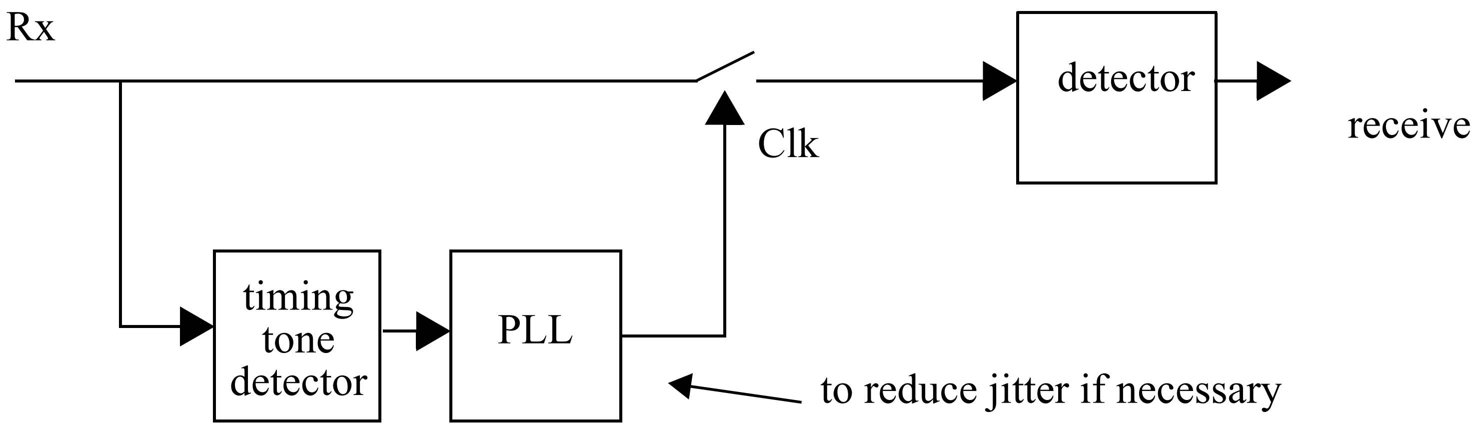
Timing Recovery

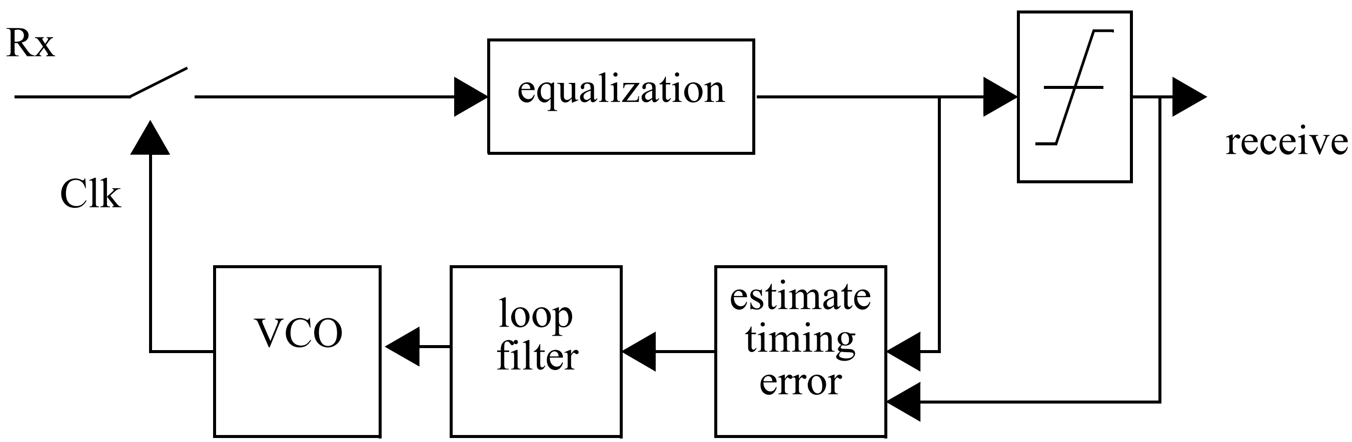
Prof. David Johns
University of Toronto

(johns@eecg.toronto.edu)
(www.eecg.toronto.edu/~johns)

Timing Recovery (two types)



deductive



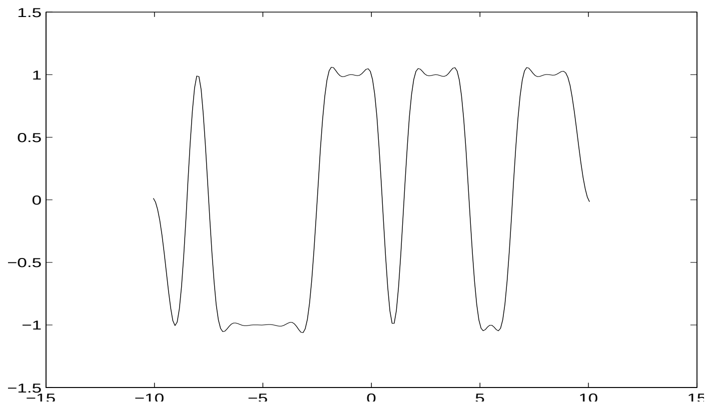
inductive

- Timing more difficult with less excess bandwidth.

Deductive Timing Recovery

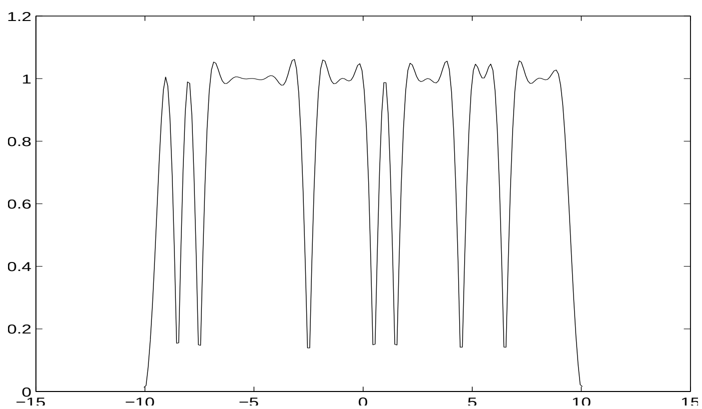
- Non-linear spectral line method most popular (linear spectral line method used if f_s tone present).
- Apply a non-linearity to receive signal and bandpass filter to recover f_s tone (usually with PLL).
- Works because receive signal is cyclostationary (i.e. its moments vary in time and are periodic).
- Common non-linearities used are squaring and absolute circuit (rectifier) (for low excess BW)
- ***Ensemble average*** of non-linear circuit output is periodic in T
- Thus, a f_s component exists (scrambled data)

Example (100% excess BW)

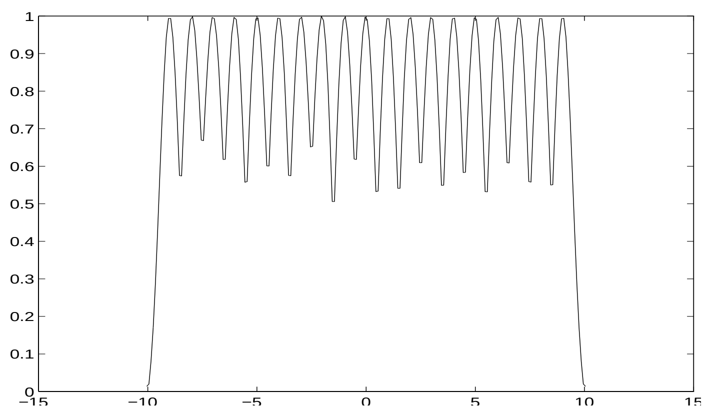


receive signal

average NOT in time but
over transmit sequences
(100 sequences in this case)

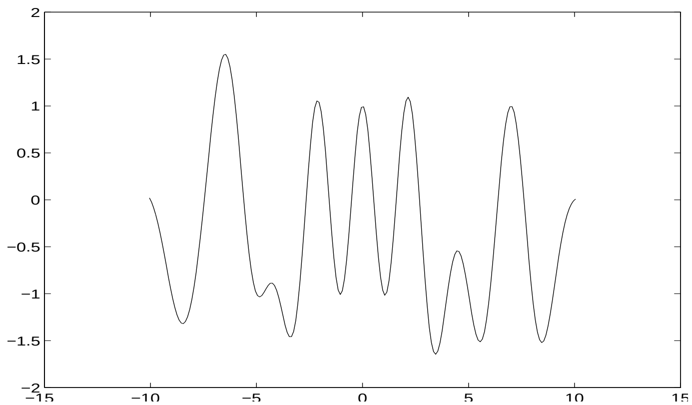


abs(receive signal)



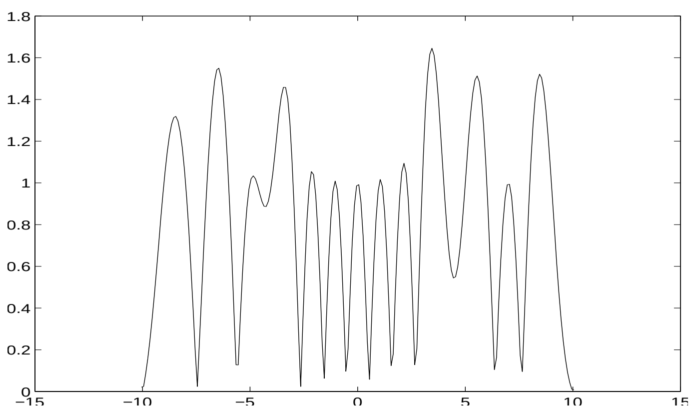
average(abs(receive signal))

Example (20% excess BW)

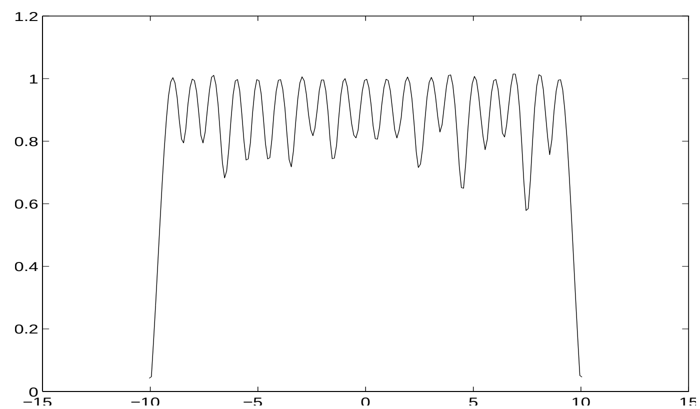


receive signal

average NOT in time but
over transmit sequences
(100 sequences in this case)



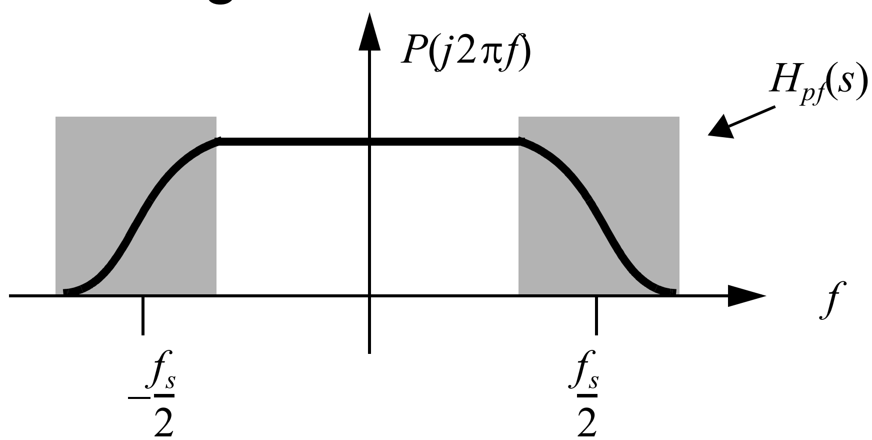
abs(receive signal)

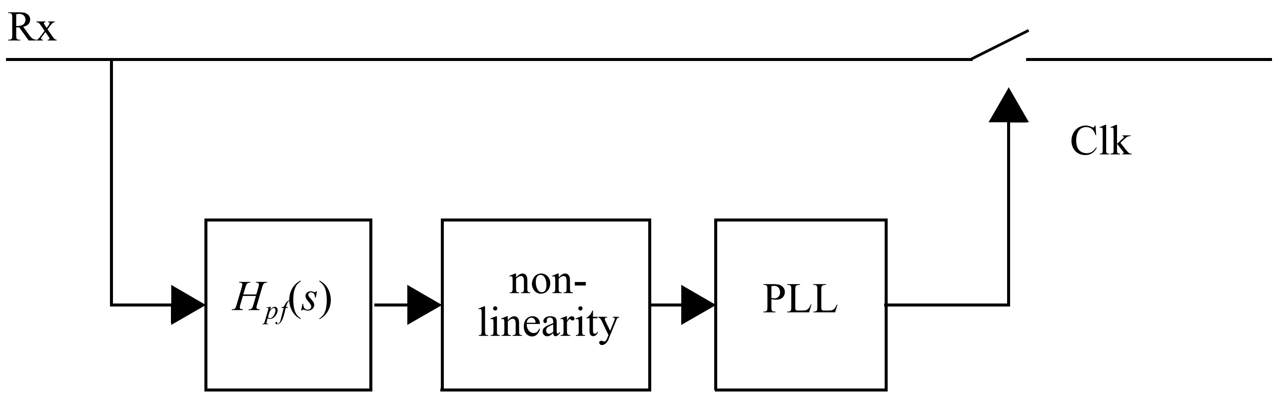


average(abs(receive signal))

Deductive Timing

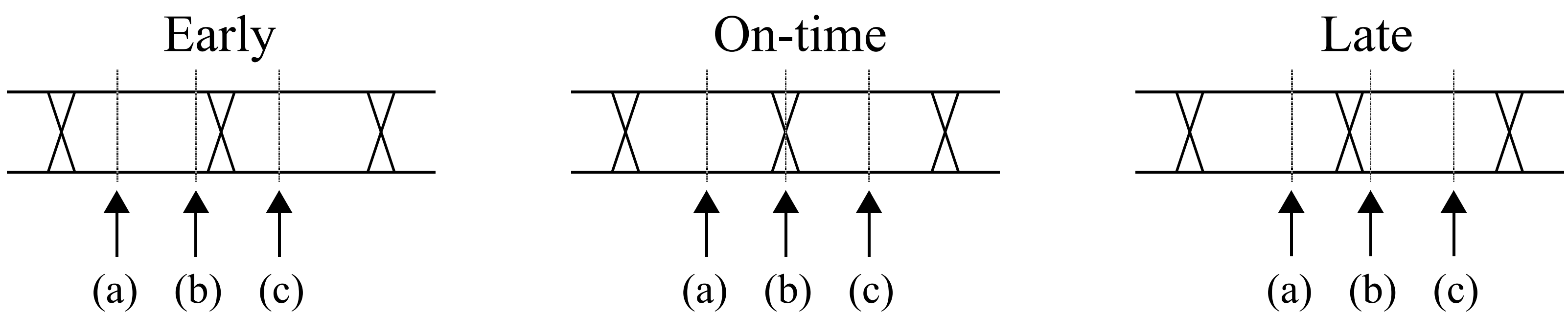
- Can pre-filter receive signal to only non-flat portion to reduce jitter — eliminate portion that does not contribute to timing tone.





Inductive Timing — Early Late

- Can sample at 2X and determine if clock is early or late when a transition occurs.



(a) = (b) ≠ (c)
Slow down clock

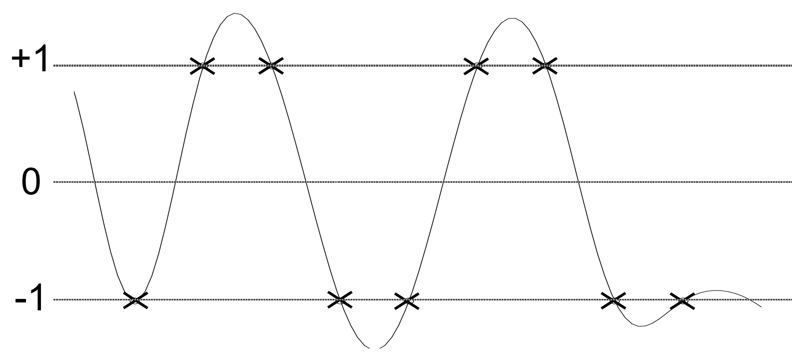
(a) ≠ (b) = (c)
Speed up clock

- If (a) = (b) = (c), do nothing
- However, (b) sample does not indicate how far away from zero crossing — can add dither to (b) to aid estimate.

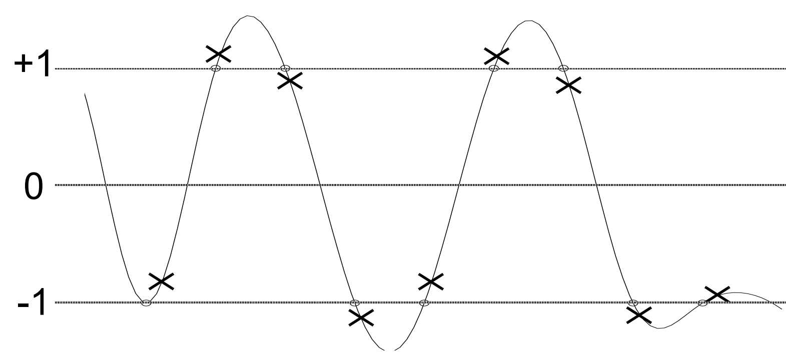
Inductive Timing (MMSE)

- Commonly realized as minimum mean-square error (i.e. MMSE timing)
- Also called LMS timing.
- Assume sample times are $kT + \tau_k$

$$A_k = \pm 1$$



Correct sampling phase



Late sampling phase $\tau_k > 0$

Inductive Timing (MMSE)

- MMSE adjusts τ_k to minimize

$$E[E_k^2(\tau_k)] = E[(Q_k(\tau_k) - A_k)^2] \quad (1)$$

where $E[\bullet]$ denotes expectation, $Q_k(\tau_k)$ is the sampled signal (it is a function of τ_k) and A_k is the ideal symbol.

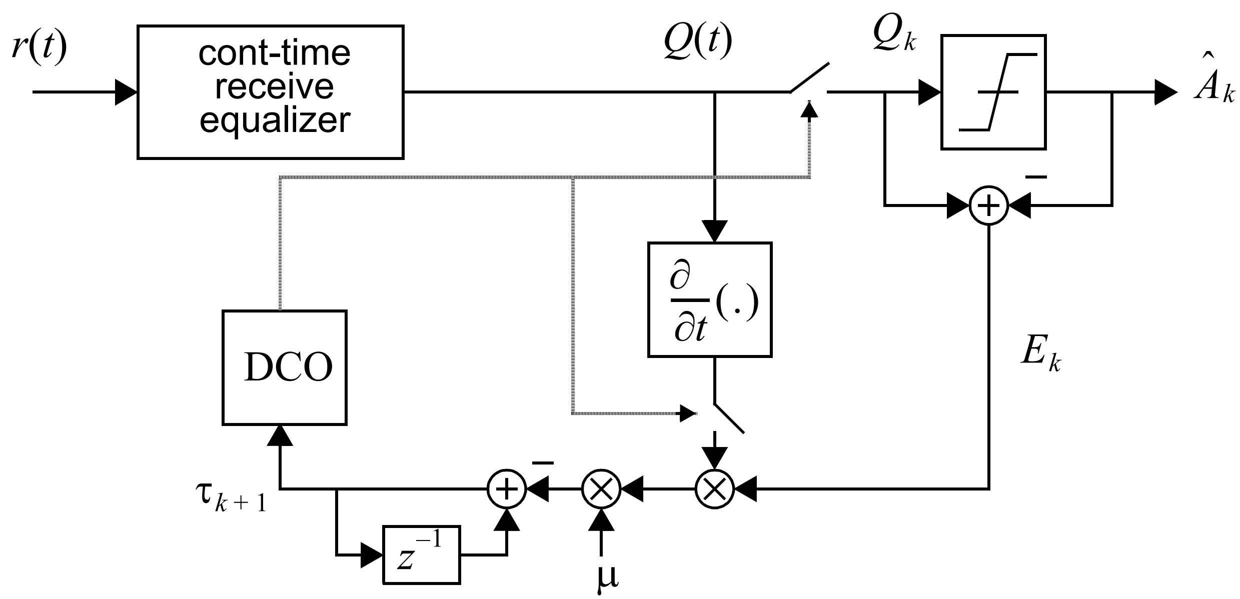
- Stochastic gradient (as in LMS algorithm) leads to

$$\tau_{k+1} = \tau_k - \mu\left(E_k(\tau_k) \times \frac{\partial Q_k(\tau_k)}{\partial \tau_k}\right) \quad (2)$$

Inductive Timing (MMSE)

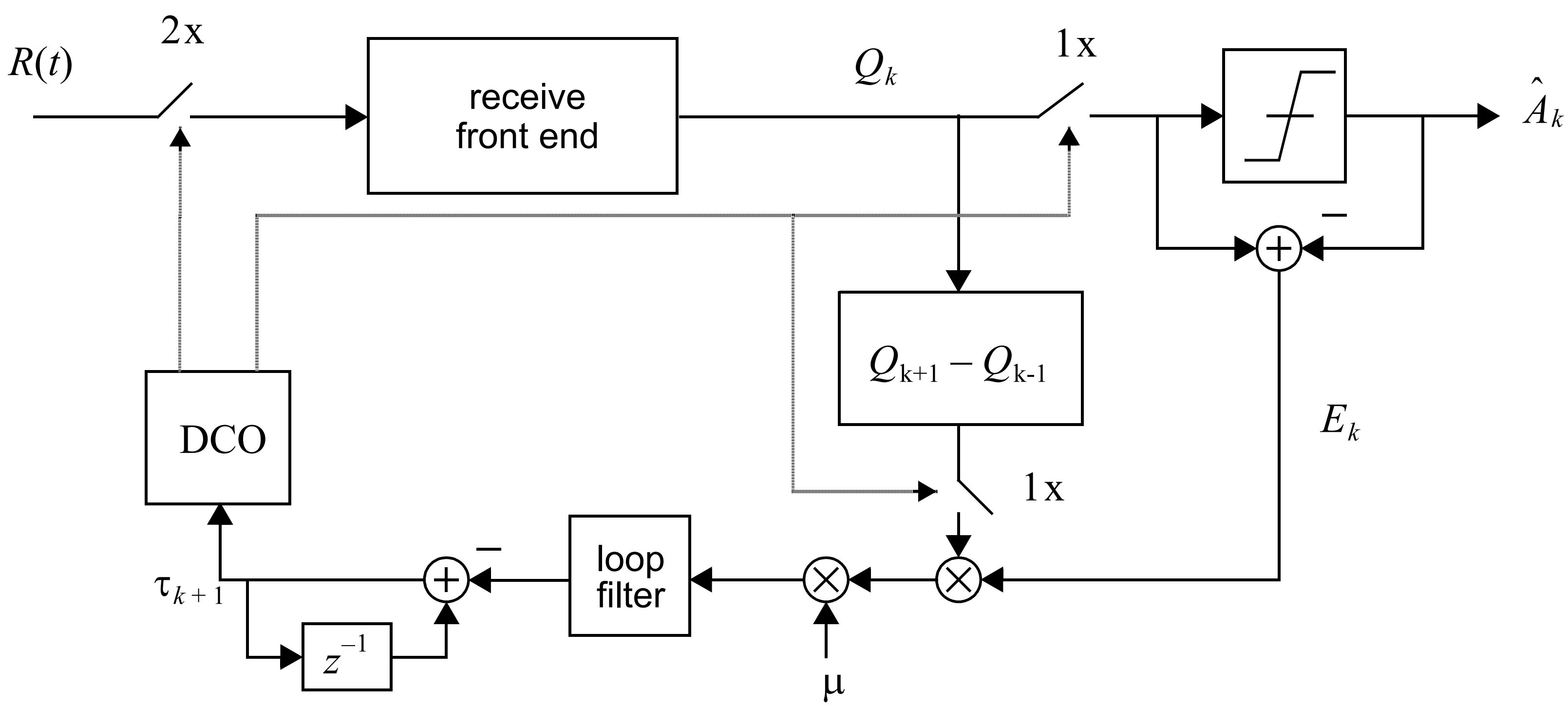
- Can replace derivative wrt τ_k by derivative wrt time since sampled at $t = kT + \tau_k$

$$\frac{\partial Q_k(\tau_k)}{\partial \tau_k} = \left.\frac{\partial Q(t)}{\partial t}\right|_{t = kT + \tau_k} \qquad (3)$$



Inductive Timing (MMSE)

- Can sample at 2X symbol-rate and perform derivative in discrete-time.

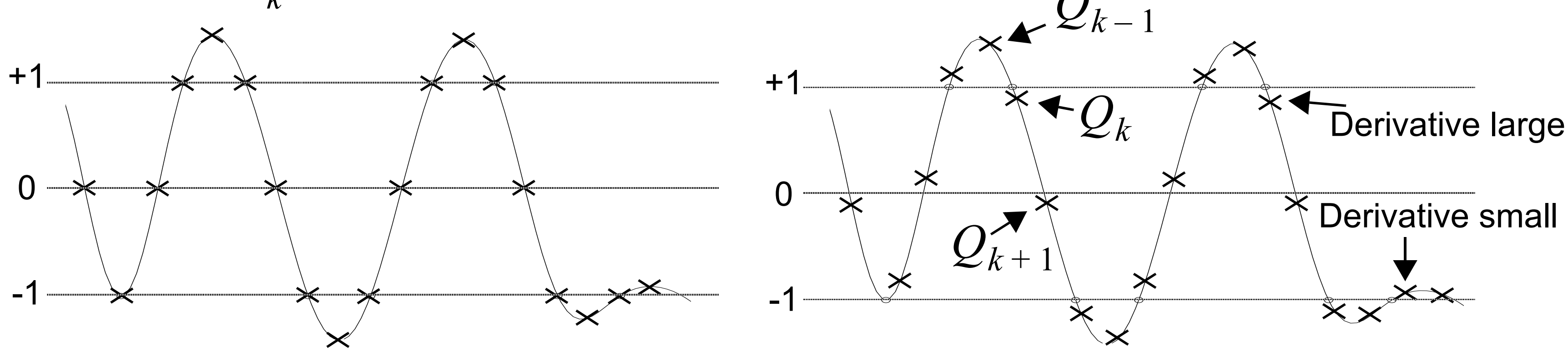


2X Timing Example

- Sample at twice symbol-rate

$$\tau_{k+1} = \tau_k - \mu(Q_k - \hat{A}_k) \times (Q_{k+1} - Q_{k-1}) \qquad (4)$$

$$A_k = \pm 1$$



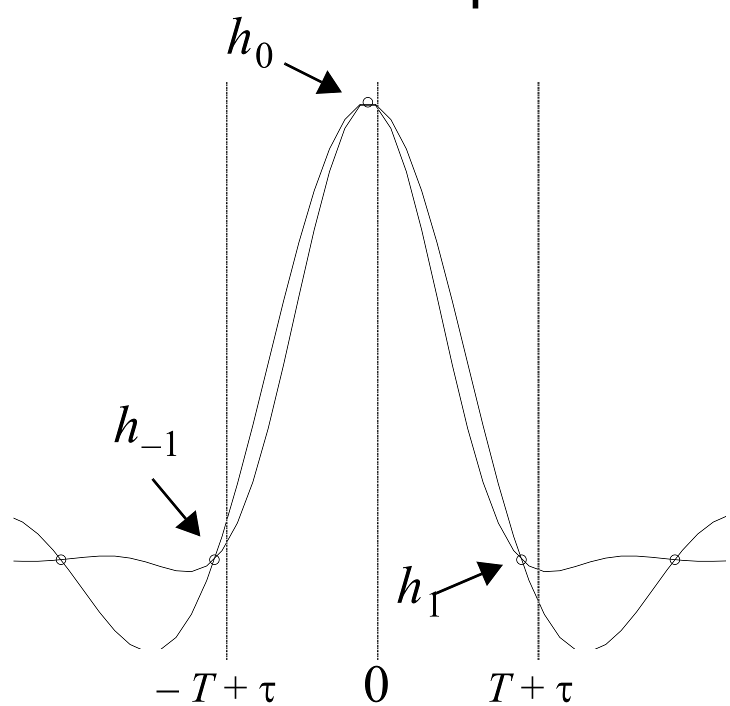
Correct sampling phase

Late sampling phase $\tau_k > 0$

- At Q_k, slope is neg, E_k is neg, so τ_k is decreased.
- Use absolute values then 50% duty cycle not needed

Inductive Timing — Baud-Rate

- If all sampling done at symbol-rate, MMSE timing can still be used — base it on impulse response.



- Early-late — adjust so $h_1 - h_{-1} = 0$
- Zero-crossing — adjust so $h_1 = 0$

Inductive Timing — Baud-Rate

- To obtain impulse response estimates, cross correlate received signals with received symbols.
- Recall

$$Q(t) = \sum_m A_m h(t - mT) + n(t) \tag{5}$$

- Sampled at time $kT + \tau$, we have

$$Q_k \equiv Q(kT + \tau)$$

$$= \ldots + A_{k-1} h(kT + \tau - (k-1)T) + A_k h(kT + \tau - kT) + \ldots$$

$$= \ldots + A_{\mathrm{k-1}} h_1(\tau) + A_k h_0(\tau) + A_{\mathrm{k+1}} h_{-1}(\tau) + \ldots \tag{6}$$

where $h_k(\tau) \equiv h(kT + \tau)$

Inductive Timing — Baud-Rate

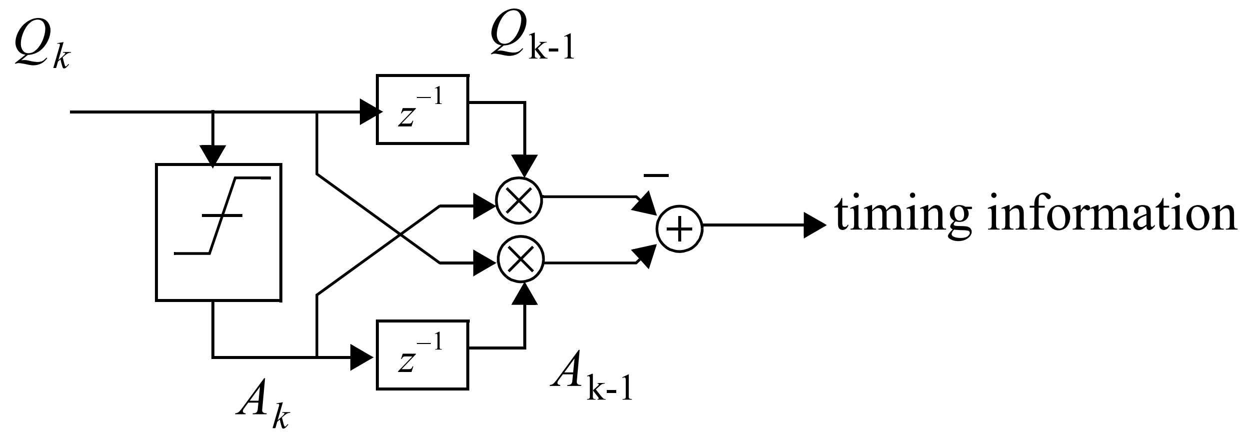
- To estimate $h_1(\tau)$, use $Q_k \times A_{\text{k-1}}$
- ***All other terms go to zero since*** A_{k-1} ***is uncorrelated with*** A_j ***when*** $k \neq j$
- To estimate $h_{-1}(\tau)$, we need to use a delayed version of Q_k

$$Q_{\text{k-1}} = \ldots + A_{\text{k-1}} h_0(\tau) + A_k h_{-1}(\tau) + A_{\text{k+1}} h_{-2}(\tau) + \ldots \qquad (7)$$

- To estimate $h_{-1}(\tau)$, use $Q_{\text{k-1}} \times A_k$

Inductive Timing — Baud-Rate

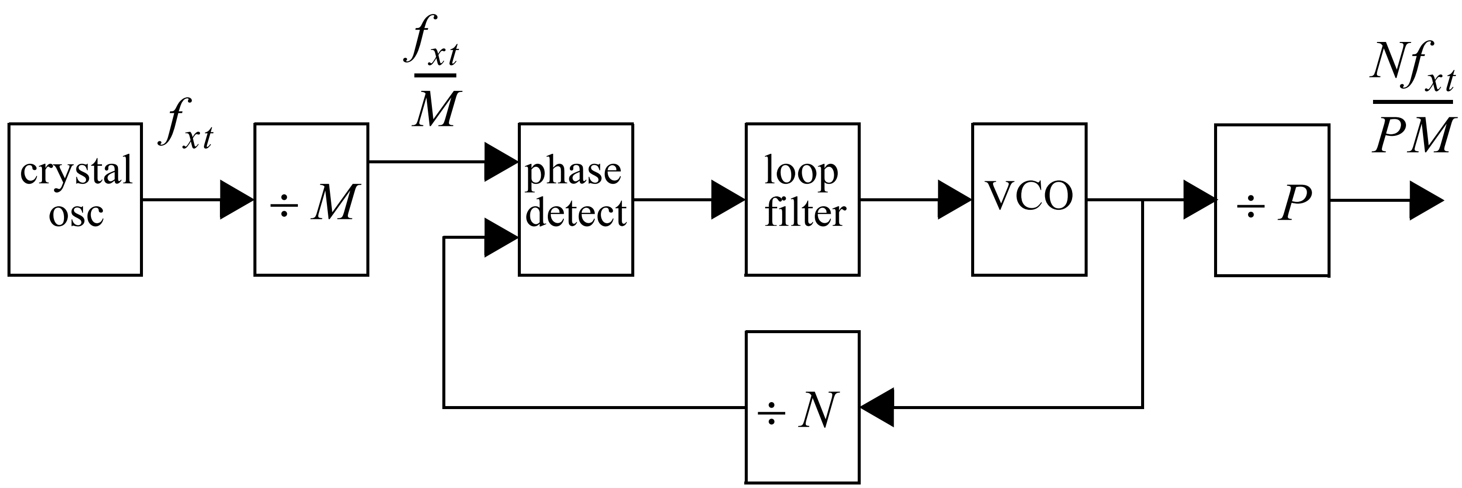
- To build early-late scheme,



- Early-late is insensitive to amplitude distortion.
- Zero-crossing is better where phase distortion dominates
- h_0 factor should be known otherwise adaptation gain will vary (can divide it out in algorithm).

A Fractional-N Frequency Synthesizer

- Often need a low jitter clock that can have arbitrary frequency.
- A voltage-controlled crystal oscillator is expensive.
- Use oversampling within a PLL



$N = \{k\text{-}1, k, k+1\}$

A digital controlled oscillator

Elastic Buffer

- Used to deal with low frequency input clock jitter
- Allows attenuation of clock jitter to next stage

Example

- Input clock rate — 1MHz but varies from 0.9MHz to 1.1MHz in sinusoidal fashion at 1kHz
- Output clock rate — fixed at 1MHz
- Input clock high — 16 extra bits stored in buffer
- Input clock low — 16 bits removed from buffer

- Keep elastic buffer half-full on-average through feedback