University of Toronto

Final Exam

Date — Dec 12, 2016

Duration — 2.5 hrs

ECE 331 — Analog Electronics

Lecturer — D. Johns

ANSWER QUESTIONS ON THESE SHEETS **USING BACKS IF NECESSARY**

- Equation sheet is on the last page of this test.
- Unless otherwise stated, use transistor parameters on equation sheet and assume $g_m r_o >> 1$
- Non-programmable calculator is allowed; No other aids are allowed
- Grading indicated by [ ]. Attempt all questions since a blank answer will certainly get 0.

Last Name: SOLUTIONS

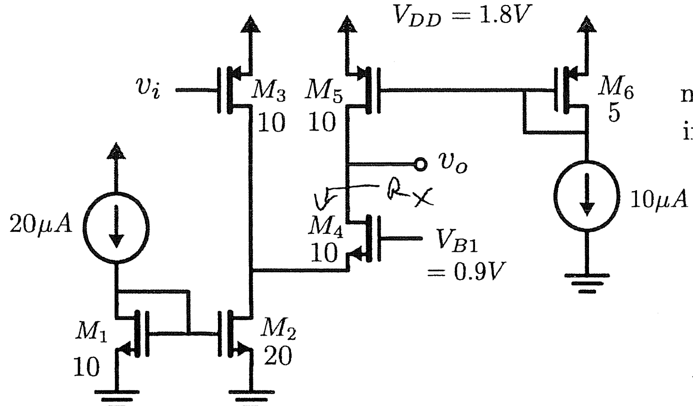
First Name: ____________________

Student #: ____________________

Question	1	2	3	4	5	6	Total
Points:	6	6	6	6	6	6	36
Score:							

Grading Table (do not write in above table)

Q1. Consider the amplifier shown below where all transistor lengths are 0.18 μm.



numbers under transitor name indicate W/L

n-CHANNEL

$$V_{OV} = \sqrt{\frac{2I_D}{\mu_n C_{ox}(W/L)}}$$

$$V_{OV} = 0.0913\sqrt{I_D}\,(W/L)^{-\frac{1}{2}}$$

[2] (a) Calculate the drain currents, overdrive voltages and r_o for all the transistors.

$I_{D1} = 20\mu A$ $I_{D2} = 40\mu A$ $I_{D6} = 10\mu A$

$I_{D5} = 20\mu A$ $I_{D4} = 20\mu A$ $I_{D3} = I_{D2} - I_{D4} = 20\mu A$

$V_{OV1} = V_{OV2} = 0.129\,V$

$V_{OV4} = 0.129\,V$

$V_{OV5} = V_{OV6} = 0.2582$

$V_{OV3} = 0.2582$

P-CHANNEL

$$V_{OV} = 0.1826\sqrt{I_D}\,(W/L)^{-\frac{1}{2}}$$

$$\lambda = \frac{\lambda'}{L} = \frac{0.05}{0.18} = 0.278$$

$$r_o = \frac{1}{\lambda I_D} = \frac{3.6}{I_D}$$

$r_{o1} = r_{o3} = r_{o4} = r_{o5} = 180k$

$r_{o2} = 90k$ $r_{o6} = 360k$

[2] (b) Find the maximum V_{omax} and minimum V_{omin} DC voltage at the output while keeping all transistors in the active region.

$$V_{OMAX} = V_{DD} - V_{OV5} = \underline{1.54\,V}$$

$$V_{OMIN} = V_{B1} - V_{tn} = 0.9 - 0.4 = \underline{0.5\,V}$$

[2] (c) Estimate the small-signal gain v_o/v_i. (CAN IGNORE)

$\underline{R_{OUT}}$ $R_{OUT} = R_x \,//\, r_{o5}$

$$R_x \cong (1 + g_{m4}(r_{o2}//r_{o3}))\,r_{o4} \approx g_{m4}(r_{o2}//r_{o3})\,r_{o4}$$

$$g_m = \frac{2I_D}{V_{OV}} \Rightarrow g_{m4} = 0.31\ mA/V$$

$$g_{m3} = 0.155\ mA/V$$

$$R_x \cong (0.31\,mA/V)(60k)(180k) \cong 3.35\,M\Omega$$

$$R_{OUT} = (3.35\,M) // 180k = 170k \approx \underline{180k}$$

$$I_{SC} = g_{m3} v_i = (0.155e\text{-}3)\,v_i$$

$$v_o = -I_{SC} R_{OUT} = (0.155e\text{-}3)(180k)\,v_i$$

$$\frac{v_o}{v_i} \cong 28\ V/V$$

Q2. Using the transistor parameters on the equation sheet, consider a transistor of size $W = 2\mu m$ and $L = 0.18\mu m$.

[2] (a) Assuming the transistor is biased in the active region and assuming $V_{sb} = 0$, find the values of C_{gs}, C_{gd}, and C_{db} for the transistor (all in fF).

$$C_{gs} = \left(\frac{2}{3}\right) W L C_{ox} + W L_{ov} C_{ox} = 2.04 + 0.68 = \underline{2.72 fF}$$

$$C_{gd} = W L_{ov} C_{ox} = \underline{0.68 fF}$$

$$C_{db} = \frac{C_{db0}}{W} \times W = 0.3 \times 2 = \underline{0.6 fF}$$

[2] (b) If $V_{ov} = 0.2V$, find the unity gain freq of the transistor in Hz (do not ignore C_{gd}).

$$g_m = \mu_N C_{ox} \left(\frac{W}{L}\right) V_{ov} = (240e\text{-}6)\left(\frac{2}{0.18}\right) 0.2 = 5.33e\text{-}4 \ A/V$$

$$f_t = \frac{g_m}{2\pi (C_{gs} + C_{gd})} = \underline{24.97 \ GHz}$$

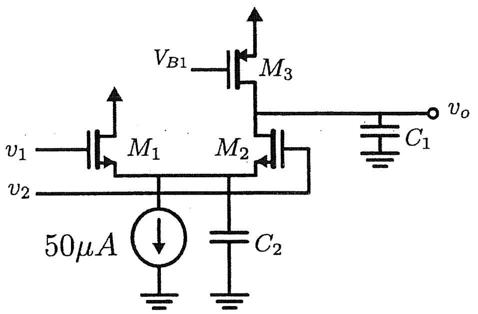
[2] (c) Besides noise, give 2 reasons why bias voltages for current mirrors are NOT sent a long distance in a microchip. (Explain clearly).

- CURRENT MISMATCH CAUSED BY EITHER

⇒ TRANSISTOR V_t MISMATCH DUE TO TRANSISTORS BEING FAR AWAY

⇒ SOURCE VOLTAGE OF TRANSISTORS BEING DIFFERENT DUE TO CURRENT IN GROUND LINES OR POWER SUPPLIES LINES

Q3. Consider the amplifier shown below.



$V_{ov1} = V_{ov2} = 0.1V$
$V_{ov3} = 0.4V$
All $r_o = 100k$
$C_1 = 1pF$
$C_2 = 2pF$

$$g_{m1} = g_{m2} = \frac{2I_D}{V_{ov}} = 0.5 mA/V = \frac{1}{2K}$$

[2] (a) Defining $v_i = v_1 - v_2$, find the dc gain v_o/v_i.

$$R_{OUT} = r_{o3} \| \left[\left(1 + g_{m2}\left(\frac{1}{g_{m1}}\right)\right) r_{o2} \right] = r_{o3} \| 2r_{o2} = 67 k\Omega$$

$$I_{SC} = \frac{v_i}{r_{s1} + r_{s2}} = \frac{v_i}{4k}$$

$$v_o = I_{SC} R_{OUT} = \frac{67k}{4k} v_i = 16.7 v_i$$

$$\frac{v_o}{v_i} = 16.7 \, V/V$$

[2] (b) Find the poles due to C_1 and C_2 in Hz.

$$C_1 \Rightarrow \omega_{P1} = \frac{1}{2\pi(R_{OUT} C_1)} = \frac{1}{2\pi(67k)(1pF)} = 2.37 MHz$$

$$C_2 \Rightarrow \omega_{P2} = \frac{1}{2\pi C_2 \left(r_{s1} \| \left[\frac{1}{g_{m2}} + \frac{r_{o3}}{g_{m2} r_{o2}}\right]\right)} = \frac{1}{2\pi C_2 (r_{s1} \| 2r_{s2})}$$

$$\omega_{P2} = 60 MHz$$

ESTIMATE

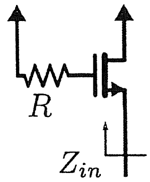
[2] (c) For the case where v_o is connected to v_2, find the dc gain and 3dB freq for v_o/v_1.

$$\beta = 1 \Rightarrow A_{CL} = \frac{A_0}{1 + A_0\beta} = \frac{16.7}{1 + 16.7} = 0.94 \, V/V$$

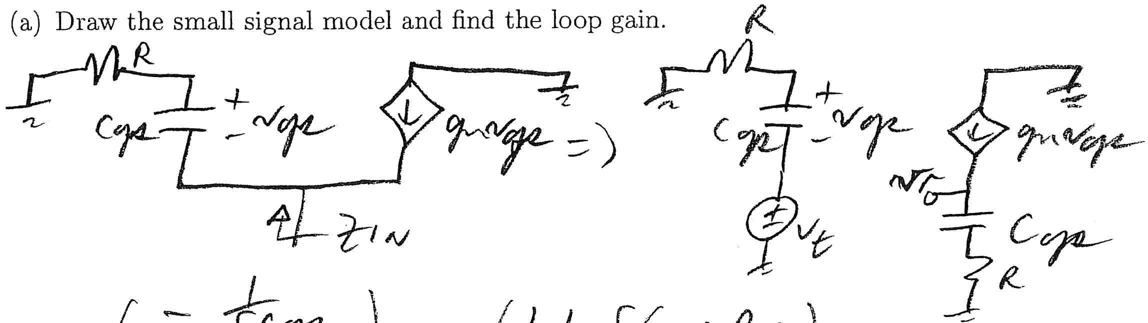
$$\text{3dB FREQ} \Rightarrow f'_{3dB} \simeq f_{3dB} \times A_0 = (2.37 MHz)(17)$$

$$\simeq 40 MHz$$

Q4. Consider the input impedance looking into the source of the circuit below. Let $r_o \to \infty$ and only consider capacitance C_{gs}.



[3] (a) Draw the small signal model and find the loop gain.



$$L_0 = -\left(\frac{-\frac{1}{sC_{gs}}}{\frac{1}{sC_{gs}}+R}\right) g_m \left(\frac{1+sC_{gs}R_6}{sC_{gs}}\right)$$

$$L_0 = \frac{g_m}{sC_{gs}} \qquad L_s = 0$$

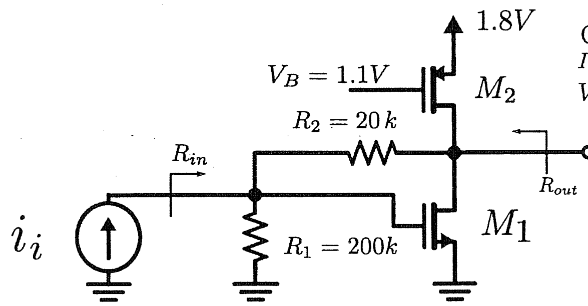
[3] (b) Using the loop gain found above, find Z_{in}.

$$Z_{P0} = R + \frac{1}{sC_{gs}} = \frac{1+sC_{gs}R}{sC_{gs}}$$

$$Z_{IN} = Z_{P0}\left[\frac{1+L_s}{1+L_0}\right] = \left(\frac{1+sC_{gs}R}{sC_{gs}}\right)\left(\frac{1}{1+\frac{g_m}{sC_{gs}}}\right)$$

$$Z_{IN} = \left(\frac{1+sC_{gs}R}{g_m+sC_{gs}}\right) = \left(\frac{1}{g_m}\right)\left(\frac{1+sC_{gs}R}{1+\frac{sC_{gs}}{g_m}}\right)$$

Q5. Consider the current to voltage amplifier shown below where i_i is nominally zero.



Given
$I_{D2} = 20\mu A$
$V_{G1} = 0.7V$

$L = 0.18\,\mu m$

[2] (a) Find the dc operating points V_{D1}, g_{m1}, g_{m2}, r_{o1}, r_{o2}. (note I_{D2} and V_{G1} given)

$I_{R_1} = \frac{V_{G1}}{R_1} = 3.5\mu A \Rightarrow V_{O1} = V_{G1} + I_{R_1}R_1 = \underline{0.77V}$

$I_{D1} = I_{D2} - I_{R_1} = 16.5\mu A$

$V_{OV1} = 0.7 - V_{tn} = 0.3V$ $\quad V_{OV2} = (V_{DD} - V_B) - |V_{tp}| = 0.3V$

$g_{m1} = \frac{2I_{D1}}{V_{OV1}} = \underline{0.11\,mA/V}$ $\quad g_{m2} = \frac{2I_{D2}}{V_{OV2}} = \underline{0.133\,mA/V}$

$r_{o1} = \frac{3.6}{I_{D1}} = \underline{218k\Omega}$ $\quad r_{o2} = \frac{3.6}{I_{D2}} = \underline{180k\Omega}$

[4] (b) Find A_∞, L, ~~d~~, A_{CL}, R_{in}, and R_{out}.

$A_\infty \equiv \frac{v_o}{i_i}\Big|_{L\to\infty} = \underline{20k}$

$L = \left(g_{m1}\left[\overbrace{((R_1+R_2)\|r_{o2})}^{99k}\|r_{o1}\right]\right)\left(\frac{R_1}{R_1+R_2}\right)$

$L = \underline{6.8}$

$A_{CL} = A_\infty\left(\frac{L}{1+L}\right) = \underline{17.4k}$

$R_{OUT} = \left[r_{o2}\|r_{o1}\|(R_1+R_2)\right]\left(\frac{1}{1+L_0}\right) = (68k)\left(\frac{1}{1+L}\right) = \underline{8.7\,k\Omega}$

$R_{IN} = \left[R_1\|\left(R_2 + (r_{o2}\|r_{o1})\right)\right]\left(\frac{1}{1+L}\right) = 74.4\left(\frac{1}{1+L}\right) = \underline{9.5\,k\Omega}$

Q6. Consider a feedback amplifier that has a low frequency gain of 10^5, a dominant pole at 10 rad/s and a **pair** of non-dominant poles at 10^5 rad/s.

[4] (a) Assuming the feedback factor, β, is independent of frequency, find the value of β that will result in a 60° phase margin.

$$A(s) = \frac{A_0}{\left(1+\frac{s}{\omega_{P1}}\right)\left(1+\frac{s}{\omega_{P2}}\right)^2} \qquad A_0 = 10^5 \quad \omega_{P1} = 10 \quad \omega_{P2} = 10^5$$

WANT TO FIND ω_{60} WHERE $\angle A(j\omega_{60}) = -120^0$ FOR PM $= 60^0$

ASSUME $\omega_{60} \gg \omega_{P1}$ SINCE $A_0 \gg 1$

$$\angle A(j\omega_{60}) = -90^0 - 2\tan^{-1}\left(\frac{\omega_{60}}{\omega_{P2}}\right) = -120^0$$

$$\Rightarrow \tan^{-1}\left(\frac{\omega_{60}}{\omega_{P2}}\right) = +15^0 \Rightarrow \omega_{60} = 0.268\,\omega_{P2} = 26.8 \text{ KRAD/S}$$

WANT $|A\beta| = 1$ AT $\omega = \omega_{60} = 26.8$ KRAD/S

$$\frac{A_0\beta}{\left(\frac{\omega_{60}}{\omega_{P1}}\right)\left(1+\left(\frac{\omega_{60}}{\omega_{P2}}\right)^2\right)} = 1 \Rightarrow A_0\beta = 2872$$

$$\underline{\beta = 0.0287}$$

[2] (b) For the β found above and a step voltage applied to the input of the closed-loop amplifier, estimate the time it takes to settle to 90% of the final value.

$$\omega'_{3dB} \cong \omega_{3dB} L_0 = \omega_{3dB} A_0 \beta = (10)(10^5)(0.0287)$$

$$\omega'_{3dB} \cong 28700 \text{ RAD/S} \Rightarrow \tau' = \frac{1}{\omega'_{3dB}} = 34.8\,\mu s$$

TO SETTLE TO 90% OF FINAL VALUE

$$\left(1 - e^{-t/\tau'}\right) = 0.9 \Rightarrow e^{-t/\tau'} = 0.1 \Rightarrow \frac{t}{\tau'} = 2.3$$

$$t = 2.3 \times \tau' = \underline{80.1\,\mu s}$$

Equation Sheet

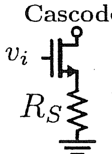
Constants: $k = 1.38 \times 10^{-23}\,\mathrm{J\,K^{-1}}$; $q = 1.602 \times 10^{-19}\,\mathrm{C}$; $V_T = kT/q \simeq 26\mathrm{mV}$ at $300\,\mathrm{K}$; $\epsilon_0 = 8.85 \times 10^{-12}\,\mathrm{F\,m^{-1}}$; $k_{ox} = 3.9$; $C_{ox} = (k_{ox}\epsilon_0)/t_{ox}$; $\omega = 2\pi f$

NMOS: $k_n = \mu_n C_{ox}(W/L)$; $V_{tn} > 0$; $v_{DS} \geq 0$; $V_{ov} = v_{GS} - V_{tn}$
(triode) $v_{DS} \leq V_{ov}$; $v_D < v_G - V_{tn}$; $i_D = k_n(V_{ov}v_{DS} - (v_{DS}^2/2))$
(active) $v_{DS} \geq V_{ov}$; $i_D = 0.5k_nV_{ov}^2(1 + \lambda v_{DS})$; $g_m = k_nV_{ov} = 2I_D/V_{ov} = \sqrt{2k_nI_D}$; $r_s = 1/g_m$; $r_o = L/(|\lambda'|I_D)$

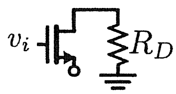
PMOS: $k_p = \mu_p C_{ox}(W/L)$; $V_{tp} < 0$; $v_{SD} \geq 0$; $V_{ov} = v_{SG} - |V_{tp}|$
(triode) $v_{SD} \leq V_{ov}$; $v_D > v_G + |V_{tp}|$; $i_D = k_p(V_{ov}v_{SD} - (v_{SD}^2/2))$
(active) $v_{SD} \geq V_{ov}$; $i_D = 0.5k_pV_{ov}^2(1 + |\lambda|v_{SD})$; $g_m = k_pV_{ov} = 2I_D/V_{ov} = \sqrt{2k_pI_D}$; $r_s = 1/g_m$; $r_o = L/(|\lambda'|I_D)$

BJT: (active) $i_C = I_Se^{(v_{BE}/V_T)}(1 + (v_{CE}/V_A))$; $g_m = \alpha/r_e = I_C/V_T$; $r_e = V_T/I_E$; $r_\pi = \beta/g_m$; $r_o = |V_A|/I_C$; $i_C = \beta i_B$; $i_E = (\beta+1)i_B$; $\alpha = \beta/(\beta+1)$; $i_C = \alpha i_E$; $R_b = (\beta+1)(r_e + R_E)$; $R_e = (R_B + r_\pi)/(\beta+1)$

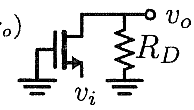
Cascode:



$R_x \simeq (1 + g_mR_S)r_o$
$i_{sc} \simeq -(1/g_m + R_S)^{-1}v_i$



$R_x \simeq 1/g_m + R_D/(g_mr_o)$
$v_{oc} \simeq v_i$



$v_o/v_i \simeq g_m(r_o \parallel R_D)$
(Approx due to $g_mr_o \gg 1$)

Diff Pair: $A_d = g_mR_D$; $A_{CM} = -(R_D/(2R_{SS}))(\Delta R_D/R_D)$; $A_{CM} = -(R_D/(2R_{SS}))(\Delta g_m/g_m)$; $V_{OS} = \Delta V_t$; $V_{OS} = (V_{OV}/2)(\Delta R_D/R_D)$; $V_{OS} = (V_{OV}/2)(\Delta(W/L)/(W/L))$

1st order: step response $y(t) = Y_\infty - (Y_\infty - Y_{0+})e^{-t/\tau}$; unity gain freq for $T(s) = \dfrac{A_M}{1 + (s/\omega_{3dB})}$ for $A_M \gg 1 \Rightarrow \omega_t \simeq |A_M|\omega_{3dB}$

Freq: for real axis poles/zeros $T(s) = k_{dc}\dfrac{(1+s/z_1)(1+s/z_2)\ldots(1+s/z_m)}{(1+s/\omega_1)(1+s/\omega_2)\ldots(1+s/\omega_n)}$
OTC estimate $\omega_H \simeq 1/(\sum \tau_i)$; dominant pole estimate $\omega_H \simeq 1/(\tau_{max})$

Miller: $Z_1 = Z/(1-K)$; $Z_2 = Z/(1-1/K)$

Mos caps: $C_{gs} = (2/3)WLC_{ox} + WL_{ov}C_{ox}$; $C_{gd} = WL_{ov}C_{ox}$; $C_{db} = C_{db0}/\sqrt{1 + V_{db}/V_0}$;
$\omega_t = g_m/(C_{gs} + C_{gd})$; for $C_{gs} \gg C_{gd} \Rightarrow f_t \simeq (3\mu V_{ov})/(4\pi L^2)$

Feedback: $A_f = A/(1 + A\beta)$; $x_i = (1/(1 + A\beta))x_s$; $dA_f/A_f = (1/(1 + A\beta))dA/A$; $\omega_{Hf} = \omega_H(1 + A\beta)$; $\omega_{Lf} = \omega_L/(1 + A\beta)$;
Loop Gain $L \equiv -s_r/s_t$; $A_f = A_\infty(L/(1+L)) + d/(1+L)$; $Z_{port} = Z_{p^o}((1 + L_S)/(1 + L_O))$: $PM = \angle L(j\omega_t) + 180$; $GM = -|L(j\omega_{180})|_{db}$;
Pole splitting $\omega'_{p1} \simeq 1/(g_mR_2C_fR_1)$; $\omega'_{p2} \simeq (g_mC_f)/(C_1C_2 + C_f(C_1 + C_2))$

Pole Pair: $s^2 + (\omega_o/Q)s + \omega_o^2$; $Q \leq 0.5 \Rightarrow$ real poles; $Q > 1/\sqrt{2} \Rightarrow$ freq resp peaking

Power Amps: Class A : $\eta = (1/4)(\hat{V}_O/IR_L)(\hat{V}_O/V_{CC})$; Class B : $\eta = (\pi/4)(\hat{V}_O/V_{CC})$; $P_{DN\_max} = V_{CC}^2/(\pi^2R_L)$;
Class AB : $i_ni_p = I_Q^2$

2-stage opamp: $\omega_{p1} \simeq (R_1G_{m2}R_2C_c)^{-1}$; $\omega_{p2} = G_{m2}/C_2$; $\omega_z = (C_c(1/G_{m2} - R))^{-1}$;
$SR = I/C_c = \omega_tV_{ov1}$; will not SR limit if $\omega_t\hat{V}_O < SR$

MOS TRANSISTOR: CMOS basic parameters. Channel length $= 0.18\mu$m

	V_t [V]	μC_{ox} [μA/V²]	λ' [μm V⁻¹]	C_{ox} [fF/μm²]	t_{ox} [nm]	L_{ov} [μm]	C_{db0}/W [fF μm⁻¹]
NMOS	0.4	240	0.05	8.5	4	0.04	0.3
PMOS	-0.4	60	-0.05	8.5	4	0.04	0.3