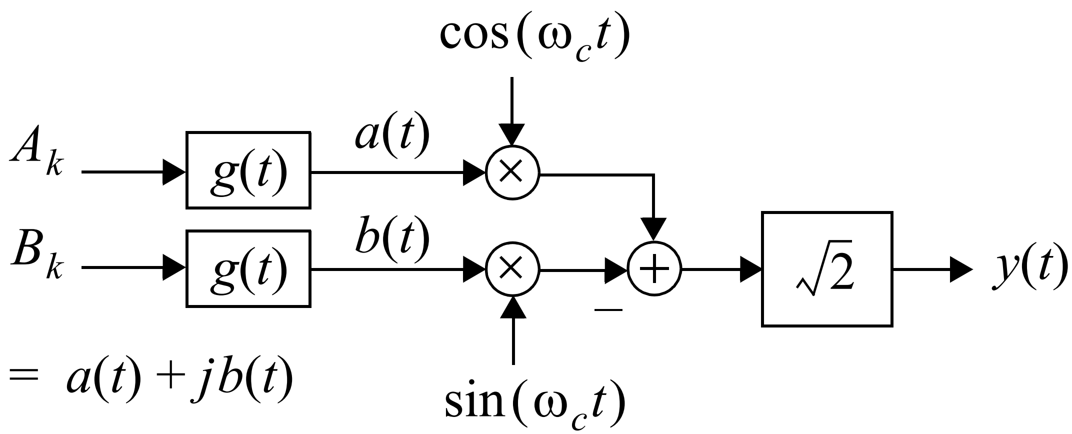
- To receive a QAM signal, use correlation receiver



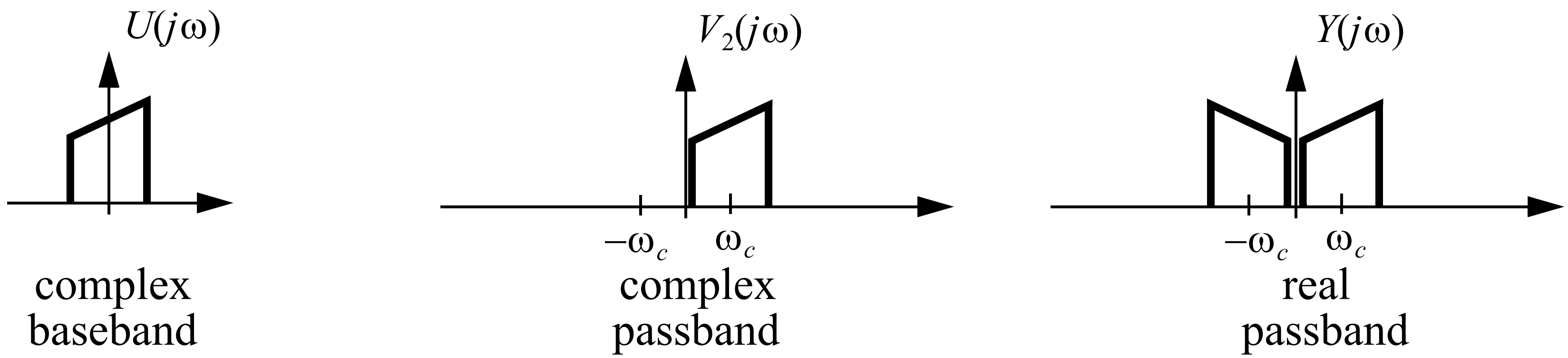
- When transmitting a small bandwidth (say 20kHz) to a large carrier freq (say 100MHz), often little need for adaptive equalization — use fixed equalizer

CAP

- Carrierless AM-PM modulation
- Essentially QAM modulated to a low carrier, f_c

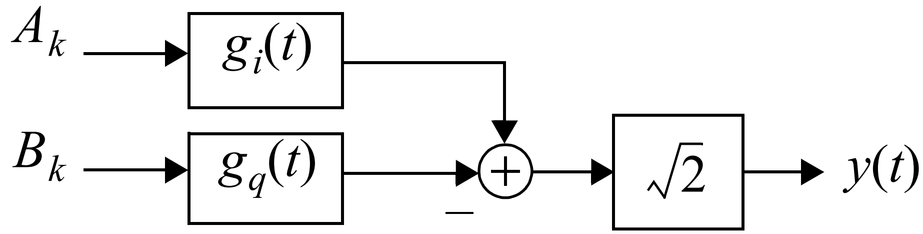


$$u(t) = a(t) + jb(t)$$



CAP

- BIG implementation difference — can directly create impulse response of two modulated signals.



where

$$g_i(t) = g(t)\cos(\omega_c t) \quad (21)$$

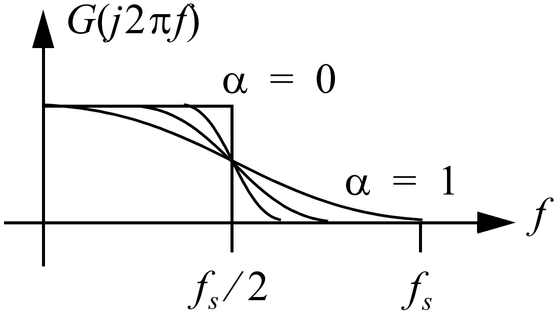
$$g_q(t) = g(t)\sin(\omega_c t) \quad (22)$$

- Not feasible if ω_c is much greater than symbol freq
- Two impulse responses are orthogonal

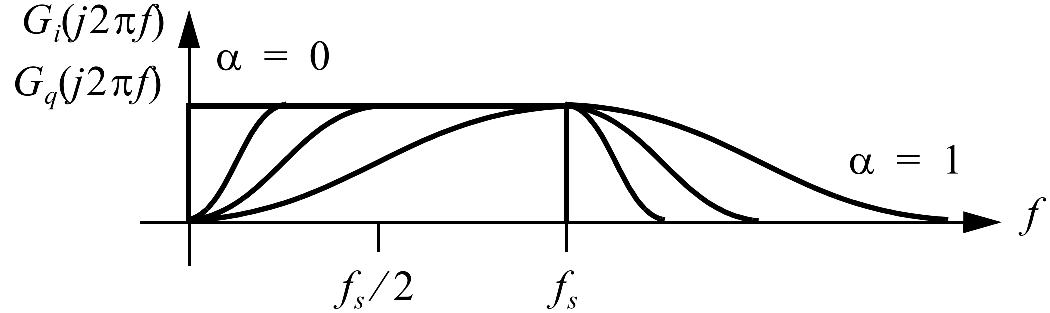
$$\int_{-\infty}^{\infty} g_i(t) g_q(t) dt = 0 \quad (23)$$

CAP

- The choice for ω_c depends on excess bandwidth



lowpass prototype

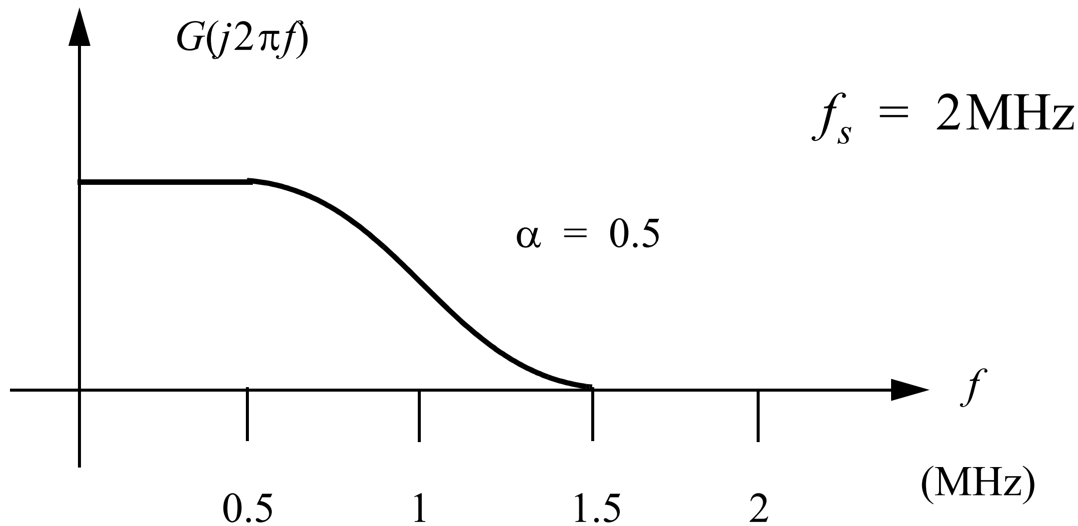


passband

- Excess bandwidth naturally gives a notch at dc
- For 100% excess bandwidth $\omega_c = f_s$
- For 0% excess bandwidth $\omega_c = f_s/2$

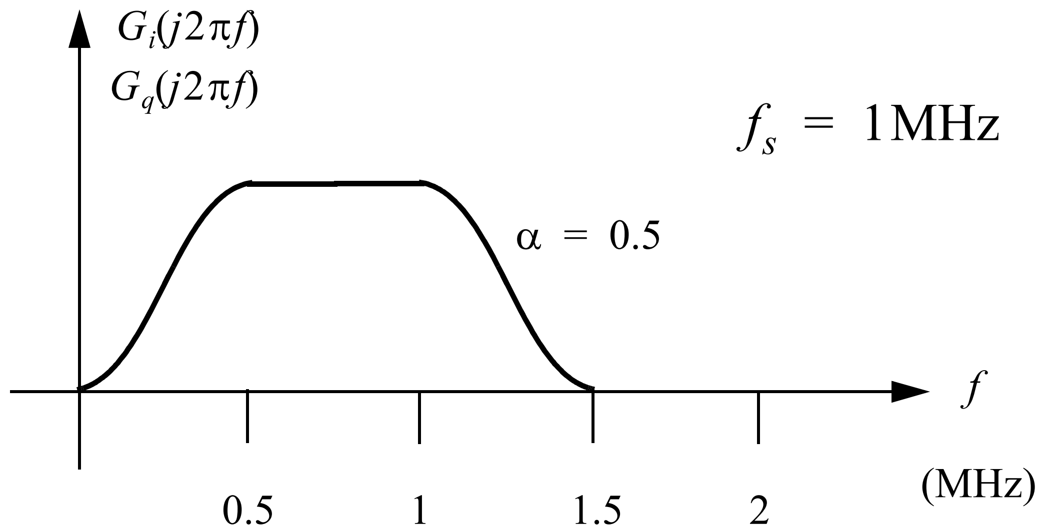
Example — Baseband PAM

- Desired Rate of 4Mb/s — Freq limited to 1.5MHz
- Use 50% excess bandwidth ($\alpha = 0.5$)
- Use 4-level signal (2-bits) and send at 2MS/s



Example — CAP

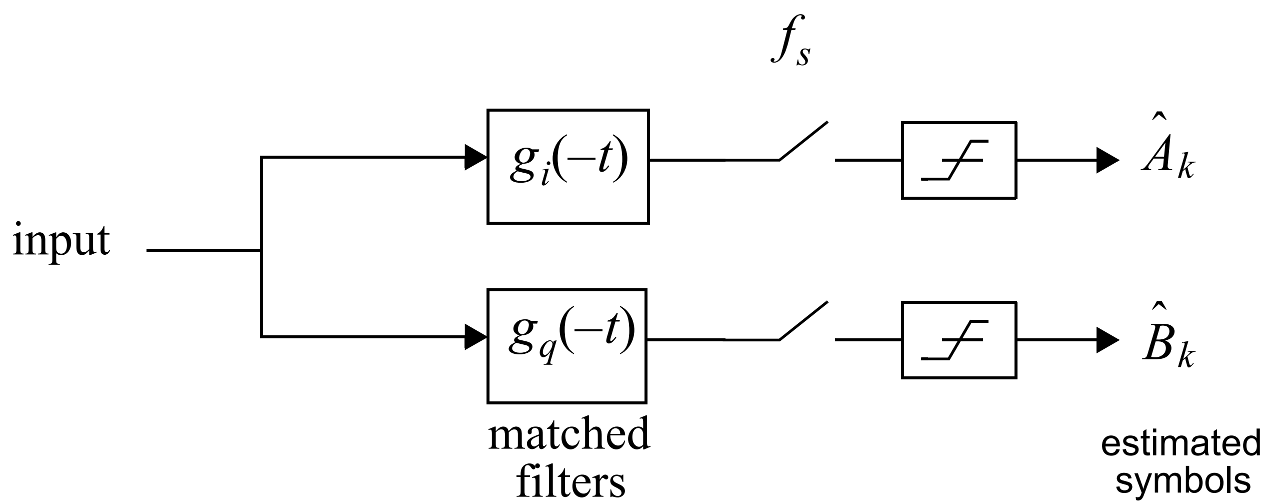
- Desired Rate of 4Mb/s — Freq limited to 1.5MHz
- Use 50% excess bandwidth ($\alpha = 0.5$)
- Use CAP-16 signalling and send at 1MS/s



- Note faster roll-off above 1MHz
- Area under two curves the same

CAP

- Two matched filters used for receiver



- When adaptive, need to adapt each one to separate impulse — should ensure they do not converge to same impulse

CAP vs. PAM

- Both have same spectral efficiency
- Carrier recovery similar? (not sure)

- CAP is a passband scheme and does not rely on signals near dc
- More natural for channels with no dc transmission

- Can always map a PAM scheme into CAP
  — 2-PAM $\leftrightarrow$ 4-CAP   4-PAM $\leftrightarrow$ 16-CAP   8-PAM $\leftrightarrow$ 64-CAP
- Cannot always map a CAP scheme into PAM
  — cannot map 32-CAP into PAM since $\sqrt{32}$ is not an integer number

DMT Modulation

- Discrete-MultiTone (DMT)
- A type of multi-level orthogonal multipulse modulation
- More tolerant to radio-freq interference
- More tolerant to impulse noise
- Can theoretically achieve closer to channel capacity
- Generally more complex demodulation
- Generally more latency

ADSL (Asymmetric DSL)

- 6Mb/s to home, 350kb/s back to central office over existing twisted-pair
- POTS splitter so telephone can coexist

Multipulse Modulation

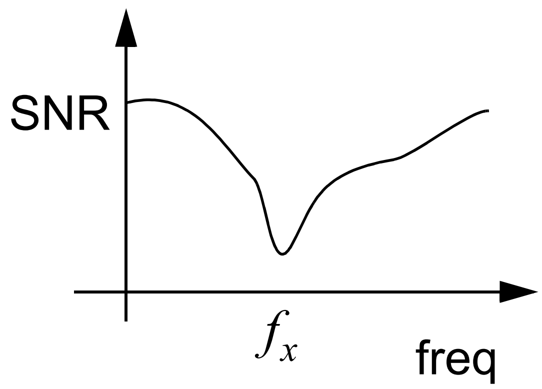
- Consider the two orthogonal signals from CAP — one transmission scheme is to transmit $g_i(t)$ for a binary 1 and $g_q(t)$ for a binary 0.
- Use a correlation receiver to detect which one was sent.
- Spectral efficiency (if $\alpha = 0$) is only $1\ \text{(symbols/s)/Hz}$ rather than $2\ \text{(symbols/s)/Hz}$ in the case of PAM
- In general, need $N\pi / T$ bandwidth to send N orthogonal pulses
- PAM, $N = 1$, minimum bandwidth: π / T
- QAM and CAP, $N = 2$, minimum bandwidth: $2\pi / T$

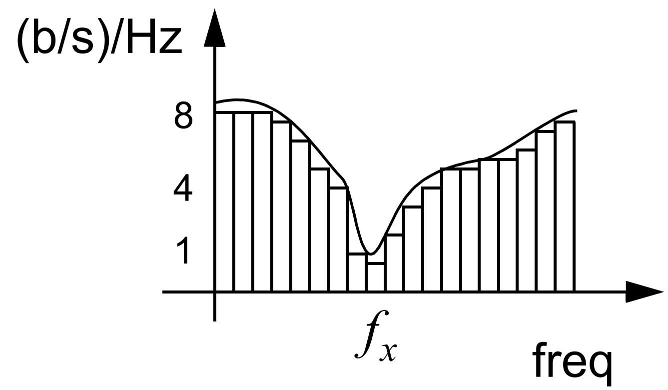
Combined PAM and Multipulse

- Changing scheme to sending $\pm g_i(t)$ and $\pm g_q(t)$ becomes a 2-level for each 2 orthogonal multipulses which is same as 4-CAP
- Multitone uses many orthogonal pulses as well as multi-levels on each (each pulse may have different and/or varying number of multi-levels)
- In discrete-form, it makes use of FFT — called Discrete MultiTone (DMT)
- Also called MultiCarrier Modulation (MCM)

Bit Allocation

- Allocate more bits where SNR is best

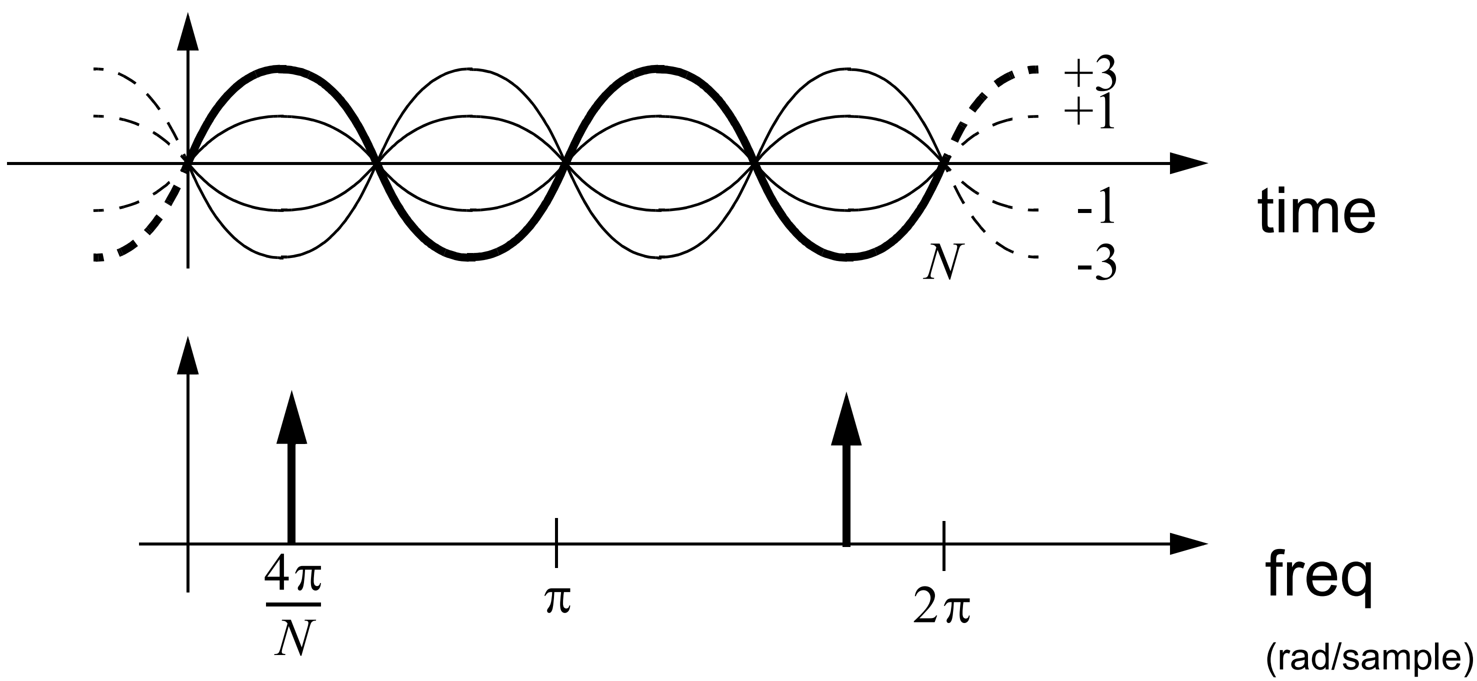




- A radio interferer causes low SNR at f_x
- Perhaps send only 1 b/s/Hz in those bands
- At high SNR send many b/s/Hz

FFT Review

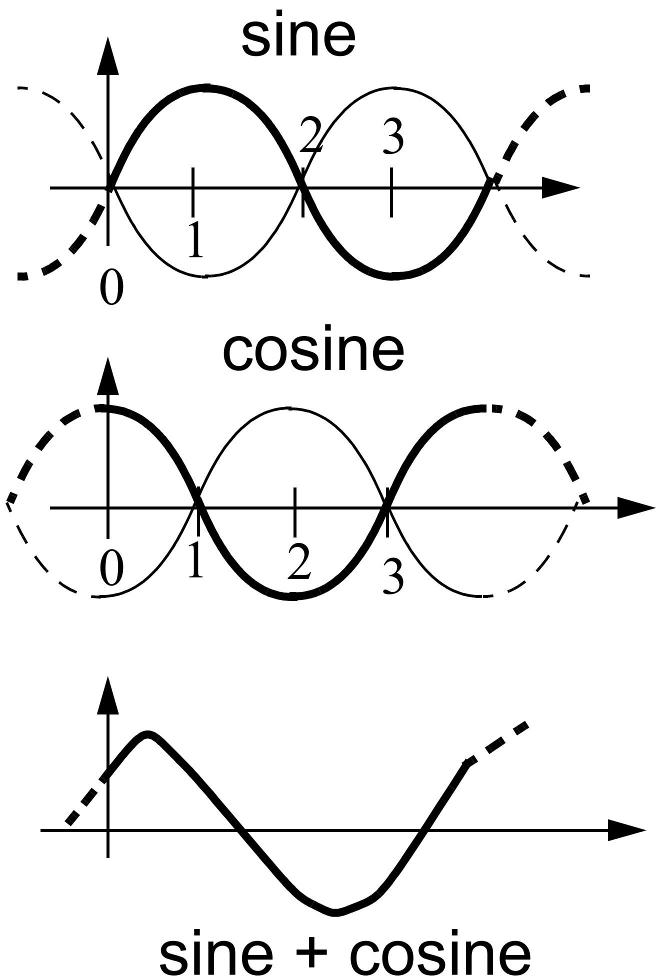
- FFT is an efficient way to build a DFT (Discrete Fourier Transform) when number of samples $N = 2^M$
- If rectangular window used and time-domain signal periodic in N, then FFT has impulses in freq domain

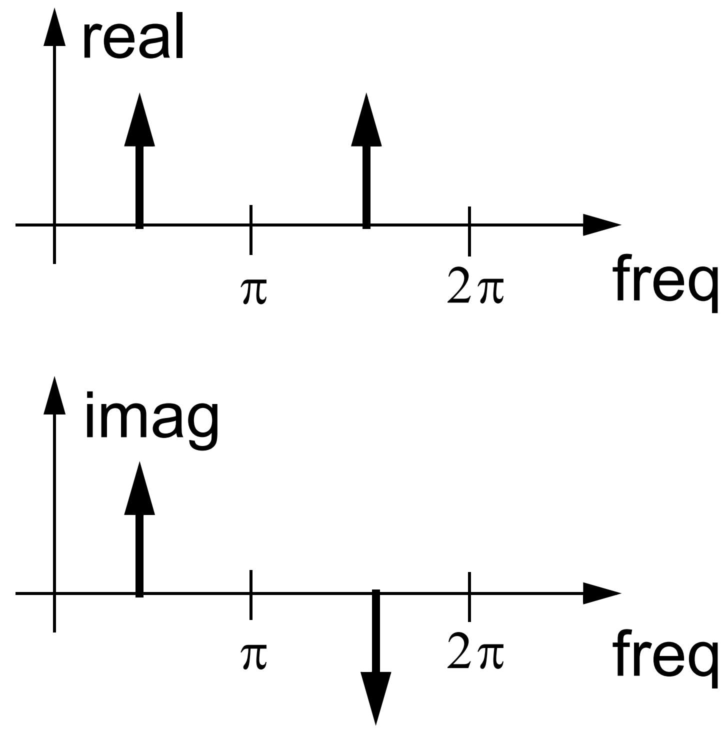


DMT Generation

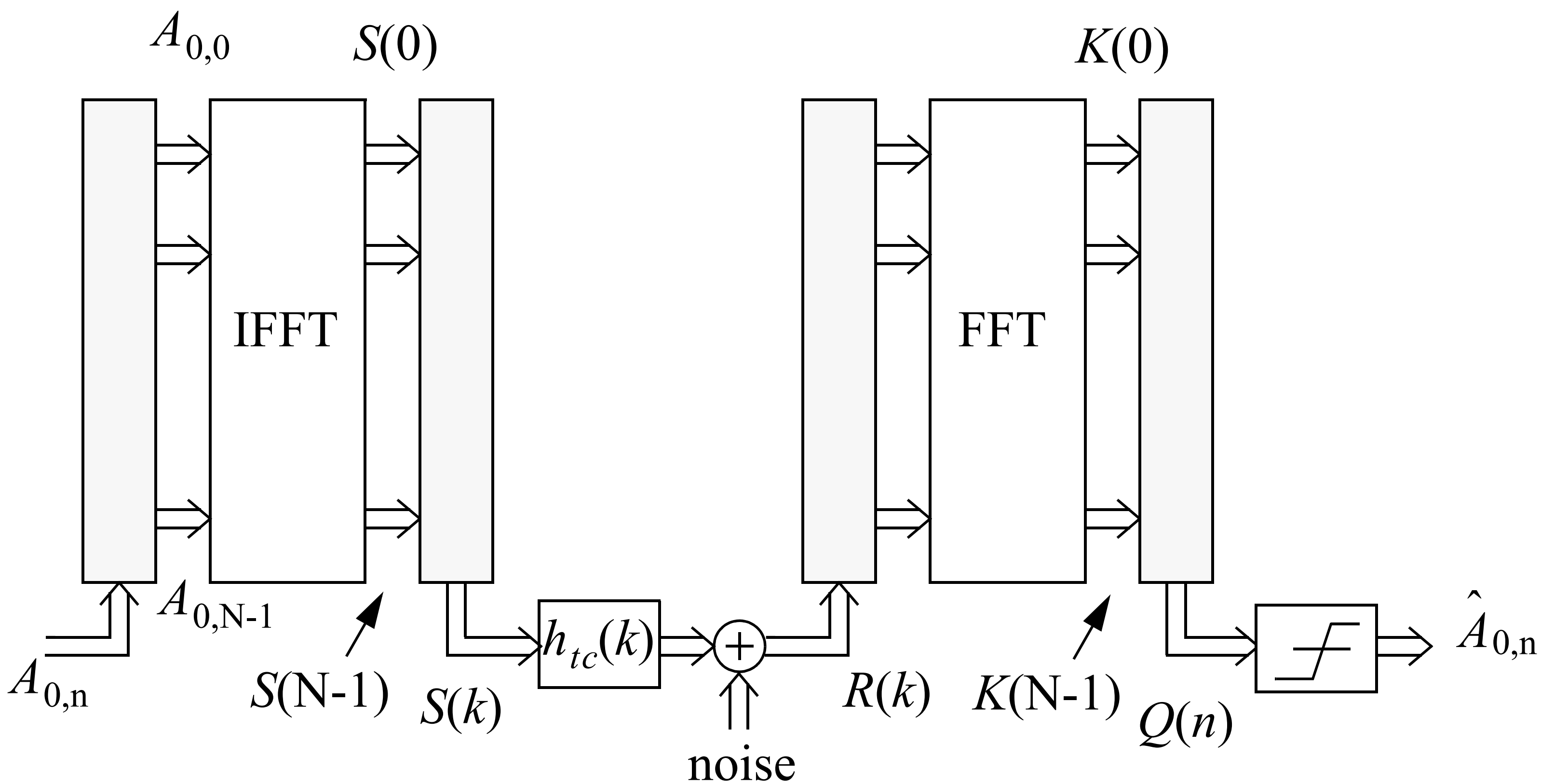
- Input to IFFT (inverse FFT) is quantized impulses at each freq (real and imag)
- Forced symmetric around π (complex conjugate)
- Output is real and is sum of quantized amplitude sinusoids
- Quantized real - quantized amplitude cosine
- Quantized imag - quantized amplitude sine
- Symbol-rate is much lower than bandwidth used

Example — N=4


sine
0
1
2
3
cosine
0
1
2
3
sine + cosine


real
π
2π
freq
imag
π
2π
freq

DMT Modulation



DMT Modulation

- Symbol Length, T
  — make symbol length as long as tolerable
  — typically need 3 symbol periods to decode
- If max channel bandwidth is f_{max}, sampling rate should be $f_{\mathrm{samp}} > 2f_{\mathrm{max}}$
- Choose $N = 2^M > f_{samp}T$ where M is an integer

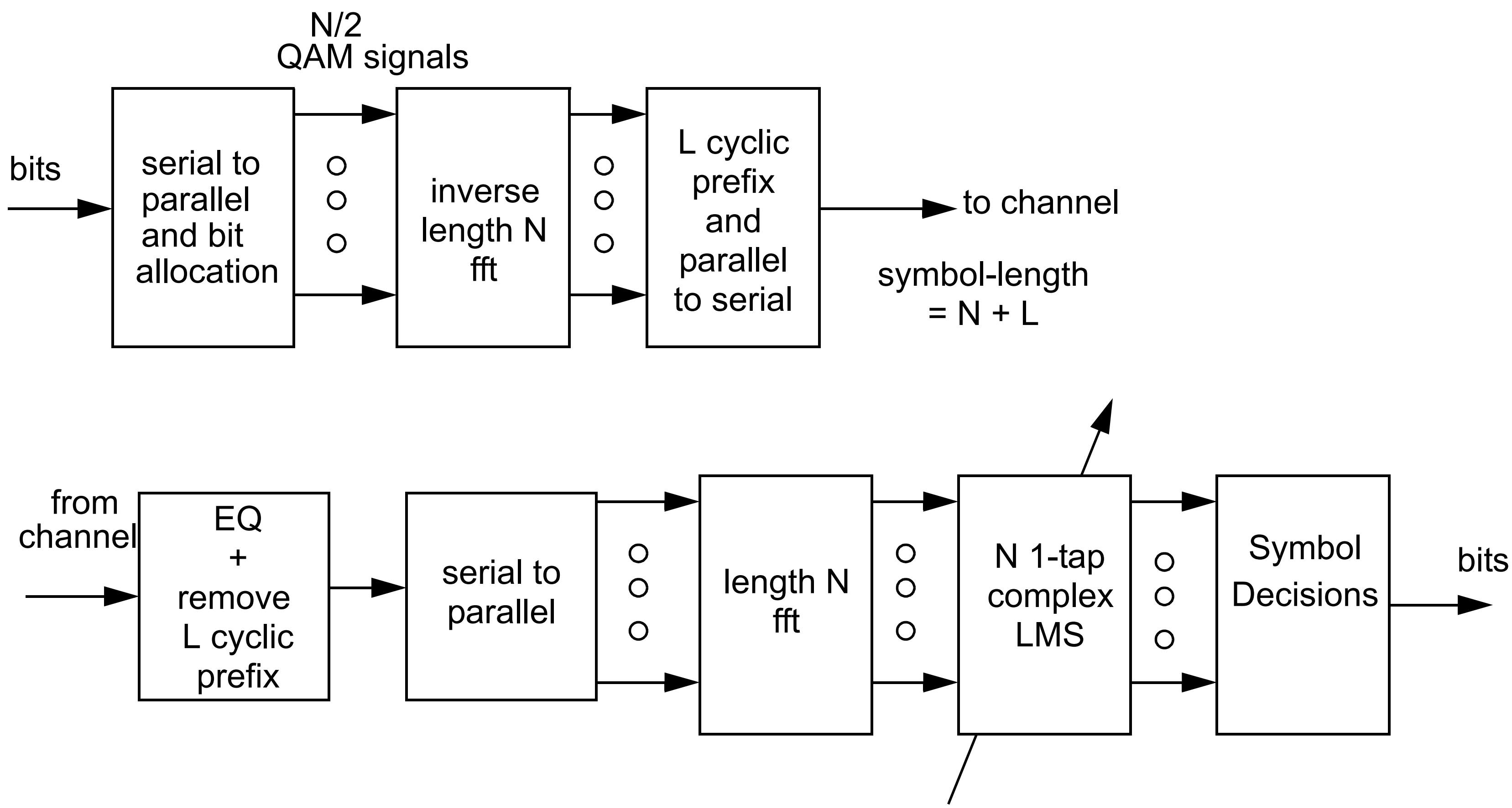
Example

- Max channel bandwidth is $1\mathrm{MHz}$,
- $f_{samp} = 2\mathrm{MHz}$, $N = 512$ results in $M = 9$, $T = 1/3.9\mathrm{kHz}$
- Channel bandwidths are $\Delta f = f_{\mathrm{max}}/(N/2) = 3.9\mathrm{kHz}$

Cyclic Prefix

- If channel is modelled as having a finite impulse response on length L, send last L samples at beginning to ignore transient portion of channel
- Could send much more but no need
- When receiving, ignore first L samples received (purge out transient part of channel)
- Each FFT bin will undergo phase and magnitude change, equalize out using a complex multiplication
- If channel model too long, pre-equalize to shorten signficant part of channel impulse response

DMT Modulation



DMT Modulation

- Clock sent in one frequency bin
- More tolerant to impulse noise because of long symbol length
  — expect around $10\log(N)$ dB improvement
  $N = 512$ implies 27 dB improvement
- Longer latency
- Can place more bits in frequency bins where more dynamic range occurs (achieve closer to capacity)
- Transmit signal appears more Gaussian-like
  — a large Crest factor
  — more difficult line driver
  — need channel with less distortion or clipping

Coding

Coding

Scrambling (Spectrum control)

- "Whiten" data statistics
- Better for dc balance and timing recovery

Line Coding (Spectrum control)

- Examples: dc removal or notch

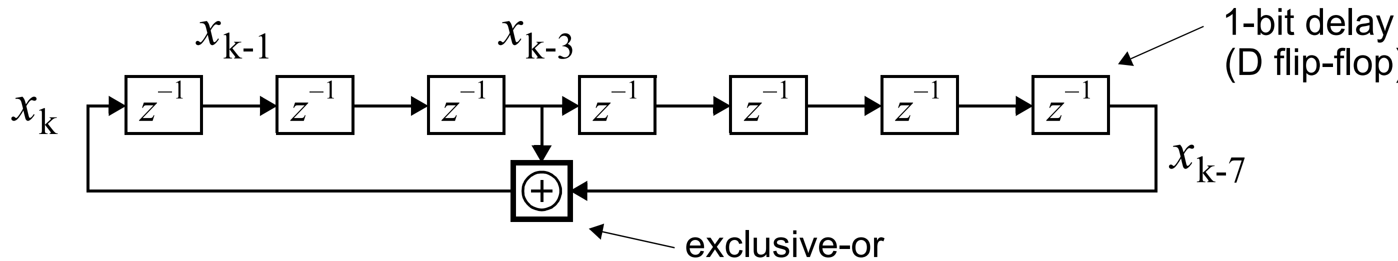
Hard-Decoding (Error Control)

- Error detection or correction — received bits used

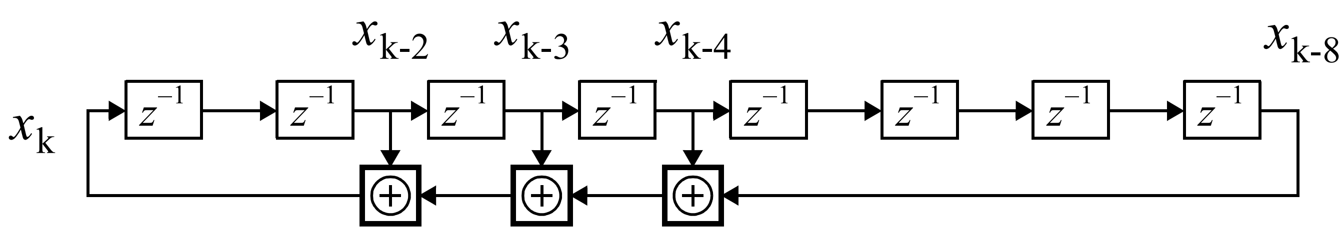
Soft-Decoding (Error Control)

- Error prevention
- Most likely sequence — received samples used

PN Sequence Generators



7-bit PN Sequence (sequence length = 127)



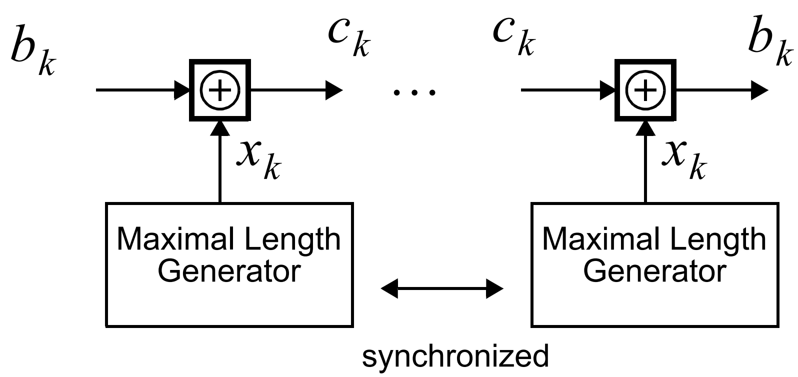
8-bit PN Sequence (sequence length = 255)

- Use n-bit shift register with feedback
- If all-zero state occurs, it remains in that state forever
- Maximal length if period is $2^n - 1$

Maximal-Length PN Sequences

Delay Length	Feedback Taps	Delay Length	Feedback Taps	Delay Length	Feedback Taps
2	1,2	13	1,3,4,13	24	1,2,7,24
3	1,3	14	1,6,10,14	25	3,25
4	1,4	15	1,15	26	1,2,6,26
5	2,5	16	1,3,12,16	27	1,2,5,27
6	1,6	17	3,17	28	3,28
7	3,7	18	8,18	29	2,29
8	2,3,4,8	19	1,2,5,19	30	1,2,23,30
9	4,9	20	3,20	31	3,31
10	3,10	21	2,21	32	1,2,22,32
11	2,11	22	1,22	33	13,33
12	1,4,6,12	23	5,23	34	1,2,27,34

Side-Stream Scrambler



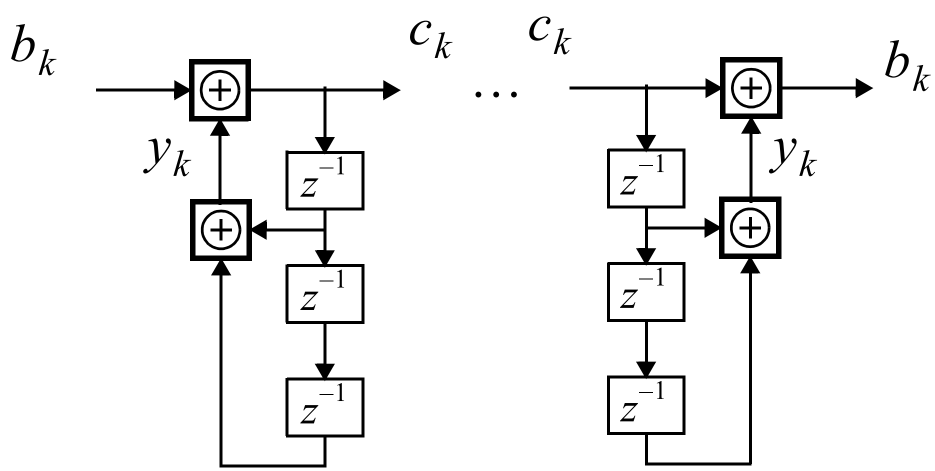
- Also called “frame-synchronized”

$$c_k = b_k \oplus x_k \tag{24}$$

$$c_k \oplus x_k = b_k \oplus x_k \oplus x_k = b_k \oplus 0 = b_k \tag{25}$$

- Advantage: no error propagation
- Disadvantage: need to synchronize scramblers
- Note that c_k would be all zeros if $b_k = x_k$ (unlikely)

Self-Synchronized Scrambler

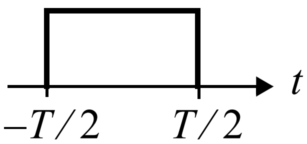


example 3-bit scrambler

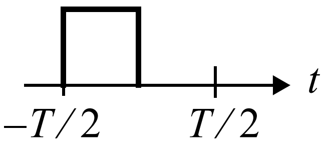
- Similar to side-stream, b_k recovered since $y_k \oplus y_k = 0$
- Advantage: no need for alignment of scramblers.
- Disadvantage: one error in received value of c_k results in three errors (one for each XOR summation)
- Can also have more problems with periodic inputs.

Line Coding

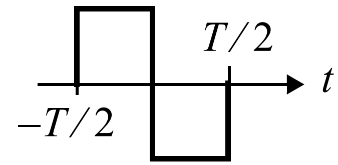
Change pulse shape



NRZ



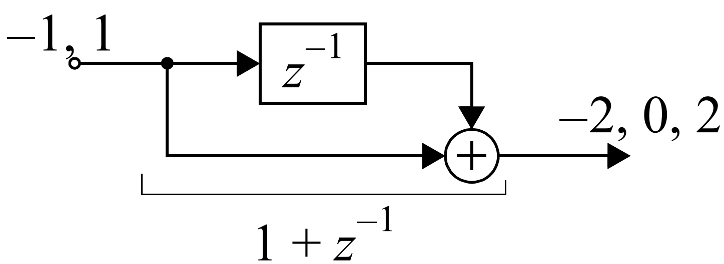
RTZ



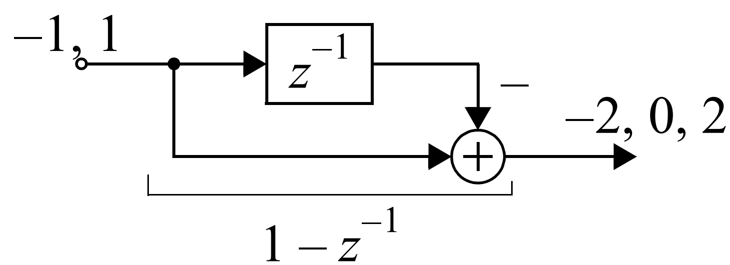
Biphase

- Remains a 2-level signal but more high-freq content

Filter data signal



zero at fs/2



Line Coding

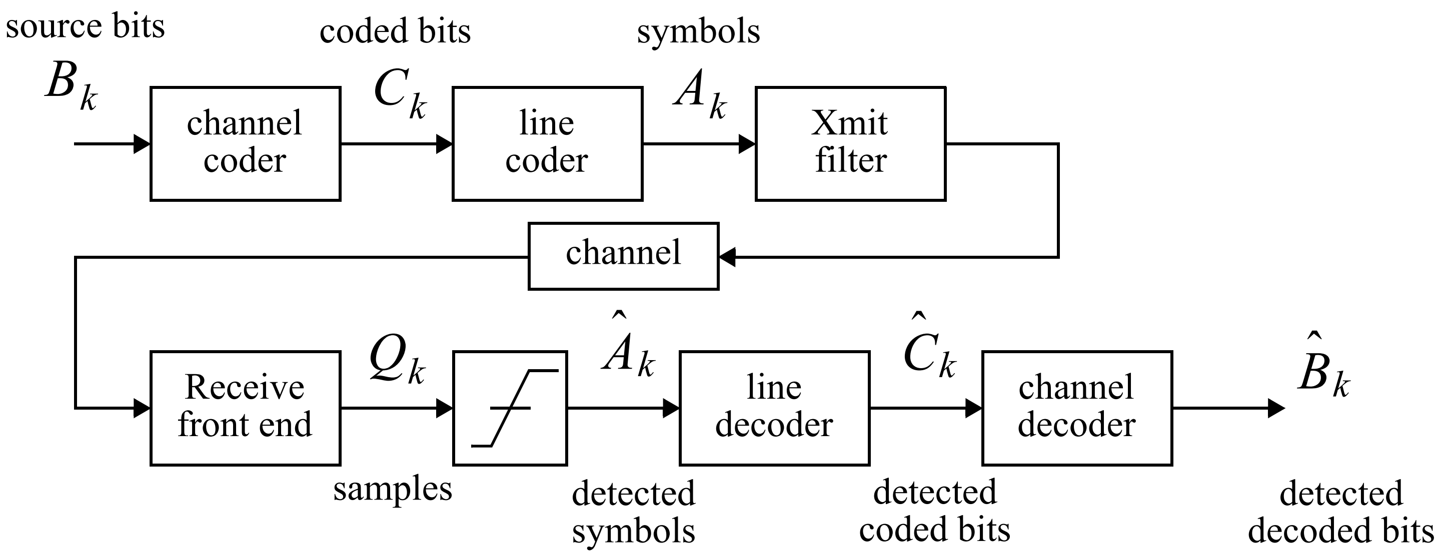
Filter data signal

- Results in more signal levels than needed for bit transmission — “correlated level coding”
- Loose 3dB in performance unless maximal likelihood detector used.

Block Line Codes

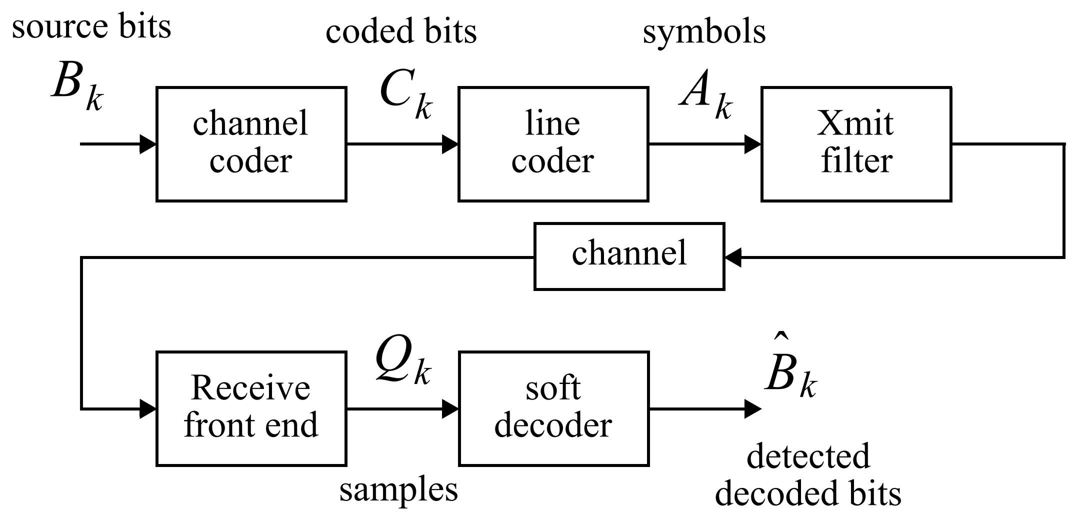
- Map block of k bits into n data symbols drawn from alphabet of size L.
- When $2^k < L^n$, redundancy occurs and can be used to shape spectrum.
- Example: blocks of 3 bits can be mapped to blocks of 2 3-level symbols.

Hard-Decoding



- Redundancy by adding extra bits
- Error detection and/or correction performed by looking ***after*** quantizer
- Examples: parity check, Reed-Solomon

Soft-Decoding



- Makes direct decisions on info bits without making intermediate decisions about transmitted symbols.
- Processes Q_k directly — combines slicing and removal of redundancy
- Can achieve better performance than hard decoding