University of Toronto

Final Exam

Date — Dec 20, 2017

Duration — 2.5 hrs

ECE 331 — Analog Electronics

Lecturer — D. Johns

ANSWER QUESTIONS ON THESE SHEETS **USING BACKS IF NECESSARY**

- Equation sheet is on the last page of this test.
- Unless otherwise stated, use transistor parameters on equation sheet and assume $g_m r_o >> 1$
- Non-programmable calculator is allowed; No other aids are allowed
- Grading indicated by [ ]. Attempt all questions since a blank answer will certainly get 0.

Last Name: SOLUTIONS

First Name: ______

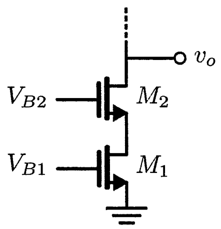
Student #: ______

Question	Points	Score
1	6	
2	6	
3	6	
4	6	
5	6	
6	6	
Total:	36	

Grading Table
(do not write in above table)

Q1.

[3] (a) Consider the wide swing current mirror below where the desired output current is $200uA$. It is desired that the minimum output voltage, v_o, be $0.5V$ while keeping M_1/M_2 active and that $V_{ov1} = 1.5V_{ov2}$.
Find V_{B1} and V_{B2}.



All $L = 0$
$I_D = 200uA$
$\lambda = 0$

$V_{ov1} + V_{ov2} = 0.5V$

$1.5V_{ov2} + V_{ov2} = 0.5V$

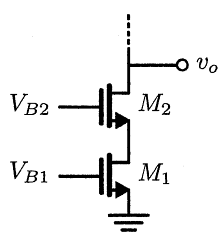
$V_{ov2} = 0.2V$ $\quad V_{ov1} = 0.3V$

$V_t = 0.4V \Rightarrow V_{B1} = V_{GS1} = V_t + V_{ov1} = \underline{\underline{0.7V}}$

$V_{GS2} = V_t + V_{ov2} = 0.6V$ FOR WIDE SWING $V_{D1} = V_{ov1} = 0.3V$

$V_{B2} = V_{ov1} + V_{GS2} = \underline{\underline{0.9V}}$

[3] (b) Consider the wide swing current mirror below where the desired output current is $20uA$, M_1 and M_2 are identical in size and the minimum output voltage is 0.4V.
Find the length of the transistors, L, such that the current mirror output resistance is $72M\Omega$.



$I_D = 20uA$

$R_o \cong (1 + g_{m2} r_{o1}) r_{o2} \approx g_{m2} r_o^2$ (1)

$V_{ov1} + V_{ov2} = 0.4V \Rightarrow V_{ov1} = V_{ov2} = 0.2V$

$g_m = \frac{2I_D}{V_{ov}} = \frac{2(20e-6)}{0.2} = 0.2\, mA/V$

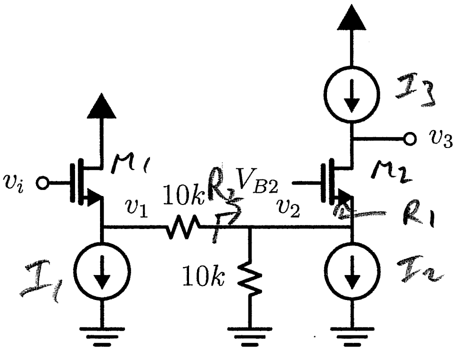
$r_o = r_{o1} = r_{o2} = \frac{L}{\lambda' I_D}$ $\quad \lambda' = 0.05\ \mu m/V$

$I_D = 20e-6\ A$

FROM (1) $\quad 72e6 = (0.2e-3) \frac{L^2}{(\lambda')^2 I_D^2} \Rightarrow L^2 = 0.36$

$L = \underline{\underline{0.6\ \mu m}}$

Q2. Consider the multistage amplifier shown below. **All** current sources and transistors have the same output impedance of $50k\Omega$. Also, all transistors have $g_m = 1mA/V$.



$$r_{S1} = r_{S2} = \frac{1}{g_m} = 1k$$

$$R_1 = \frac{1}{g_{m2}} + \frac{r_{oI3}}{g_m r_{o2}} = \frac{2}{g_m} = 2k$$

$$R_2 = 10k \,||\, r_{oI_2} \,||\, R_1 \approx 10k||2k = 1.67k$$

[2] (a) Find small-signal gain v_1/v_i.

$$R_x \equiv r_{oI_1} \,||\, r_{o1} \,||\, (10k + R_2) = 50k \,||\, 50k \,||\, 11.67k \cong 8k$$

$$\frac{v_1}{v_i} = \frac{R_x}{R_x + r_{S1}} = \frac{8}{8+1} = \underline{\underline{0.889}}\ V/V$$

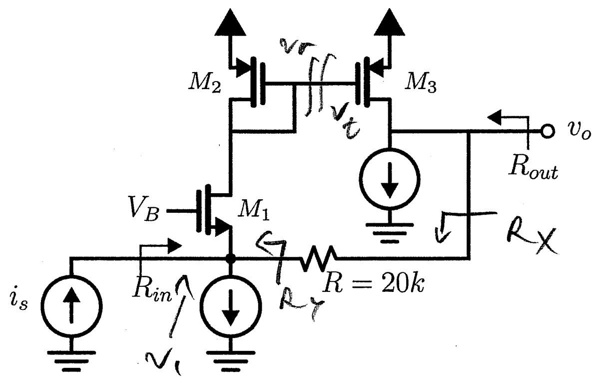
[2] (b) Find small-signal gain v_2/v_1.

$$\frac{v_2}{v_1} = \frac{R_2}{R_2 + 10k} = \frac{1.67}{1.67 + 10} = \underline{\underline{0.143}}\ V/V$$

[2] (c) Find small-signal gain v_3/v_2.

$$\frac{v_3}{v_2} = g_{m2}(r_{o2} \,||\, r_{oI_3}) = \frac{25k}{1k} = \underline{\underline{25}}\ V/V$$

Q3. Consider the feedback amp shown below where the input signal, i_s is a current.



All current sources ideal
$g_{m1} = g_{m2} = 1mA/V$
$g_{m3} = 10mA/V$
$r_{o1} = r_{o2} = r_{o3} = 40k$

$R_Y = \frac{1}{g_{m1}} + \frac{\frac{1}{g_{m2}}}{g_{m1} r_{o1}} \simeq \frac{1}{g_{m1}} = 1k$

$R_X = R_Y + R = 21K$

[3] (a) Using loop-gain analysis, find L, A_∞, d and v_o/i_s.

$\frac{v_o}{v_t} = -g_{m3}(r_{o3} \| R_X) = (10e-3)(40k \| 21k) = -138\ V/V$

$\frac{v_1}{v_o} = \frac{R_Y}{R_Y + R} = \frac{1}{1+20} = 0.0476$

$\frac{v_r}{v_1} = g_{m1}\left(\frac{1}{g_{m2}} \| r_{o1}\right) \simeq 1 \Rightarrow L = -\frac{v_r}{v_t} = \underline{6.57}\ V/V$

$A_\infty = \frac{v_o}{i_s}\Big|_{L\to\infty} = \underline{-20k}$

$d \equiv \frac{v_o}{i_s}\Big|_{L=0} \Rightarrow \frac{v_1}{i_s} = \frac{1}{g_{m1}} \| (R + r_{o3}) \simeq \frac{1}{g_{m1}} = 1k$

$\frac{v_o}{v_1} = \frac{r_{o3}}{r_{o3} + R} = \frac{2}{3}$

$d = \underline{0.667k}$

$\frac{v_o}{i_s} = A_\infty\left(\frac{L}{1+L}\right) + d\left(\frac{1}{1+L}\right)$

$= 20k\left(\frac{6.57}{7.57}\right) + 0.667k\left(\frac{1}{7.57}\right)$

$\frac{v_o}{i_s} = \underline{-17.4\ k\Omega}$

[3] (b) Find R_{in} and R_{out}.

$L_s = 0 \quad L_o = L = 6.57$

$R_i^0 = \frac{1}{g_{m1}} \| (R + r_{o3}) \simeq \frac{1}{g_{m1}} = 1k$

$R_{in} = R_i^0\left(\frac{1+L_s}{1+L_o}\right) = \underline{132\ \Omega}$

$R_o^0 = R_X \| r_{o3} = (21k \| 40k) = 13.8k$

$R_{OUT} = R_o^0\left(\frac{1+L_s}{1+L_o}\right) = \underline{1.82K}$

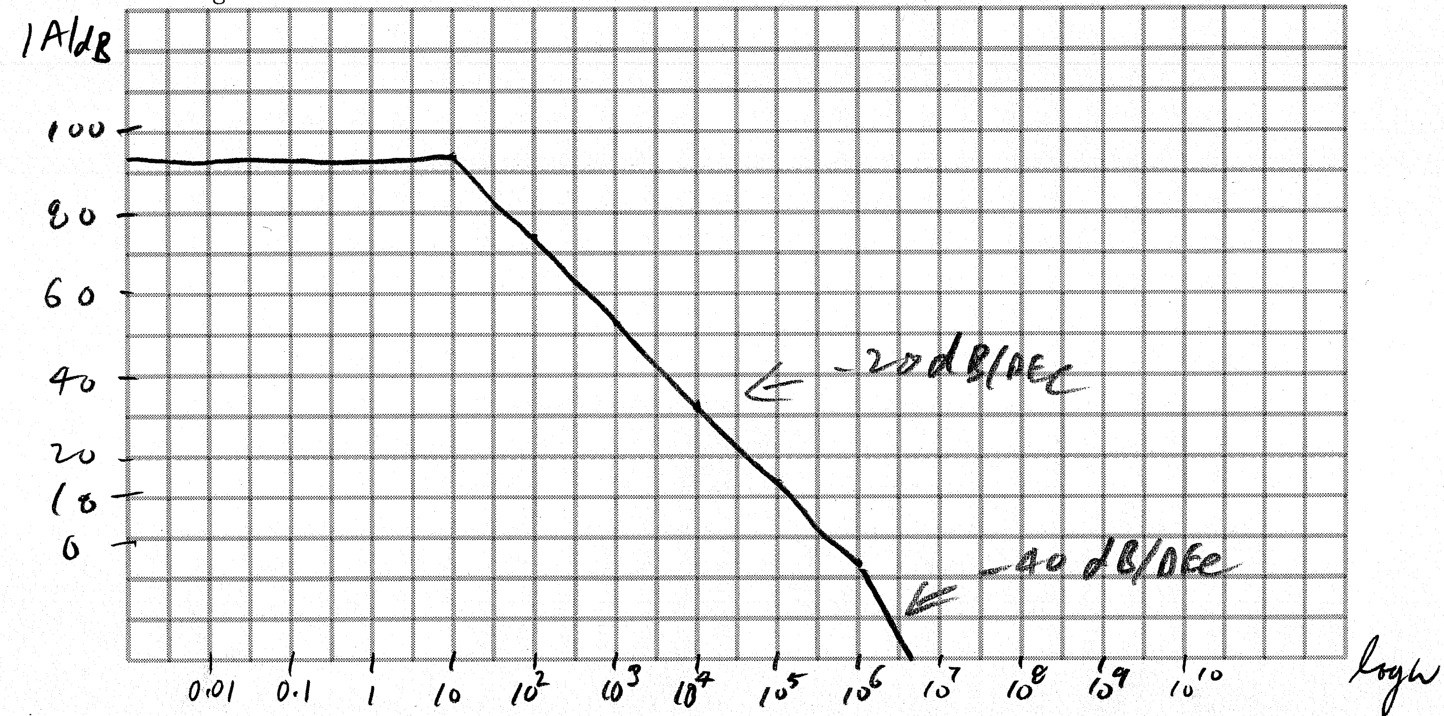
Q4. Assume an opamp is ideal but has the following open-loop gain.

$$A(s) = \frac{0.5 \times 10^5}{(1+s/\omega_{p1})(1+s/\omega_{p2})(1+s/\omega_{p3})} \text{ where } \omega_{p1} = 10^1,\ \omega_{p2} = 10^6 \text{ and } \omega_{p3} = 10^9$$

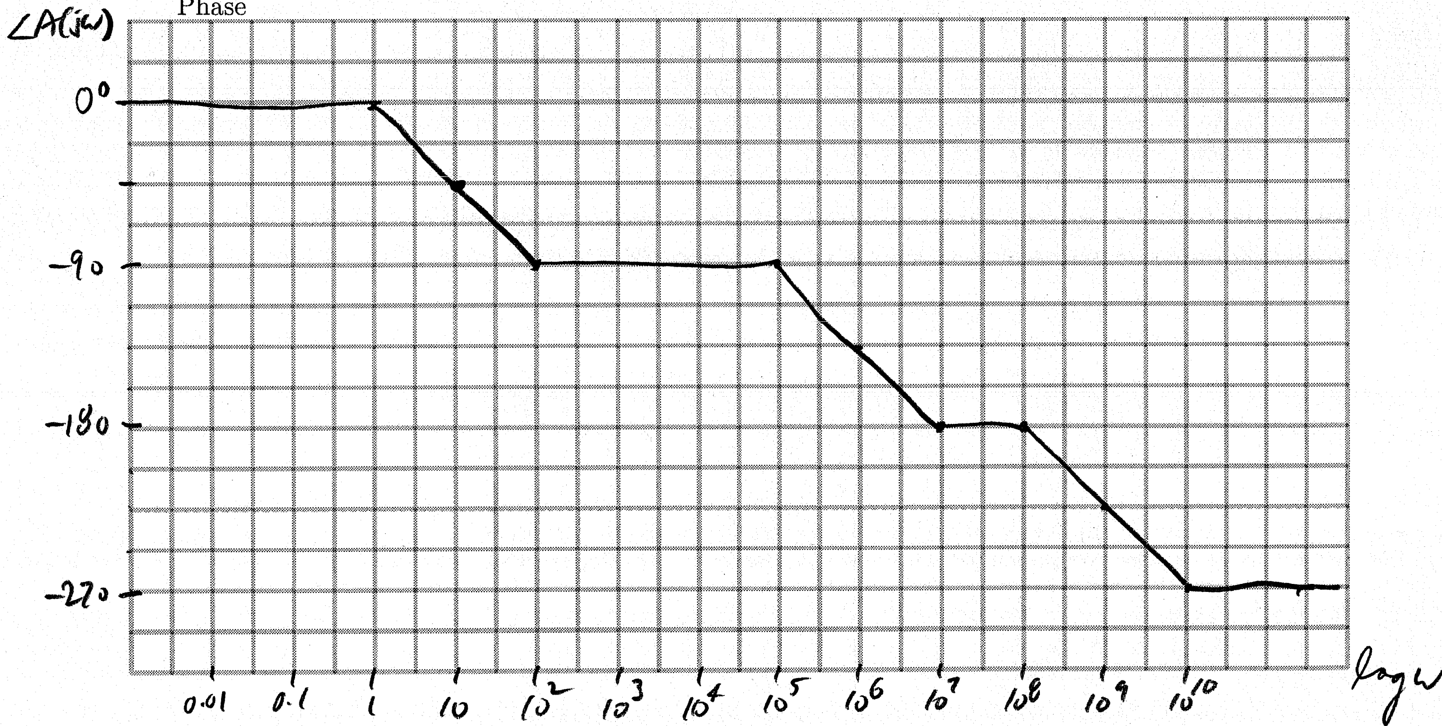
$A_0|_{dB} = 94dB$

[3] (a) Draw the Bode plot for the above open loop gain (Label all plot axis).

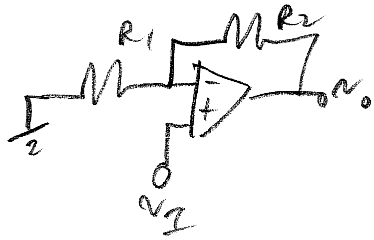
Magnitude



Phase



[3] (b) Estimate the phase-margin (PM) if the above opamp is used to create a gain of +2 using 2 resistors (a non-inverting configuration) (Hint: Note that the unity-gain frequency is much greater than ω_{p1} and much less than ω_{p3}.)



$$\frac{v_o}{v_I} = \left(1 + \frac{R_2}{R_1}\right) = 2 \Rightarrow R_2 = R_1 \Rightarrow \beta = 0.5$$

$$L(s) = \beta A(s) = \frac{0.25e5}{\left(1+\frac{s}{10}\right)\left(1+\frac{s}{10^6}\right)\left(1+\frac{s}{10^9}\right)}$$

$|L(j\omega_t)| = 1$ & $10 \ll \omega_t \ll 10^9$ GIVEN

$$|L(j\omega_t)| \simeq \frac{(0.25e5)}{\left|\frac{j\omega_t}{10}\right| \left|1 + \frac{j\omega_t}{10^6}\right|} = 1$$

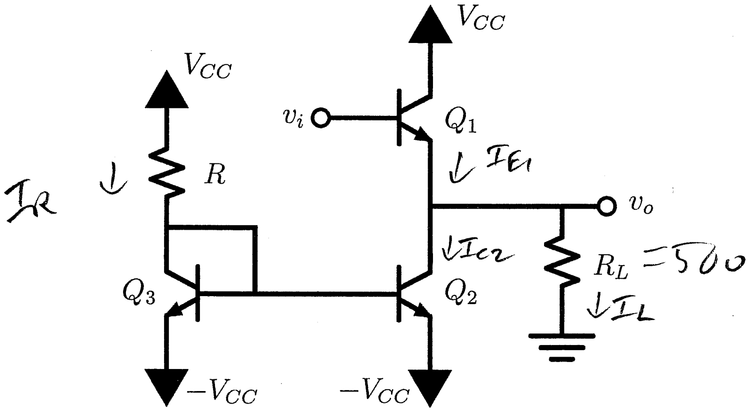
$$\frac{(0.25e5)^2}{\left(\frac{\omega_t}{10}\right)^2\left(1+\left(\frac{\omega_t}{10^6}\right)^2\right)} = 1 \Rightarrow \frac{(10^2)(10^{12})(0.25e5)^2}{\omega_t^2\,(10^{12}+\omega_t^2)} = 1$$

$$(\omega_t^2)^2 + 10^{12}\omega_t^2 - 6.25e22 = 0 \Rightarrow \omega_t^2 = 5.9e10$$
$$\omega_t = 2.4e5$$

$$\angle L(j\omega_t) \simeq -90^\circ - \tan^{-1}\left(\frac{2.5e5}{1e6}\right) = -90^\circ - 13.7^\circ = -103.7^\circ$$

$$PM = \angle L(j\omega_t) - (-180^\circ) = \underline{\underline{76^\circ}}$$

Q5. Consider the class A BJT power amp shown below where $V_{CC} = 10V$, all transistors are matched and have large β (such that base currents can be ignored) and $V_{cesat} = 0.2V$ for all transistors. The load resistance is 500Ω.



[2] (a) Assuming the desired output swing is $\pm 8V$, find R for the minimum power consumption.

FOR $v_o = -8V \Rightarrow I_{E1} = 0$ & $I_{C2} = -I_L = \frac{8}{500} = 16mA = I_R$

$$I_R = \frac{2V_{CC} - 0.7}{R} \Rightarrow R = \frac{2V_{CC} - 0.7}{I_R} = \frac{19.3}{16mA} = \underline{\underline{1.21 k\Omega}}$$

[2] (b) What is the efficiency of the above design when driving an $8V$ sinusoidal waveform?

LOAD POWER $P_L = \frac{(8/\sqrt{2})^2}{R_L} = 64 mW$

VCC POWER $P_{CC} = (I_{C3} + I_{C2}) 2V_{CC} = (32mA)(20) = 640mW$

$$\eta = \frac{P_L}{P_{CC}} \times 100 = \underline{\underline{10\%}}$$

[2] (c) Assuming a large input signal, v_i, for the above design, what value of R_L would cause Q_2 to saturate?

Q_2 SATURATES WHEN $v_o = -10V + 0.2 = -9.8V$
& $I_{E1} = 0$

& $v_o = -9.8V$ OCCURS FOR

$$v_o = i_L R_L = -I_{C2} R_L = -9.8V$$

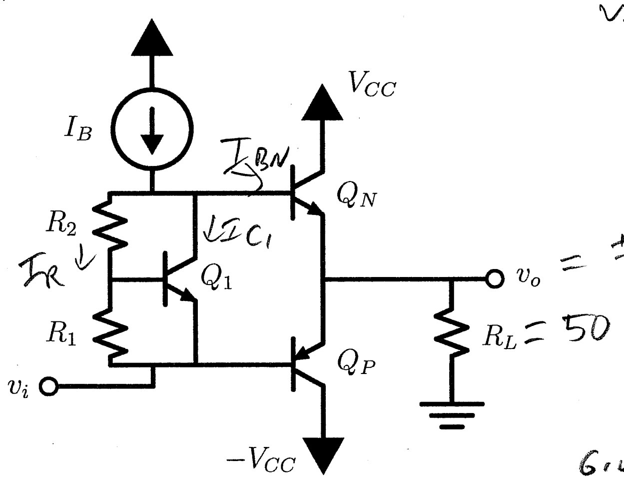
$$R_L = \frac{9.8}{I_{C2}} = \frac{9.8}{16mA} = \underline{\underline{612.5\ \Omega}}$$

Q6.

~~[4]~~ [6] ~~(a)~~ Consider a class AB BJT output stage with a V_{be} multiplier used for biasing as shown below. The load resistor is 50Ω and $V_{CC} = 12V$. Assume the output is sinusoidal with a maximum amplitude of $10V$ and the power transistors Q_N/Q_P are matched with $I_S = 10^{-13}A$ and $\beta = 40$. The bias transistor has $I_S = 10^{-14}A$ and $\beta = 200$. Design the bias circuit for a quiescent current of $10mA$ and a minimum current of $1mA$ through the V_{be} multiplier circuit (Find I_B, R_1 and R_2).

$\beta_N = 40$

$\alpha_N = \frac{40}{41} = 0.975$



$V_T = 25\,mV$

$$I_{BN\text{-}MAX} = \left(\frac{\hat{V}_o}{R_L} \times \frac{\alpha_N}{\beta}\right) = \frac{10}{50} \times \frac{0.975}{40} \simeq 5\,mA$$

$v_o = \pm 10V$ SINUSOID

GIVEN

$$I_B = 5mA + 1mA = \underline{\underline{6\,mA}}$$

GIVEN

$$\text{AT QUIESCENT} \Rightarrow I_N = I_P = I_Q = 10\,mA \Rightarrow I_{BN}' = \frac{10}{40} = 0.25\,mA$$

$$Q_N \;\&\; Q_P \text{ MATCHED} \Rightarrow V_{BE_N} = V_{EB_P} = \frac{V_{BB}}{2}$$

$$I_Q = \left(\frac{I_{SN}}{\alpha_N}\right) e^{\frac{V_{BB}}{2V_T}} \Rightarrow V_{BB} = 2V_T \ln\left(\frac{I_Q \alpha_N}{I_{SN}}\right) = (50e\text{-}3)\ln\left(\frac{10e\text{-}3 \times 0.975}{1e\text{-}13}\right)$$

$$V_{BB} = 1.265\ V$$

$$\text{CHOOSE } I_R = 0.5\,mA \Rightarrow I_{C_1} = 6 - 0.5 - 0.25 = 5.25\,mA \text{ AT QUIESCENT}$$

$$V_{BE_1} = V_T \ln\left(\frac{I_{C1}}{I_{S1}}\right) = 25e\text{-}3 \ln\left(\frac{5.25e\text{-}3}{1e\text{-}14}\right) = 0.6747\,V$$

$$I_R = \frac{V_{BE_1}}{R_1} \Rightarrow R_1 = \frac{0.6747}{0.5\,mA} = \underline{\underline{1.35\,k\Omega}}$$

ASSUMING $I_B \approx 0$ SINCE $\beta_1 = 200$

$$V_{BB} \doteq V_{BE_1}\left(1 + \frac{R_2}{R_1}\right) \Rightarrow R_2 = \left(\frac{V_{BB}}{V_{BE_1}} - 1\right) R_1 = \underline{\underline{1.18\,k\Omega}}$$

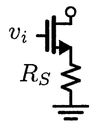
Equation Sheet

Constants: $k = 1.38 \times 10^{-23}\,\mathrm{J\,K^{-1}}$; $q = 1.602 \times 10^{-19}\,\mathrm{C}$; $V_T = kT/q \approx 26\mathrm{mV}$ at $300\,\mathrm{K}$; $\epsilon_0 = 8.85 \times 10^{-12}\,\mathrm{F\,m^{-1}}$; $k_{ox} = 3.9$; $C_{ox} = (k_{ox}\epsilon_0)/t_{ox}$; $\omega = 2\pi f$

NMOS: $k_n = \mu_n C_{ox}(W/L)$; $V_{tn} > 0$; $v_{DS} \geq 0$; $V_{ov} = V_{GS} - V_{tn}$
(triode) $v_{DS} \leq V_{ov}$; $v_D < v_G - V_{tn}$; $i_D = k_n(V_{ov}v_{DS} - (v_{DS}^2/2))$
(active) $v_{DS} \geq V_{ov}$; $i_D = 0.5k_nV_{ov}^2(1 + \lambda v_{DS})$; $g_m = k_nV_{ov} = 2I_D/V_{ov} = \sqrt{2k_nI_D}$; $r_s = 1/g_m$; $r_o = L/(|\lambda'|I_D)$

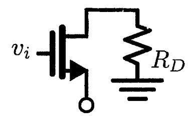
PMOS: $k_p = \mu_p C_{ox}(W/L)$; $V_{tp} < 0$; $v_{SD} \geq 0$; $V_{ov} = V_{SG} - |V_{tp}|$
(triode) $v_{SD} \leq V_{ov}$; $v_D > v_G + |V_{tp}|$; $i_D = k_p(V_{ov}v_{SD} - (v_{SD}^2/2))$
(active) $v_{SD} \geq V_{ov}$; $i_D = 0.5k_pV_{ov}^2(1 + |\lambda| v_{SD})$; $g_m = k_pV_{ov} = 2I_D/V_{ov} = \sqrt{2k_pI_D}$; $r_s = 1/g_m$; $r_o = L/(|\lambda'|I_D)$

BJT: (active) $i_C = I_S e^{(v_{BE}/V_T)}(1 + (v_{CE}/V_A))$; $g_m = \alpha/r_e = I_C/V_T$; $r_e = V_T/I_E$; $r_\pi = \beta/g_m$; $r_o = |V_A|/I_C$; $i_C = \beta i_B$; $i_E = (\beta+1)i_B$; $\alpha = \beta/(\beta+1)$; $i_C = \alpha i_E$; $R_b = (\beta+1)(r_e + R_E)$; $R_e = (R_B + r_\pi)/(\beta+1)$

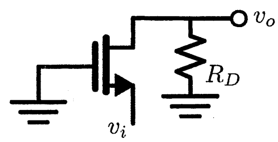


$i_{sc} \approx -(1/g_m + R_S)^{-1}v_i$
$R_x \approx (1 + g_mR_S)r_o$

(Approx due to $g_mr_o \gg 1$)



$v_{oc} \approx v_i$
$R_x \approx 1/g_m + R_D/(g_mr_o)$



$v_o/v_i \approx g_m(r_o||R_D)$

Diff Pair: $A_d = g_mR_D$; $A_{CM} = -(R_D/(2R_{SS}))(\Delta R_D/R_D)$; $A_{CM} = -(R_D/(2R_{SS}))(\Delta g_m/g_m)$; $V_{OS} = \Delta V_t$; $V_{OS} = (V_{OV}/2)(\Delta R_D/R_D)$; $V_{OS} = (V_{OV}/2)(\Delta(W/L)/(W/L))$

1st order: step response $y(t) = Y_\infty - (Y_\infty - Y_{0+})e^{-t/\tau}$; unity gain freq for $T(s) = \dfrac{A_M}{1 + (s/\omega_{3dB})}$ for $A_M \gg 1 \Rightarrow \omega_t \simeq |A_M|\omega_{3dB}$

Freq: for real axis poles/zeros $T(s) = k_{dc}\dfrac{(1 + s/z_1)(1 + s/z_2)\ldots(1 + s/z_m)}{(1 + s/\omega_1)(1 + s/\omega_2)\ldots(1 + s/\omega_n)}$
OTC estimate $\omega_H \simeq 1/(\sum \tau_i)$; dominant pole estimate $\omega_H \simeq 1/(\tau_{max})$

Miller: $Z_1 = Z/(1 - K)$; $Z_2 = Z/(1 - 1/K)$

Mos caps: $C_{gs} = (2/3)WLC_{ox} + WL_{ov}C_{ox}$; $C_{gd} = WL_{ov}C_{ox}$; $C_{db} = C_{db0}/\sqrt{1 + V_{db}/V_0}$; $\omega_t = g_m/(C_{gs} + C_{gd})$; for $C_{gs} \gg C_{gd} \Rightarrow f_t \simeq (3\mu V_{ov})/(4\pi L^2)$

Feedback: $A_f = A/(1 + A\beta)$; $x_i = (1/(1 + A\beta))x_s$; $dA_f/A_f = (1/(1 + A\beta))dA/A$; $\omega_{Hf} = \omega_H(1 + A\beta)$; $\omega_{Lf} = \omega_L/(1 + A\beta)$;
Loop Gain $L \equiv -s_r/s_t$; $A_f = A_\infty(L/(1 + L)) + d/(1 + L)$; $Z_{port} = Z_{p^o}((1 + L_S)/(1 + L_O))$: $PM = \angle L(j\omega_t) + 180$; $GM = -|L(j\omega_{180})|_{db}$;
Pole splitting $\omega'_{p1} \simeq 1/(g_mR_2C_fR_1)$; $\omega'_{p2} \simeq (g_mC_f)/(C_1C_2 + C_f(C_1 + C_2))$

Pole Pair: $s^2 + (\omega_o/Q)s + \omega_o^2$; $Q \leq 0.5 \Rightarrow$ real poles; $Q > 1/\sqrt{2} \Rightarrow$ freq resp peaking

Power Amps: Class A : $\eta = (1/4)(\hat{V}_O/IR_L)(\hat{V}_O/V_{CC})$; Class B : $\eta = (\pi/4)(\hat{V}_O/V_{CC})$; $P_{DN\_max} = V_{CC}^2/(\pi^2R_L)$; Class AB : $i_ni_p = I_Q^2$

2-stage opamp: $\omega_{p1} \simeq (R_1G_{m2}R_2C_c)^{-1}$; $\omega_{p2} = G_{m2}/C_2$; $\omega_z = (C_c(1/G_{m2} - R))^{-1}$;
$SR = I/C_c = \omega_tV_{ov1}$; will not SR limit if $\omega_t\hat{V}_O < SR$

MOS TRANSISTOR: CMOS basic parameters. Minimum channel length $= 0.18\mu$m

	V_t [V]	μC_{ox} [µA/V²]	λ' [µm V⁻¹]	C_{ox} [fF/µm²]	t_{ox} [nm]	L_{ov} [µm]	C_{db0}/W [fF µm⁻¹]
NMOS	0.4	240	0.05	8.5	4	0.04	0.3
PMOS	-0.4	60	-0.05	8.5	4	0.04	0.3