Problem Set 2 - Small Signal Model

Question 1

An NMOS transistor is operated with a small v_{DS} voltage in the triode region and the drain source resistance is measured to be r_{DS}. What will be the new r'_{DS} under each of the following situations? (give r'_{DS} relationship to r_{DS}).

Assume the only change is the one(s) discussed in each situation.

(a) The overdrive voltage is increased by a factor of 1.5.

(b) The transistor width is increased by a factor of 1.8.

(c) The transistor width and length are both increased by a factor of 3.

(d) The transistor gate oxide thickness is reduced by a factor of 2.

Solution

For a small v_{DS} voltage, the transistor is in triode and the drain-to-source resistance, r_{DS}, can be approximated by

$$r_{ds} = \frac{1}{\mu_n C_{ox}(W/L)V_{OV}}$$

(a) If $V'_{OV} = 1.5V_{OV}$, then $r'_{DS} = r_{DS}/1.5$

(b) If $W' = 1.8W$, then $r'_{DS} = r_{DS}/1.8$

(c) If $W' = 3W$ and $L' = 3L$, then $r'_{DS} = r_{DS}$

(d) If the oxide thickness $t`_{ox} = t_{ox}/2$, then $C_{ox} = \epsilon_{ox}/t_{ox}$ is multiplied by 2 resulting in $r'_{DS} = r_{DS}/2$

Question 2

Consider a CMOS technology with the following parameters:

NMOS: $V_{tn} = 0.4\text{V}$; $\mu_n C_{ox} = 240\mu\text{A/V}^2$; $\lambda`_n = 40\text{nm/V}$

(a) For an NMOS transistor with $W_n = 2\mu\text{m}$ and $L_n = 200\text{nm}$, find I_{Dn} when the overdrive voltage is 0.3V and $V_{DS} = 0.5\text{V}$. For this question, do NOT assume $\lambda = 0$.

(b) Find the value of r_o for the transistor (a)

(c) For the transistor in (a), find the change in I_{Dn} if V_{DS} is increased by 0.4V by using r_o found in (b)

Solution

(a) $I_{Dn} = \frac{\mu_n C_{ox}}{2}\left(\frac{W_n}{L_n}\right)V_{ov}^2(1 + \lambda_n(V_{DS} - V_{ov}))$

so we need to find λ_n from

$$\lambda_n = \lambda`_n/L_n = (40e-9)/(200e-9) = 0.2\text{V}^{-1}$$

resulting in

$$I_{Dn} = \frac{240e-6}{2}\left(\frac{2e-6}{200e-9}\right)0.3^2(1 + 0.2 \times (0.5 - 0.3))$$

$$I_{Dn} = 112.3\mu\text{A}$$

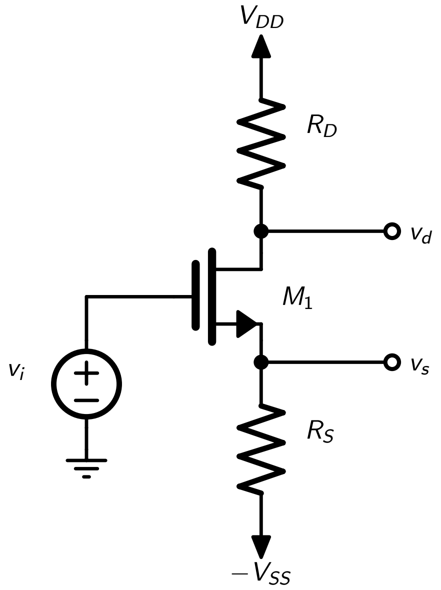
(b) $r_o = L_n/(\lambda`_n * I_{Dn}) = (200e-9)/((40e-9) * (112.3e-6)) = 44.52\text{k}\Omega$

(c) $\Delta I_{Dn} = \Delta V_{DS}/r_o = (0.4)/(44.52e3) = 8.986\mu\text{A}$

COMMENT: Note that if $\lambda_n = 0$ was used for part (a), it would only change the answers by less than 5% and this is often done for dc bias analysis. However, λ_n must then be taken into account for parts (b) and (c) otherwise gross errors would occur.

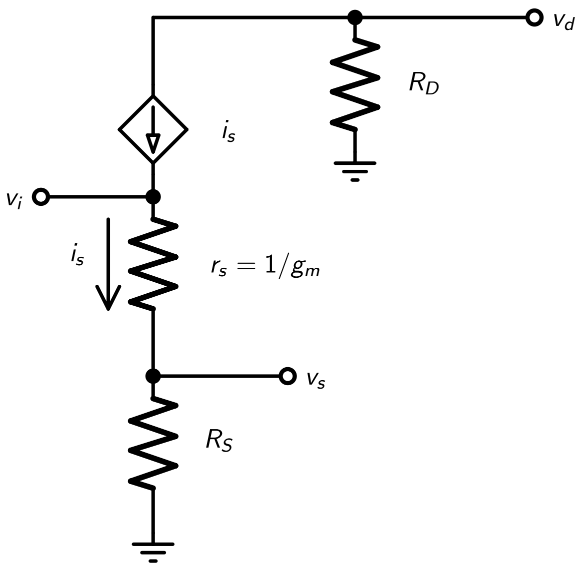
Question 3

For the NMOS amplifier below, replace the transistor with its T equivalent circuit and assume $\lambda = 0$. Derive expressions for small-signal voltage gains v_s/v_i and v_d/v_i given g_m for the transistor.



Solution

For small-signal analysis, all dc independent sources are set to zero. As a result, we have the following circuit



For v_s/v_i, there is a simple resistor divider between the applied voltage v_i and the measured output voltage v_s. As a result, we have

$$\frac{v_s}{v_i} = \frac{R_S}{R_S + r_s} = \frac{R_S}{R_S + (1/g_m)}$$

$$\frac{v_s}{v_i} = \frac{R_S}{R_S + (1/g_m)}$$

For v_d/v_i, we first find i_s and then since all of i_s goes through R_D, we can find v_d.

$$i_s = \frac{v_i}{r_s + R_S}$$

$$v_d = (-i_s) \times R_D$$

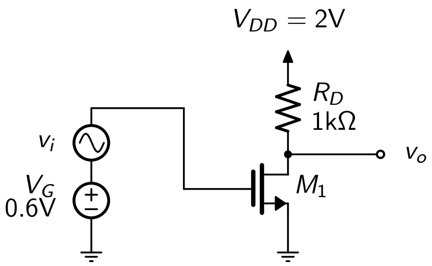
$$v_d = \frac{-v_i}{r_s + R_S} \times R_D$$

$$\frac{v_d}{v_i} = \frac{-R_D}{r_s + R_S} = \frac{-R_D}{R_S + (1/g_m)}$$

$$\frac{v_d}{v_i} = \frac{-R_D}{R_S + (1/g_m)}$$

Question 4

For the common-source amplifier shown below, find the small signal gain, v_o/v_i.



$V_{tn} = 0.3\text{V}$
$\mu_n C_{ox} = 240\mu\text{A/V}^2$
$W = 2\mu\text{m}$
$L = 200\text{nm}$
$\lambda`_n = 40\text{nm/V}$

Solution

We first need to find the dc operating values.

$$V_{GS} = V_G = (0.6) = 0.6\text{V}$$

$$V_{ov} = V_G - V_{tn} = (0.6) - (0.3) = 0.3\text{V}$$

and for dc analysis, we let $\lambda = 0$ so

$$I_D = 0.5 * \mu_n C_{ox} * (W/L) * {V_{ov}}^2 = 0.5 * (240e-6) * ((2e-6)/(200e-9)) * (0.3)^2 = 108\mu\text{A}$$

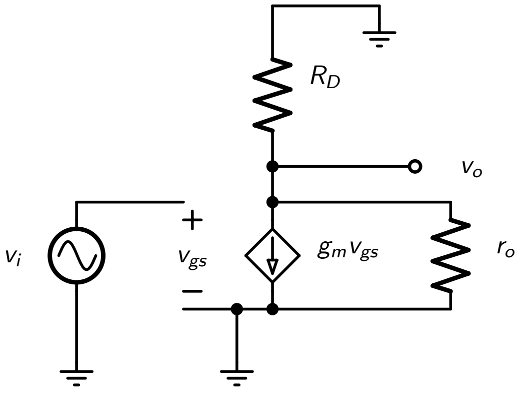
$$V_D = V_{DD} - I_D * R_D = (2) - (108e-6) * (1e3) = 1.892\text{V}$$

Since $V_{DS} > V_{ov}$, the transistor is indeed in the active region.
Now, we find the small signal parameters:

$$g_m = 2 * I_D / V_{ov} = 2 * (108e-6)/(0.3) = 720e-6$$

$$r_o = L/(\lambda`_n * I_D) = (200e-9)/((40e-9) * (108e-6)) = 46.3\text{k}\Omega$$

and we have the following small signal circuit



$$v_o = -g_m v_{gs} (R_D || r_o)$$

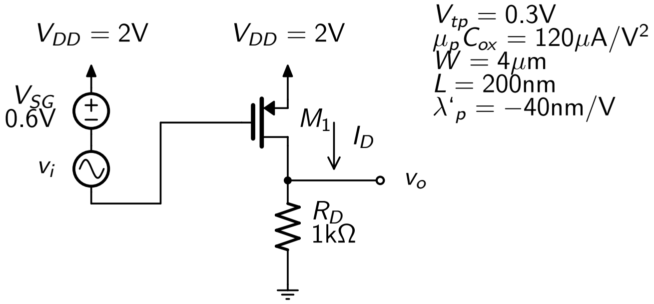
$$v_{gs} = v_i$$

$$v_o/v_i = -g_m(R_D||r_o)$$

$$v_o/v_i = -g_m * (R_D||r_o) = -(720e-6) * ((1e3)||(46.3e3)) = -0.7048\text{V/V}$$

Question 5

For the common-source PMOS amplifier shown below, find the small signal gain, v_o/v_i.



Solution

We first need to find the dc operating values.

$$V_{ov} = V_{SG} - |V_{tp}| = (0.6) - |(0.3)| = 0.3\text{V}$$

and for dc analysis, we let $\lambda = 0$ so

$$I_D = 0.5 * \mu_p C_{ox} * (W/L) * {V_{ov}}^2 = 0.5 * (120e-6) * ((4e-6)/(200e-9)) * (0.3)^2 = 108\mu\text{A}$$

$$V_D = I_D * R_D = (108e-6) * (1e3) = 0.108\text{V}$$

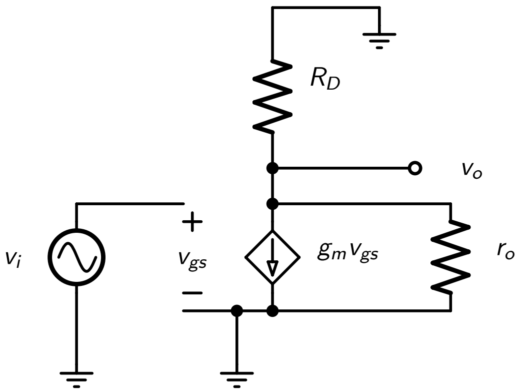
$$V_{SD} = V_{DD} - V_D = (2) - (0.108) = 1.892\text{V}$$

Since $V_{SD} > V_{ov}$, the transistor is indeed in the active region.
Now, we find the small signal parameters:

$$g_m = 2 * I_D/V_{ov} = 2 * (108e-6)/(0.3) = 720e-6$$

$$r_o = L/(|\lambda'_p| * I_D) = (200e-9)/(|(-40e-9)| * (108e-6)) = 46.3\text{k}\Omega$$

and we have the following small signal circuit



$$v_o = -g_m v_{gs}(R_D||r_o)$$

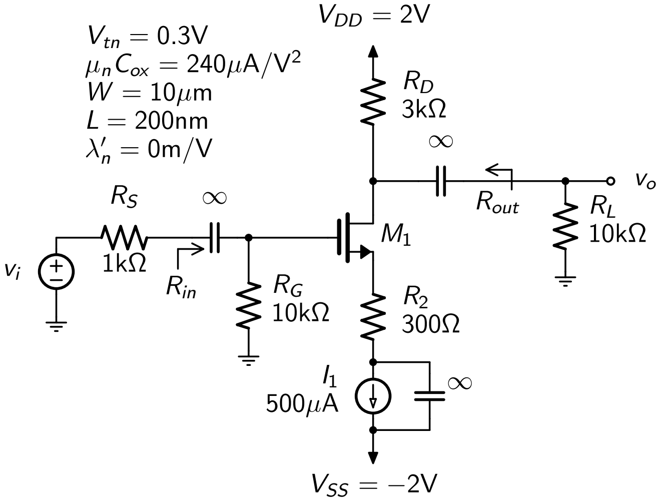
$$v_{gs} = v_i$$

$$v_o/v_i = -g_m(R_D||r_o)$$

$$v_o/v_i = -g_m * (R_D||r_o) = -(720e-6) * ((1e3)||(46.3e3)) = -0.7048\text{V/V}$$

Question 6

For the common-source amplifier shown below, find the small signal gain, v_o/v_i, R_{in} and R_{out}.



Solution

We begin by finding the dc operating values assuming M_1 is in the active region. Since we are given $\lambda = 0$, therefore

$$I_1 = I_D = 0.5\mu_n C_{ox}(W/L)V_{ov}^2$$

$$V_{ov} = \sqrt{I_1/(0.5\mu_n C_{ox}(W/L))} = 0.2887\text{V}$$

The gate voltage is given by $V_G = 0$ and since

$$V_{GS} = V_{ov} + V_{tn} = (0.2887) + (0.3) = 0.5887\text{V}$$

we have $V_S = V_G - V_{GS} = (0) - (0.5887) = -0.5887\text{V}$. We also have

$$V_D = V_{DD} - I_D * R_D = (2) - (500e-6) * (3e3) = 0.5\text{V}$$

So that

$$V_{DS} = V_D - V_S = (0.5) - (-0.5887) = 1.089\text{V}$$

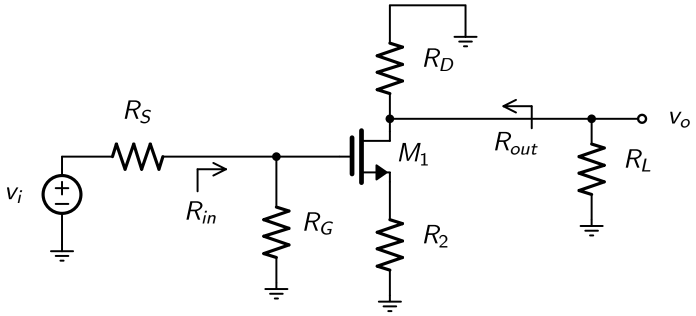
and since $V_{DS} > V_{ov}$, the transistor is indeed in the active region.
Now, we find the small signal parameters:

$$g_m = 2 * I_D/V_{ov} = 2 * (500e-6)/(0.2887) = 3.464e-3$$

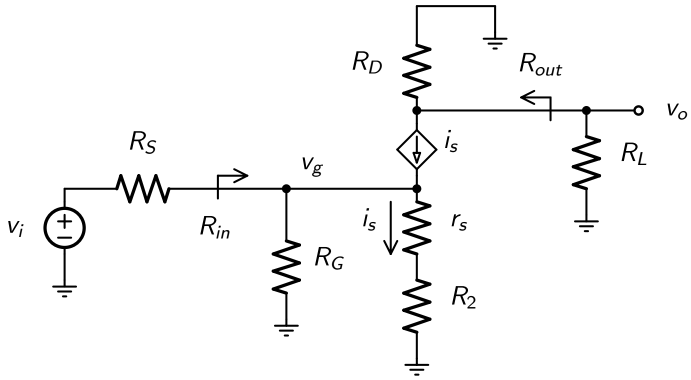
$$r_s = 1/g_m = 1/(3.464e-3) = 288.7\Omega$$

$$r_o = L/(\lambda'_n * I_D) \to \infty$$

and we have the following small signal circuit



and replacing M_1 with its small signal model, we have



Since there is no current going into the gate of the transistor,

$$R_{in} = R_G = (10e3) = 10\text{k}\Omega$$

For R_{out}, we set $v_i = 0$ which results in $i_s = 0$ so

$$R_{out} = R_D = (3e3) = 3\text{k}\Omega$$

Now we have

$$v_g/v_i = R_{in}/(R_{in} + R_S) = (10e3)/((10e3) + (1e3)) = 0.9091\text{V/V}$$

We also have

$$i_s = v_g/(r_s + R_2)$$

$$v_o = -i_s(R_D||R_L)$$

So

$$v_o/v_g = -(R_D||R_L)/(r_s + R_2) = -((3e3)||(10e3))/((288.7) + (300)) = -3.92\text{V/V}$$

So the overall gain is given by

$$v_o/v_i = (v_g/v_i) * (v_o/v_g) = ((0.9091)) * ((-3.92)) = -3.564\text{V/V}$$
