University of Toronto

Final Exam

Date — Dec 19, 2019: 2pm

Duration — 2.5 hrs

ECE 331 — Analog Electronics

Lecturer — D. Johns

ANSWER QUESTIONS ON THESE SHEETS **USING BACKS IF NECESSARY**

- Equation sheet is on the last page of this test.
- Unless otherwise stated, assume $g_m r_o \gg 1$
- Notation: 1.5e+04 is equivalent to 1.5×10^4
- Non-programmable calculator is allowed; No other aids are allowed
- Grading indicated by [ ]. Attempt all questions since a blank answer will certainly get 0.

Question	Points	Score
1	6	
2	6	
3	6	
4	6	
5	6	
6	6	
Total:	36	

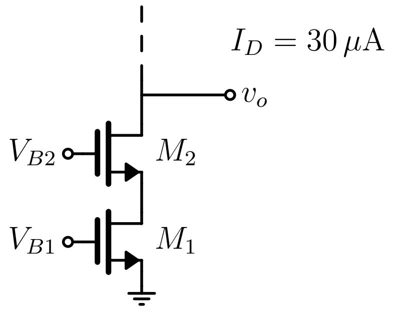
Grading Table
(do not write in above table)

Last Name: ____________________

First Name: ____________________

Student #: ____________________

[6] **Q1.** Consider the wide-swing current mirror shown below where the desired output current is 30 μA. Given that M_1 and M_2 are identical in size and the minimum output voltage is 0.5 V, find the length of the transistors such that the current mirror output resistance is 60 MΩ



V_{tn}	0.3 V
$u_n C_{ox}$	160 μA/V^2
λ'_n	0.04 μm/V

Solution

$Vo(min) = 0.5\,\text{mV} = 2V_{ov} \Rightarrow V_{ov} = 0.25\,\text{V}$

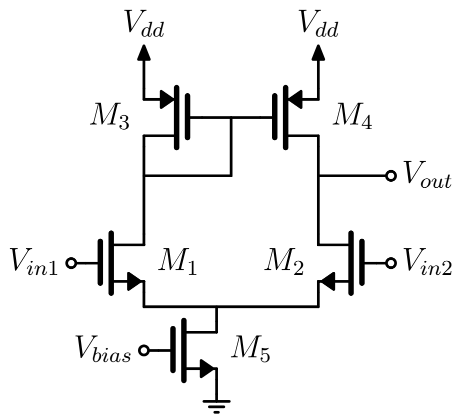
$g_m = \frac{2I_D}{V_{ov}} = \frac{2(30e-6)}{0.25} = 240.0\,\mu\text{A/V}$

$R_{out} \approx g_m r_o^2 \Rightarrow r_o = \sqrt{\frac{R_{out}}{g_m}} = \sqrt{\frac{60e6}{240.0e-6}} = 0.5\,\text{M}\Omega$

$r_o = \frac{L}{|\lambda'_n| I_D} \Rightarrow L = r_o|\lambda'_n|I_D = (0.5e6)(0.04e-6)(30e-6) = 0.6\,\mu\text{m}$

$L = 0.6\,\mu\text{m}$

Q2. Consider the differential to single ended amplifier shown below. All transistor lengths are $0.2\,\mu$m and have $V_{ov} = 0.15\,$V. Also, $I_{D5} = 80\,\mu$A and $V_{dd} = 1.8\,$V.



V_{tn}	0.25 V
u_nC_{ox}	200 μA/V^2
λ'_n	0.05 μm/V
V_{tp}	−0.3 V
u_pC_{ox}	60 μA/V^2
λ'_p	−0.04 μm/V

[3] (a) Find the small-signal gain V_{out}/v_{id} where $v_{id} \equiv V_{in2} - V_{in1}$

Solution

Since $I_{D5} = 80\,\mu$A, I_{D1} to I_{D4} all equal $40.0\,\mu$A

$r_{o2} = \frac{L_2}{|\lambda'_n| I_{D2}} = 100.0\,\text{k}\Omega$

$r_{o4} = \frac{L_4}{|\lambda'_p| I_{D4}} = 125.0\,\text{k}\Omega$

$R_{out} = r_{o2} || r_{o4} = 55.556\,\text{k}\Omega$

$g_{m2} = \frac{2I_{D2}}{V_{ov2}} = 533.333\,\mu\text{A/V}$

$V_{out}/v_{id} = -g_{m2}R_{out}$

$V_{out}/v_{id} = -29.63\,\text{V/V}$

[3] (b) Find the maximum ($V_{cm(max)}$) and minimum ($V_{cm(min)}$) common-mode input voltages that keep all transistors in the active region.

Solution

$V_{cm(min)} = V_{ov5} + V_{tn} + V_{ov2} = 0.15 + 0.25 + 0.15 = 0.55\,\text{V}$

$V_{cm(min)} = 0.55\,\text{V}$

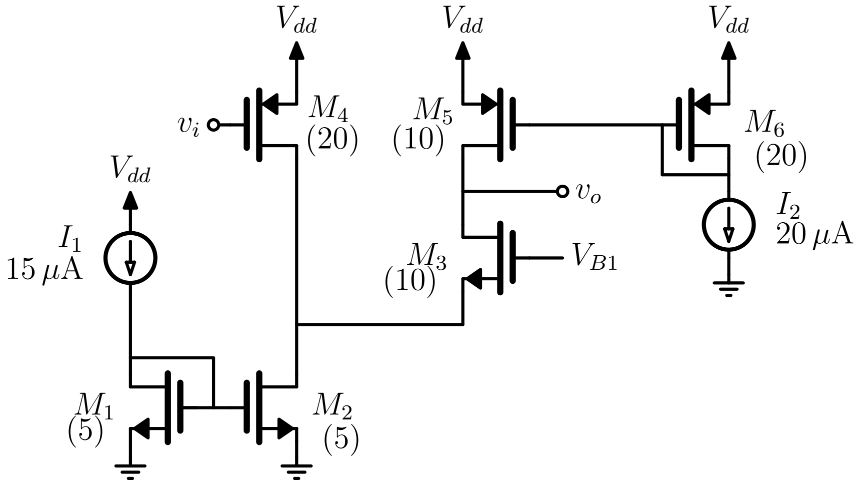
For $V_{cm(max)}$, first find the bias voltage for V_{D1}

$V_{D1} = V_{dd} - |V_{tp}| - V_{ov3} = 1.8 - 0.3 - 0.15 = 1.35\,\text{V}$

V_{in1} can go up to V_{tn} above V_{D1} so

$V_{cm(max)} = V_{D1} + V_{tn} = 1.6\,\text{V}$

Q3. Consider the amplifier shown below where all transistor lengths are 0.18 μm.
The bracketed numbers are the transistor's (W/L) value:



V_{tn}	0.25 V
u_nC_{ox}	200 μA/V^2
λ'_n	0.05 μm/V
V_{tp}	−0.3 V
u_pC_{ox}	60 μA/V^2
λ'_p	−0.04 μm/V

[3] (a) Calculate the drain currents, overdrive voltages and r_o for $M_2/M_3/M_4/M_5$ transistors.

Solution

$I_{D1} = I_1 = 15\,\mu\text{A}$ $\quad I_{D2} = ((5)/(5))I_{D1} = 15.0\,\mu\text{A}$

$I_{D6} = I_2 = 20\,\mu\text{A}$ $\quad I_{D5} = ((10)/(20))I_{D1} = 10.0\,\mu\text{A}$ $\quad I_{D3} = I_{D5} = 10.0\,\mu\text{A}$

$I_{D4} = I_{D2} - I_{D3} = 5.0\,\mu\text{A}$

$\lambda_n = \lambda'_n/L = \frac{0.05}{0.18} = 0.278\,\text{V}^{-1}$ $\quad r_{o2} = \frac{1}{\lambda_n I_{D2}} = 240.0\,\text{k}\Omega$ $\quad r_{o3} = \frac{1}{\lambda_n I_{D3}} = 360.0\,\text{k}\Omega$

$\lambda_p = \lambda'_p/L = \frac{-0.04}{0.18} = -0.222\,\text{V}^{-1}$ $\quad r_{o4} = \frac{1}{|\lambda_p| I_{D4}} = 900.0\,\text{k}\Omega$ $\quad r_{o5} = \frac{1}{|\lambda_p| I_{D5}} = 450.0\,\text{k}\Omega$

For nmos: $V_{ov} = \sqrt{\frac{2I_D}{\mu_n C_{ox}(W/L)}} = 100.0\sqrt{\frac{I_D}{(W/L)}}$

$V_{ov2} = 0.173\,\text{V}$ $\quad V_{ov3} = 0.1\,\text{V}$

For pmos: $V_{ov} = \sqrt{\frac{2I_D}{\mu_p C_{ox}(W/L)}} = 182.574\sqrt{\frac{I_D}{(W/L)}}$

$V_{ov4} = 0.091\,\text{V}$ $\quad V_{ov5} = 0.183\,\text{V}$

[3] (b) Estimate the small-signal gain v_o/v_i
Hint: Impedance looking into the M_3 drain is much higher than r_{o5}

Solution

$g_{m4} = \frac{2I_{D4}}{V_{ov4}} = 109.545\,\mu\text{A/V}$

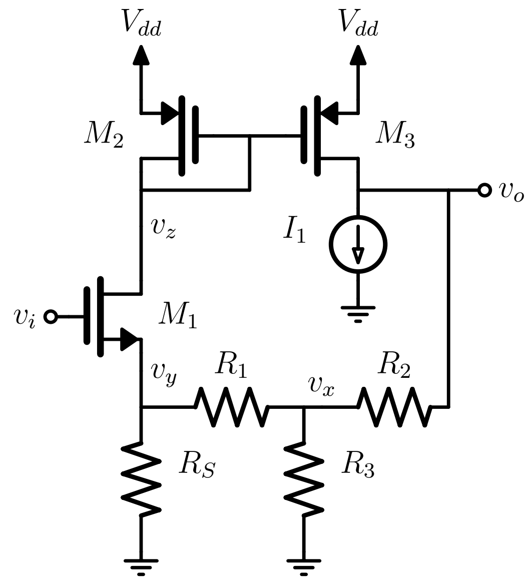
$I_{sc} = g_{m4}v_i$

$R_{out} \approx r_{o5}$

$v_o = -I_{sc}R_{out} \Rightarrow v_o/v_i = -g_{m4}r_{o5} = -49.295\,\text{V/V}$

$v_o/v_i = -49.295\,\text{V/V}$

[6] **Q4.** Consider the feedback amp shown below with the input signal, vi. All current sources are ideal (i.e. infinite output resistance)



$g_{m1} = g_{m2} = 0.1\,\mathrm{mA/V}$
$g_{m3} = 5\,\mathrm{mA/V}$
$r_{o1} = r_{o2} = r_{o3} \to \infty$
$R_1 = 10\,\mathrm{k\Omega}$ $R_2 = 10\,\mathrm{k\Omega}$
$R_3 = 10\,\mathrm{k\Omega}$ $R_S = 10\,\mathrm{k\Omega}$

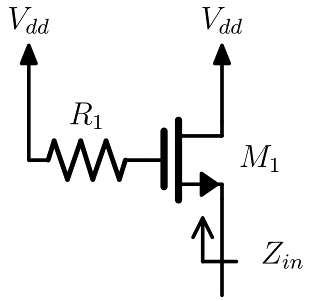
Using loop-gain analysis, find L, A_∞ and v_o/v_i.
(assume $d = 0$).

Solution

Define R_y to be the impedance looking into the source of M_1
Define R_x to be the impedance looking into R_1 from v_x side
Define R_o to be the impedance looking into R_2 from v_o side
$R_y \equiv (1/g_{m1}) = 10.0\,\mathrm{k\Omega}$
$R_x \equiv (R_y||R_S + R_1) = 15.0\,\mathrm{k\Omega}$
$R_o \equiv ((R_x||R_3 + R_2) = 16.0\,\mathrm{k\Omega}$

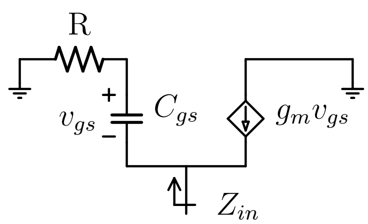
Breaking the loop at the M_3 gate, we have
$v_o/v_{g3} = -g_{m3}R_o = -80.0$ $\quad v_x/v_o = \frac{R_x||R_3}{(R_x||R_3)+R_2} = 0.375$
$vy/vx = \frac{R_y||R_S}{(R_y||R_S)+R_1} = 0.333$ $\quad v_z/v_y = \frac{g_{m1}}{g_{m2}} = 1.0$
$L = -v_o/v_{g3} \times v_x/v_o \times v_y/v_x \times v_z/v_y \Rightarrow$ $L = 10.0$
For A_∞, $L \to \infty$ which results in $v_z \to 0$. Since $v_z = 0$, $i_{D1} = 0$ resulting in $v_y = v_i$ and no current flows into the source of M_1
As a result, $v_x/v_i = 1 + R_1/R_S = 2.0$ and $v_o/v_x = 1 + R_2/(R_3||(R_1 + R_S) = 2.5$
Resulting in $A_\infty = v_o/v_x \times v_x/v_i \Rightarrow$ $A_\infty = 5.0$
$v_o/v_i = A_\infty \frac{L}{1+L} = 5.0\frac{10.0}{1+10.0}$
$v_o/v_i = 4.545$

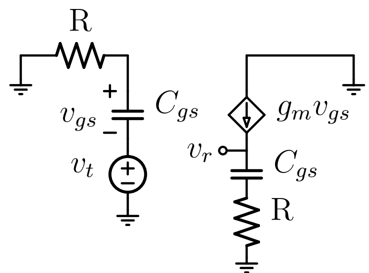
Q5. Consider the input impedance looking into the source of the circuit below. Let $r_o \to \infty$ and only consider capacitance C_{gs}.



[3] (a) Draw the small signal model and find the loop gain.

Solution





$L_O \equiv -v_r/v_t = -(\frac{-1/sC_{gs}}{1/sC_{gs}+R})g_m(\frac{1+sC_{gs}R}{sC_{gs}}) = \frac{g_m}{sC_{gs}}$

$$L_O = \frac{g_m}{sC_{gs}}$$

$$L_S = 0$$

[3] (b) Using the loop gain found above, find Z_{in}.

Solution

$Z_{P0} = R + \frac{1}{sC_{gs}} = \frac{1+sC_{gs}R}{sC_{gs}}$

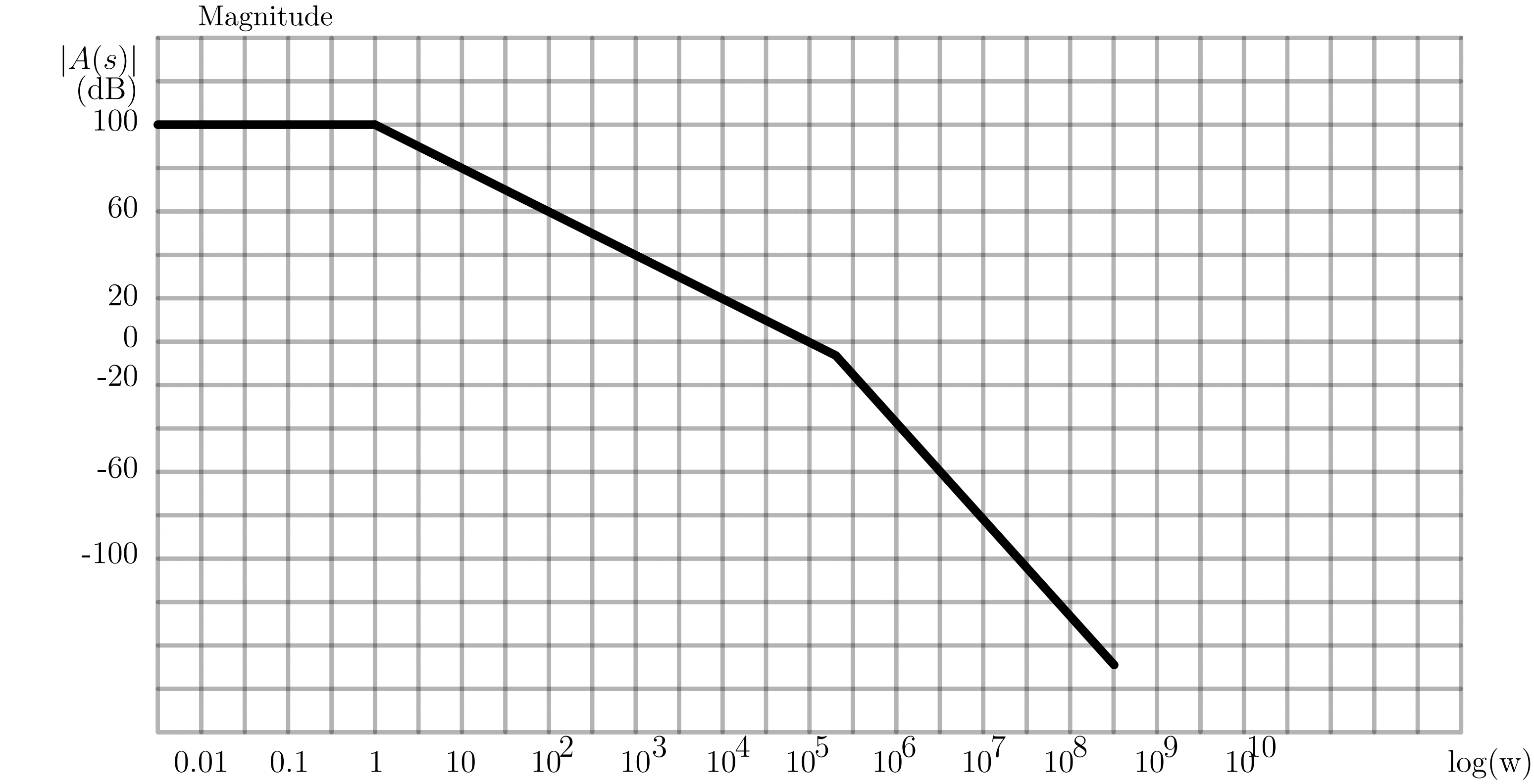
$Z_{in} = Z_{P0}\left[\frac{1+L_S}{1+L_O}\right] = \left(\frac{1+sC_{gs}R}{sC_{gs}}\right)\left(\frac{1}{1+\frac{g_m}{sC_{gs}}}\right)$

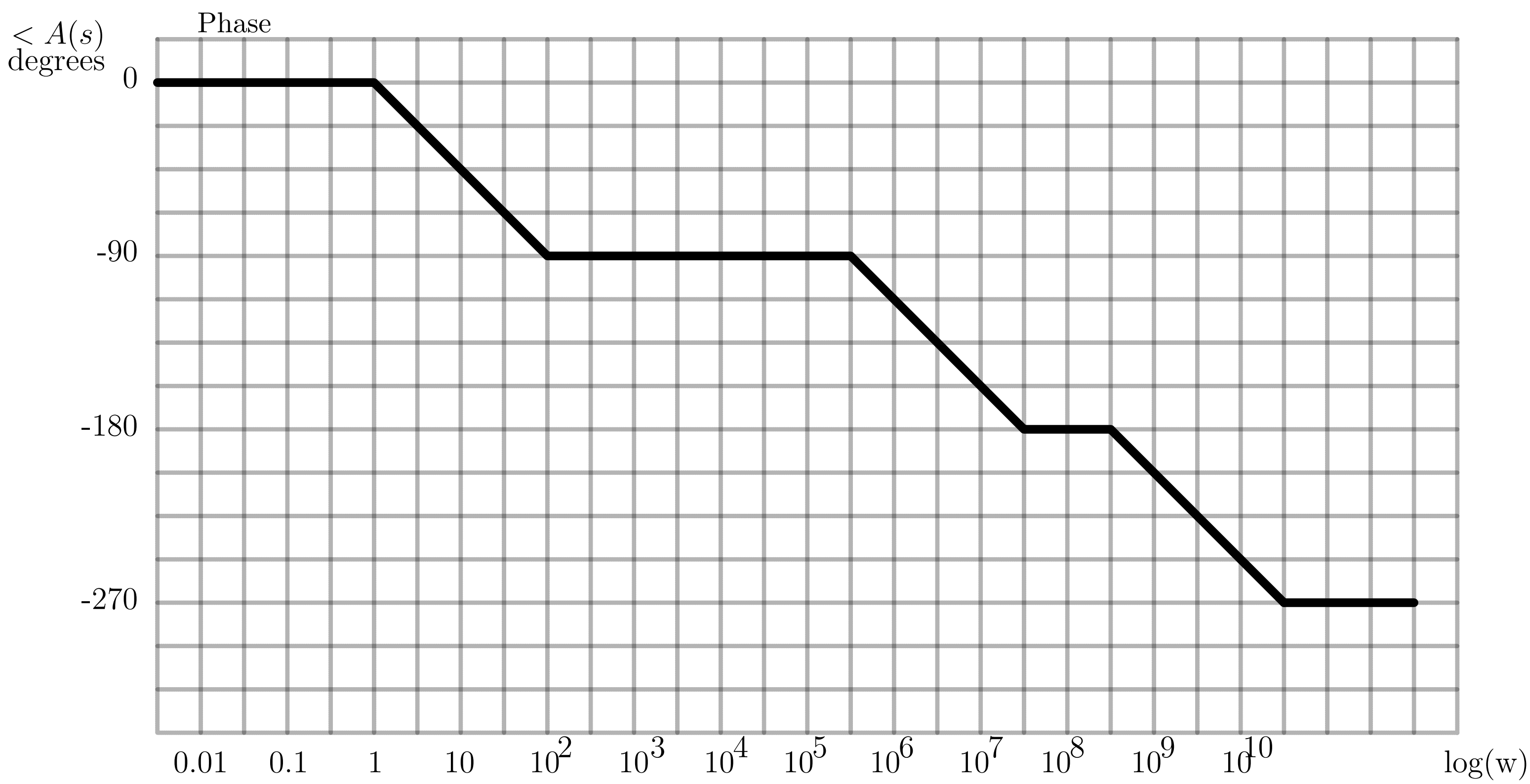
$$Z_{in} = \left(\frac{1}{g_m}\right)\left(\frac{1+sC_{gs}R}{1+sC_{gs}/g_m}\right)$$

Q6. Assume an opamp is ideal but has the following open-loop gain.

$$A(s) = \frac{1.0e{+}05}{(1+s/\omega_{p1})(1+s/\omega_{p2})(1+s/\omega_{p3})} \text{ where } \omega_{p1} = 1.0e+01,\ \omega_{p2} = 2.0e+06 \text{ and } \omega_{p3} = 2.0e+09$$

[3] (a) Draw the Bode plot for the above open loop gain (Label all plot axis).





[3] (b) Estimate the phase-margin (PM) if the above opamp is used to create a gain of +3 using 2 resistors (a non-inverting configuration) (Hint: Note that the unity-gain frequency is much greater than ω_{p1} and much less than ω_{p3}.)

Solution

For a non-inverting opamp gain of +3, $\beta = 1/3 = 0.333$ resulting in the loop gain equal to $L(s) = \beta A(s) = \frac{L_0}{(1+s/\omega_{p1})(1+s/\omega_{p2})(1+s/\omega_{p3})}$ where $L_0 = 3.33e+04$ and $\omega_{p1}, \omega_{p2}, \omega_{p3}$ given above.

For frequencies near ω_t where $\omega_{p1} \ll \omega_t \ll \omega_{p3}$, we can approximate $L(s)$ as

$L(s) \approx \frac{L_0}{(s/\omega_{p1})(1+s/\omega_{p2})}$ and making use of $|L(j\omega_t)| = 1$, we have

$\frac{L_0^2}{(\omega_t/\omega_{p1})^2(1+(\omega_t/\omega_{p2})^2)} = 1 \Rightarrow \frac{\omega_{p1}^2\omega_{p2}^2 L_0^2}{\omega_t^2(\omega_t^2+\omega_{p2}^2)} = 1 \Rightarrow (\omega_t^2)^2 + \omega_{p2}^2(\omega_t^2) - \omega_{p1}^2\omega_{p2}^2 L_0^2 = 0$

$(\omega_t^2)^2 + 4.000e+12(\omega_t^2) - 4.444e+23 = 0$ Solving for ω_t^2, we have $\omega_t^2 = 1.082e+11\,\text{rad/s}$ resulting in $\omega_t = 3.289e+05\,\text{rad/s}$

So now, we can find the phase of $L(j\omega_t)$ as ($-90°$ is due to the ω_{p1})

$< L(j\omega_t) \approx -90° - \tan^{-1}(\frac{\omega_t}{\omega_{p2}}) = -90° - 9.339° = -99.339°$

Finally, the phase-margin can be found as

$PM \equiv < L(j\omega_t) - (-180°) = 80.661°$

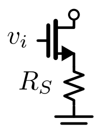
Equation Sheet

Constants: $k = 1.38 \times 10^{-23}\,\mathrm{J\,K^{-1}}$; $q = 1.602 \times 10^{-19}\,\mathrm{C}$; $V_T = kT/q \approx 26\mathrm{mV}$ at $300\,\mathrm{K}$; $\epsilon_0 = 8.85 \times 10^{-12}\,\mathrm{F\,m^{-1}}$; $k_{ox} = 3.9$; $C_{ox} = (k_{ox}\epsilon_0)/t_{ox}$; $\omega = 2\pi f$

NMOS: $k_n = \mu_n C_{ox}(W/L)$; $V_{tn} > 0$; $v_{DS} \geq 0$; $V_{ov} = V_{GS} - V_{tn}$
(triode) $v_{DS} \leq V_{ov}$; $v_D < v_G - V_{tn}$; $i_D = k_n(V_{ov}v_{DS} - (v_{DS}^2/2))$
(active) $v_{DS} \geq V_{ov}$; $i_D = 0.5k_nV_{ov}^2(1 + \lambda v_{DS})$; $g_m = k_nV_{ov} = 2I_D/V_{ov} = \sqrt{2k_nI_D}$; $r_s = 1/g_m$; $r_o = L/(|\lambda'|I_D)$

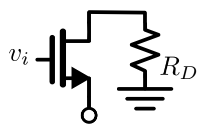
PMOS: $k_p = \mu_p C_{ox}(W/L)$; $V_{tp} < 0$; $v_{SD} \geq 0$; $V_{ov} = V_{SG} - |V_{tp}|$
(triode) $v_{SD} \leq V_{ov}$; $v_D > v_G + |V_{tp}|$; $i_D = k_p(V_{ov}v_{SD} - (v_{SD}^2/2))$
(active) $v_{SD} \geq V_{ov}$; $i_D = 0.5k_pV_{ov}^2(1 + |\lambda|v_{SD})$; $g_m = k_pV_{ov} = 2I_D/V_{ov} = \sqrt{2k_pI_D}$; $r_s = 1/g_m$; $r_o = L/(|\lambda'|I_D)$

BJT: (active) $i_C = I_S e^{(v_{BE}/V_T)}(1 + (v_{CE}/V_A))$; $g_m = \alpha/r_e = I_C/V_T$; $r_e = V_T/I_E$; $r_\pi = \beta/g_m$; $r_o = |V_A|/I_C$; $i_C = \beta i_B$; $i_E = (\beta+1)i_B$; $\alpha = \beta/(\beta+1)$; $i_C = \alpha i_E$; $R_b = (\beta+1)(r_e + R_E)$; $R_e = (R_B + r_\pi)/(\beta+1)$

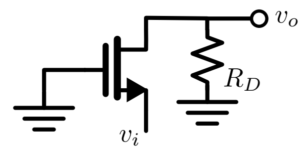


$i_{sc} \approx -(1/g_m + R_S)^{-1}v_i$
$R_x \approx (1 + g_mR_S)r_o$

(Approx due to $g_mr_o \gg 1$)



$v_{oc} \approx v_i$
$R_x \approx 1/g_m + R_D/(g_mr_o)$



$v_o/v_i \approx g_m(r_o||R_D)$

Diff Pair: $A_d = g_mR_D$; $A_{CM} = -(R_D/(2R_{SS}))(\Delta R_D/R_D)$; $A_{CM} = -(R_D/(2R_{SS}))(\Delta g_m/g_m)$; $V_{OS} = \Delta V_t$; $V_{OS} = (V_{OV}/2)(\Delta R_D/R_D)$; $V_{OS} = (V_{OV}/2)(\Delta(W/L)/(W/L))$

1st order: step response $y(t) = Y_\infty - (Y_\infty - Y_{0+})e^{-t/\tau}$; unity gain freq for $T(s) = \dfrac{A_M}{1 + (s/\omega_{3dB})}$ for $A_M \gg 1 \Rightarrow \omega_t \simeq |A_M|\omega_{3dB}$

Freq: for real axis poles/zeros $T(s) = k_{dc}\dfrac{(1 + s/z_1)(1 + s/z_2)\ldots(1 + s/z_m)}{(1 + s/\omega_1)(1 + s/\omega_2)\ldots(1 + s/\omega_n)}$
OTC estimate $\omega_H \simeq 1/(\sum\tau_i)$; dominant pole estimate $\omega_H \simeq 1/(\tau_{max})$

Miller: $Z_1 = Z/(1 - K)$; $Z_2 = Z/(1 - 1/K)$

Mos caps: $C_{gs} = (2/3)WLC_{ox} + WL_{ov}C_{ox}$; $C_{gd} = WL_{ov}C_{ox}$; $C_{db} = C_{db0}/\sqrt{1 + V_{db}/V_0}$;
$\omega_t = g_m/(C_{gs} + C_{gd})$; for $C_{gs} \gg C_{gd} \Rightarrow f_t \simeq (3\mu V_{ov})/(4\pi L^2)$

Feedback: $A_f = A/(1 + A\beta)$; $x_i = (1/(1 + A\beta))x_s$; $dA_f/A_f = (1/(1 + A\beta))dA/A$; $\omega_{Hf} = \omega_H(1 + A\beta)$; $\omega_{Lf} = \omega_L/(1 + A\beta)$;
Loop Gain $L \equiv -s_r/s_t$; $A_f = A_\infty(L/(1 + L)) + d/(1 + L)$; $Z_{port} = Z_{p^o}((1 + L_S)/(1 + L_O))$: $PM = \angle L(j\omega_t) + 180$; $GM = -|L(j\omega_{180})|_{db}$;
Pole splitting $\omega'_{p1} \simeq 1/(g_mR_2C_fR_1)$; $\omega'_{p2} \simeq (g_mC_f)/(C_1C_2 + C_f(C_1 + C_2))$

Pole Pair: $s^2 + (\omega_o/Q)s + \omega_o^2$; $Q \leq 0.5 \Rightarrow$ real poles; $Q > 1/\sqrt{2} \Rightarrow$ freq resp peaking

Power Amps: Class A : $\eta = (1/4)(\hat{V}_O/IR_L)(\hat{V}_O/V_{CC})$; Class B : $\eta = (\pi/4)(\hat{V}_O/V_{CC})$; $P_{DN\_max} = V_{CC}^2/(\pi^2R_L)$;
Class AB : $i_ni_p = I_Q^2$; $I_Q = (I_S/\alpha)e^{V_{BB}/(2V_T)}$; $i_n^2 - i_Li_n - I_Q^2 = 0$

2-stage opamp: $\omega_{p1} \simeq (R_1G_{m2}R_2C_c)^{-1}$; $\omega_{p2} = G_{m2}/C_2$; $\omega_z = (C_c(1/G_{m2} - R))^{-1}$;
$SR = I/C_c = \omega_tV_{ov1}$; will not SR limit if $\omega_t\hat{V}_O < SR$