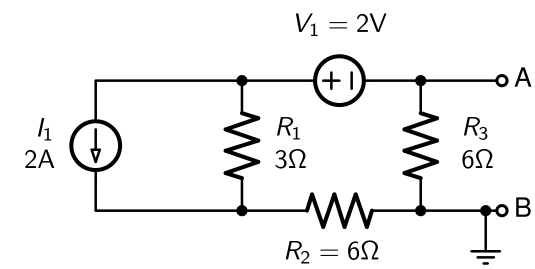
Problem Set 1 - Circuit Review

Question 1

Consider the circuit shown below where it is desired to find the Norton and Thevenin equivalent circuits between nodes A/B. Use i_{sc} for the short circuit output current and v_{oc} for the open circuit output voltage and R_{out} for the output resistance.
Solve by using Thevenin/Norton source transformations.

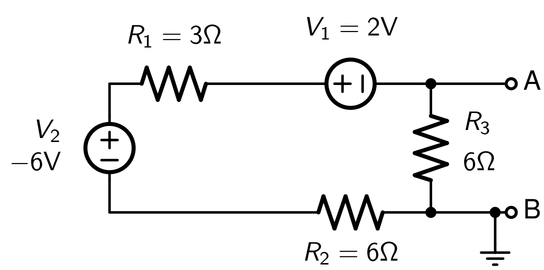


Solution

$R_1 = 3\Omega$, $R_2 = 6\Omega$, $R_3 = 6\Omega$,
We replace I_1 and R_1 with the thevenin equivalent such that

$$V_2 = -I_1 * R_1 = -(2) * (3) = -6V$$

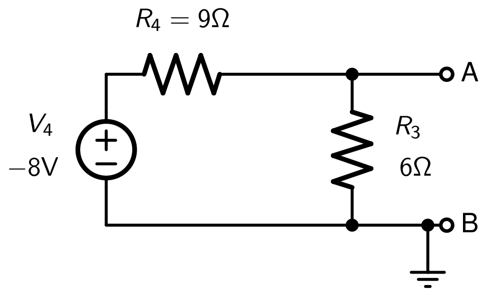
and R_1 is moved in series with V_2.



Combine resistors R_1 and R_2 as well as voltages V_2 and V_1 resulting in the following equivalent circuit

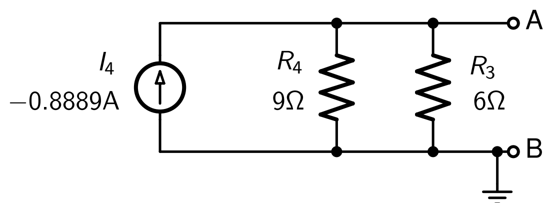
$$R_4 = R_1 + R_2 = (3) + (6) = 9\Omega$$

$$V_4 = V_2 - V_1 = (-6) - (2) = -8V$$



Replace V_4 and R_4 with their Norton equivalent circuit

$$I_4 = V_4/R_4 = (-8)/(9) = -0.8889\text{A}$$

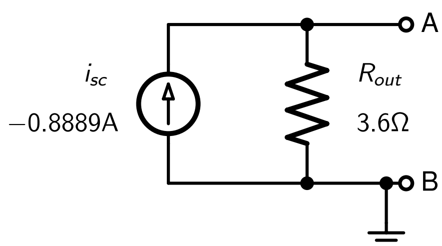


Combine R_4 and R_3 into one resistor which is R_{out}

$$R_{out} = R_4||R_3 = (9)||(6) = 3.6\Omega$$

and the short circuit current is then

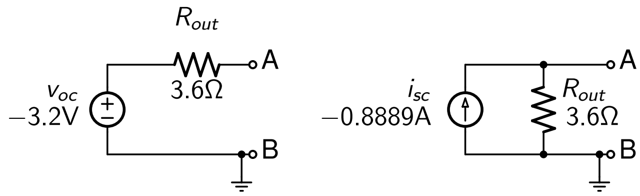
$$i_{sc} = I_4 = (-0.8889) = -0.8889\text{A}$$



Now, we find the output open circuit voltage, v_{oc}

$$v_{oc} = i_{sc} * R_{out} = (-0.8889) * (3.6) = -3.2\text{V}$$

So the 2 equivalent circuits are ...



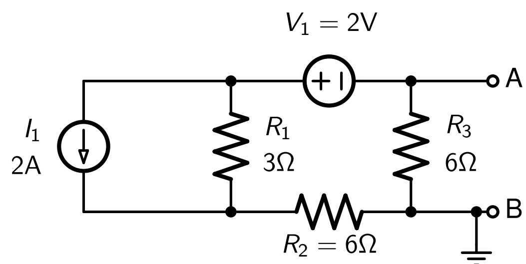
Thevenin Equivalent Norton Equivalent

Note that the output resistance, R_{out} could have also been found directly from the first circuit by zeroing the 2 independent sources. In this case, we would have

$$R_{out} = R_3||(R_1 + R_2) = (6)||((3) + (6)) = 3.6\Omega$$

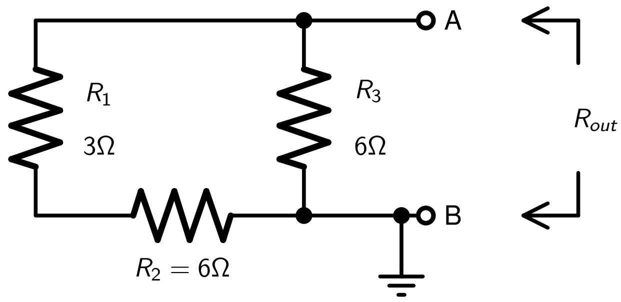
Question 2

Consider the circuit shown below where it is desired to find the Norton and Thevenin equivalents circuits for the port A/B. Use i_{sc} for the short circuit output current and v_{oc} for the open circuit output voltage and R_{out} for the output resistance.
Solve by using superposition to find v_{oc} and find R_{out} directly from the above circuit. Then find i_{sc}.



Solution

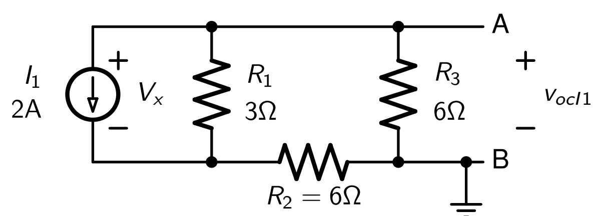
The output resistance, R_{out} can been found directly from the above circuit by zeroing the 2 independent sources. In this case, we have the circuit below...



so R_{out} is found as

$$R_{out} = R_3||(R_1 + R_2) = (6)||((3) + (6)) = 3.6\Omega$$

For v_{oc}, we are going to use superposition. We first find the voltage at AB due to I_1 alone (with $V_1 = 0$), then find the voltage at AB due to V_1 alone (with $I_1 = 0$) and then combine the 2 results to find the voltage at AB due to both I_1 and V_1.
Setting $V_1 = 0$ (V_1 is then a short circuit), results in the following circuit ...



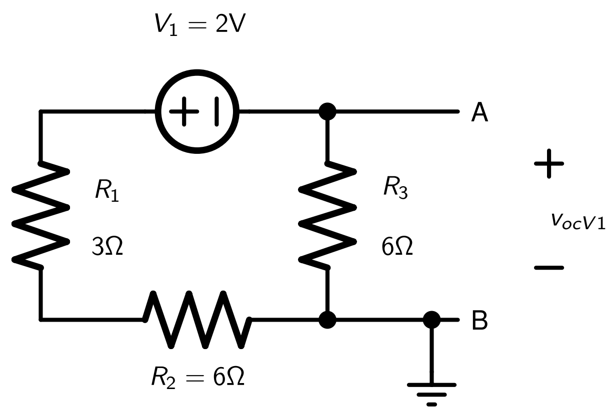
From the above circuit,

$$V_x = -I_1 * R_1||(R_2 + R_3) = -(2) * (3)||((6) + (6)) = -4.8\text{V}$$

which leads to v_{ocI1} being found as

$$v_{ocI1} = V_x * (R_3/(R_3 + R_2)) = (-4.8) * ((6)/((6) + (6))) = -2.4\text{V}$$

Now, setting $I_1 = 0$ (I_1 is then an open circuit) we have the following circuit ...



which leads to v_{ocV1} being found as

$$v_{ocV1} = -V_1 * (R_3/(R_1 + R_2 + R_3)) = -(2) * ((6)/((3) + (6) + (6))) = -0.8\text{V}$$

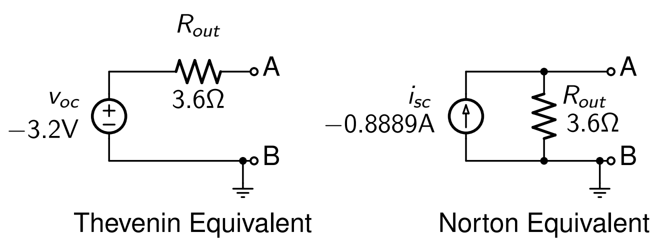
Combining the two v_{oc} results we have

$$v_{oc} = v_{ocI1} + v_{ocV1} = (-2.4) + (-0.8) = -3.2\text{V}$$

Finally, the relationship between v_{oc}, R_{out} and i_{sc} is $i_{sc} = v_{oc}/R_{out}$ which leads to ...

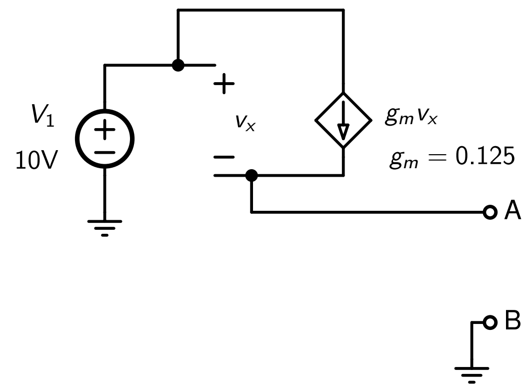
$$i_{sc} = v_{oc}/R_{out} = (-3.2)/(3.6) = -0.8889\text{A}$$

So, the 2 equivalent circuits are ...



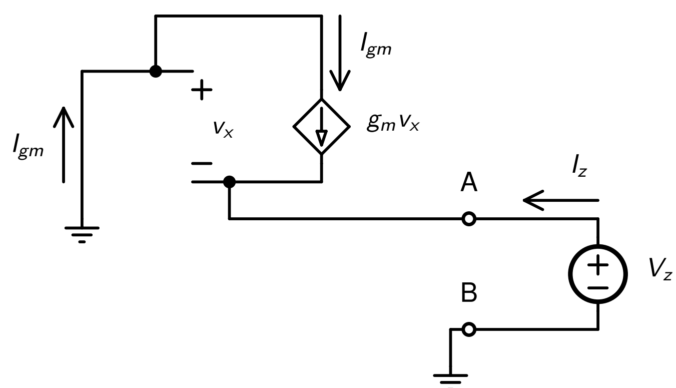
Question 3

Find the Norton equivalent circuit and the Thevenin equivalent circuit for the circuit shown below between nodes A and B. Use i_{sc} for the short circuit output current and v_{oc} for the open circuit output voltage and R_{out} for the output resistance.



Solution

To find R_{out} at port AB, we zero independent sources which in this case is V_1, so we set $V_1 = 0$ which is equivalent to V_1 being a short circuit. Then we apply a voltage V_z at port AB and determine the resulting current, I_z as seen below. The output resistance is then defined to be $R_{out} = V_z/I_z$



I_{gm} is defined to be the current through the voltage-controlled current source. So we have

$$I_{gm} = g_m v_x = -I_z$$

and we also have

$$v_x = -V_z$$

Combining the above 2 equations, we have

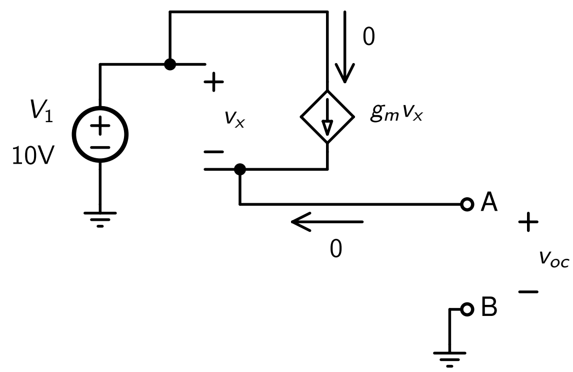
$$g_m(-V_z) = -I_z$$

which gives

$$V_z/I_z = 1/g_m = R_{out}$$

$$R_{out} = 1/g_m = 1/(0.125) = 8\Omega$$

To find V_{oc}, we leave port AB open circuit and find the output voltage as shown below



Here, the current through port A is 0 since it is an open circuit which means the current $g_m v_x = 0$ as well. Since g_m is not zero, then $v_x = 0$ From nodal analysis, we have

$$V_1 - v_x - v_{oc} = 0$$

and with $v_x = 0$, we have

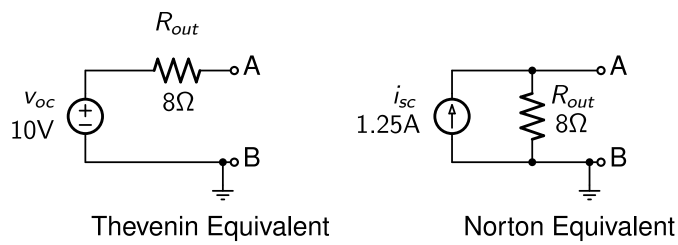
$$v_{oc} = V_1 = (10) = 10\text{V}$$

Finally, we can find i_{sc} though the relationship,

$$i_{sc} = v_{oc}/R_{out}$$

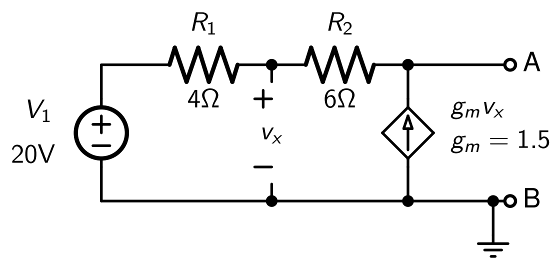
$$i_{sc} = v_{oc}/R_{out} = (10)/(8) = 1.25\text{A}$$

So the 2 equivalent circuits are ...



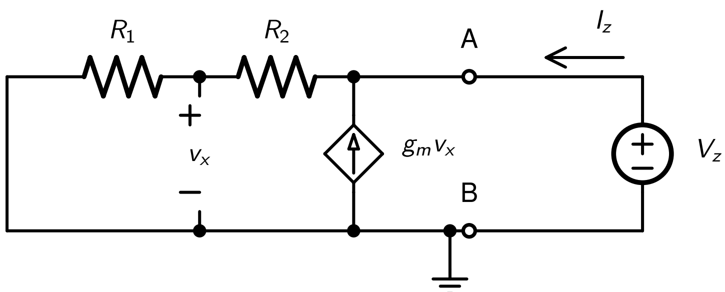
Question 4

Find the Norton equivalent circuit and the Thevenin equivalent circuit for the circuit shown below between nodes A and B. Use i_{sc} for the short circuit output current and v_{oc} for the open circuit output voltage and R_{out} for the output resistance.



Solution

To find R_{out} at port AB, we zero independent sources (which in this case is V_1) so we set $V_1 = 0$ which is equivalent to V_1 being a short circuit. Then we apply a voltage V_z at port AB and determine the resulting current, I_z as seen below. The output resistance is then defined to be $R_{out} = V_z/I_z$



$$I_z = \frac{V_z}{R_1 + R_2} - g_m v_x$$

$$v_x = (\frac{R_1}{R_1 + R_2})V_z$$

$$I_z = V_z(\frac{1}{R_1 + R_2}) - g_m(\frac{R_1}{R_1 + R_2})V_z$$

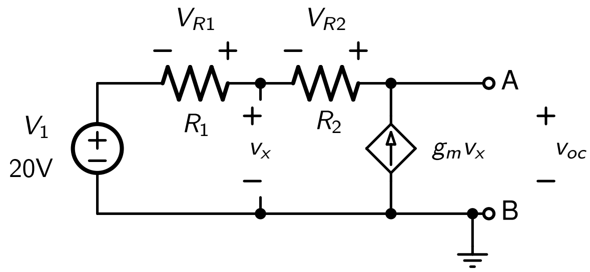
$$I_z = V_z(\frac{1 - g_m R_1}{R_1 + R_2})$$

$$R_{out} = V_z/I_z$$

$$R_{out} = \frac{R_1 + R_2}{1 - g_m * R_1} = \frac{(4) + (6)}{1 - (1.5) * (4)} = -2\Omega$$

We see here that R_out is negative which means that current flows in the opposite direction of what would normally occur when a voltage is attached to the port.

To find V_{oc}, we leave port AB open circuit and find the output voltage as shown below



$$V_1 + V_{R1} - v_x = 0$$

and since $V_{R1} = g_m v_x R_1$, we have

$$V_1 + g_m v_x R_1 - v_x = 0$$

which leads to

$$v_x = \frac{V_1}{1 - g_m R_1}$$

In a similar way, we find a second equation

$$v_x + g_m v_x R_2 - v_{oc} = 0$$

and rearranging, we have

$$v_{oc} = v_x(1 + g_m R_2)$$

Finally, substituting in for v_x found earlier, we have

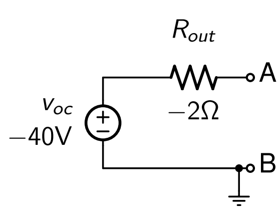
$$v_{oc} = \left(\frac{1 + g_m R_2}{1 - g_m R_1}\right) V_1$$

$$v_{oc} = \frac{1 + g_m * R_2}{1 - g_m * R_1} * V_1 = \frac{1 + (1.5) * (6)}{1 - (1.5) * (4)} * (20) = -40\text{V}$$

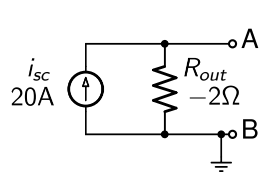
Finally, we can find i_{sc} though the relationship, $i_{sc} = v_{oc}/R_{out}$

$$i_{sc} = v_{oc}/R_{out} = (-40)/(-2) = 20\text{A}$$

So the 2 equivalent circuits are ...



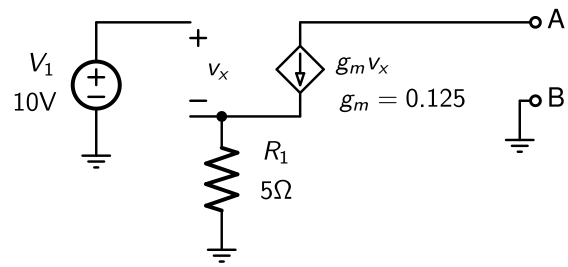
Thevenin Equivalent



Norton Equivalent

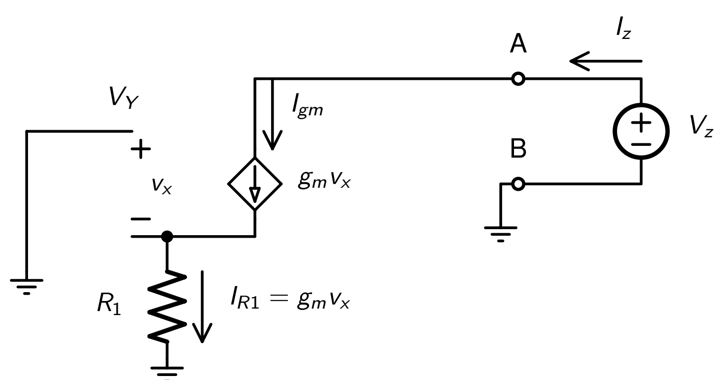
Question 5

Find the Norton equivalent circuit and the Thevenin equivalent circuit for the circuit shown below between nodes A and B. Use i_{sc} for the short circuit output current and v_{oc} for the open circuit output voltage and R_{out} for the output resistance.



Solution

To find R_{out} at port AB, we zero independent sources which in this case is V_1, so we set $V_1 = 0$ which is equivalent to V_1 being a short circuit. Then we apply a voltage V_z at port AB and determine the resulting current, I_z as seen below. The output resistance is then defined to be $R_{out} = V_z/I_z$



Defining V_Y as shown (although it is clear here that $V_Y = 0$ we shall leave it as V_Y for generality and then set it equal to zero), we find the current through R_1 has 2 equations. First, we have

$$I_{R1} = g_m v_x$$

and second, we have

$$I_{R1} = \frac{V_Y - v_x}{R_1}$$

Combining, we have

$$\frac{V_Y - v_x}{R_1} = g_m v_x$$

Rearranging to find v_x, we have

$$v_x = \frac{1/g_m}{(1/g_m) + R_1} \times V_Y$$

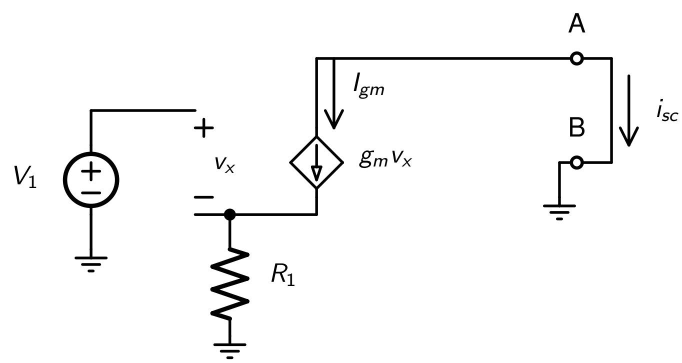
So v_x is a scaled version of V_Y where the scaling is a resistance divider equation between $1/g_m$ and R_1.
In this case, $V_Y = 0$ so $v_x = 0$. Since $v_x = 0$, then $I_{gm} = 0$ so $I_z = 0$. Since $I_z = 0$,

$$R_{out} \to \infty$$

Since $R_{out} \to \infty$, it makes little sense to find v_{oc} as that will also go to ∞.

$$v_{oc} \to \infty$$

For i_{sc}, we use the circuit below



First, we can find v_x from the general equation we found above

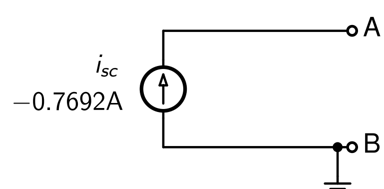
$$v_x = \frac{1/g_m}{(1/g_m) + R_1} * V_1 = \frac{1/(0.125)}{(1/(0.125)) + (5)} * (10) = 6.154\text{V}$$

Next, we have $i_{sc} = -I_{gm}$ leading to

$$i_{sc} = -g_m * v_x = -(0.125) * (6.154) = -0.7692\text{A}$$

So the 2 equivalent circuits are ...

Not Applicable



Thevenin Equivalent

Norton Equivalent