Chapter 3 - Problems

3.1) Nominal $I_{out} = \frac{25\,\mu m}{100\,\mu m} \times 80\,\mu A = \underline{20\,\mu A}$

$$\therefore \quad R_{out} = r_{DS2} = 8000 \times \frac{1.6\,\mu m}{0.02\,mA}$$
$$= \underline{640\,k\Omega}$$

Minimum output voltage $= V_{eff}$

$$= \sqrt{\frac{2 I_D}{\mu_n C_{ox} W/L}}$$
$$= \sqrt{\frac{2 \times 20 \times 10^{-6}}{92 \times 10^{-6} \times 25/1.6}}$$
$$= \underline{170\,mV}$$

3.2) DC gain, $A_V = -g_{m1}(r_{DS1} // r_{DS2})$

where $r_{DS1} = \frac{8000 L}{I_{bias}}$, $r_{DS2} = \frac{12000 L}{I_{bias}}$

$$\therefore \quad r_{DS1} // r_{DS2} = \frac{96000 L^2}{20000 L\, I_{bias}}$$
$$= \underline{\frac{4800 L}{I_{bias}}}$$

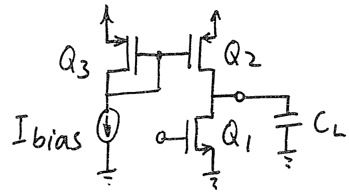
Also,

$$g_{m1} = \sqrt{2 \mu_n C_{ox} (W/L)_1 I_{bias}}$$

$$\therefore \quad \underline{A_V = -4800 \sqrt{\frac{2 \mu_n C_{ox} W L}{I_{bias}}}}$$

This result suggests that higher gains are obtained by using smaller bias currents. and/or larger transistor sizes in both width and length.

3.3) $\because$ C_L dominates the frequency response,



$$\omega_{3dB} \approx \frac{1}{R_{out} C_L}$$

for

$$R_{out} = r_{DS1} // r_{DS2}$$
$$= \frac{4800L}{I_{bias}} \quad \text{from Problem 3.2}$$

$$\therefore \ \underset{\sim\sim\sim\sim\sim\sim\sim\sim\sim\sim\sim\sim\sim\sim}{\omega_{3dB} = \frac{I_{bias}}{4800L \times C_L}}$$

Increasing the bias current speeds up the circuit.

3.4) $V_1(G_{in} + sC_{gs1} + sC_{gd1}) - V_{in}G_{in} - V_{out} \cdot sC_{gd1} = 0$ (3.101)

Isolating V_1 gives $V_1 = \dfrac{V_{in}G_{in} + V_{out}\, s\, C_{gd1}}{G_{in} + sC_{gs1} + sC_{gd1}}$ Ⓐ

sub Ⓐ $\rightarrow$ (3.102)

$$V_{out}(G_2 + sC_{gd1} + sC_2) - (sC_{gd1} - g_{m1}) \times \frac{V_{in}G_{in} + V_{out}\, s\, C_{gd1}}{G_{in} + sC_{gs} + sC_{gd1}} = 0$$

or $\gamma(s)\, V_{out} = G_{in}(sC_{gd1} - g_{m1})\, V_{in}$

where
$$\gamma(s) = (G_2 + s(C_{gd1} + C_2))(G_{in} + s(C_{gs1} + C_{gd1})) - sC_{gd1}(sC_{gd1} - g_{m1})$$
$$= G_2 G_{in} + s(G_2 C_{gs1} + G_2 C_{gd1} + G_{in}C_{gd1} + G_{in}C_2 + g_{m1}C_{gd1})$$
$$+ s^2(C_{gd1} + C_{gs1} + \cancel{C_{gd1}^2} + C_2 C_{gs1} + C_2 C_{gd1} - \cancel{C_{gd1}^2})$$
$$= G_2 G_{in}[1 + s(R_{in}(C_{gs1} + C_{gd1}(1 + g_{m1}R_2)) + R_2(C_{gd1} + C_2))$$
$$+ s^2 R_{in} R_2 (C_{gd1} C_{gs1} + C_{gs1} C_2 + C_{gd1} C_2)]$$

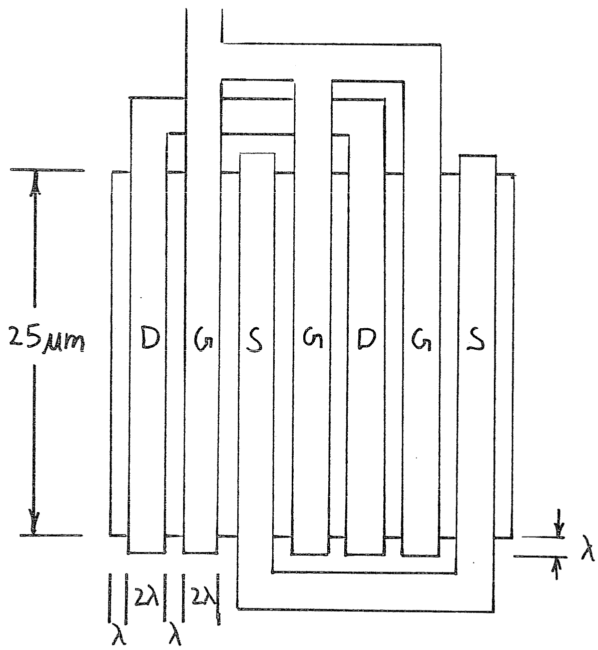
$$\therefore \frac{V_{out}}{V_{in}} = \frac{G_{in}(sC_{gd1} - g_{m1})}{\gamma(s)} = \frac{sC_{gd1} - g_{m1}}{G_2(1 + sa + s^2 b)} = \frac{-g_{m1}R_2\left(1 - s\frac{C_{gd1}}{g_{m1}}\right)}{1 + sa + s^2 b}$$

where
$$a = R_{in}[C_{gs} + C_{gd1}(1 + g_{m1}R_2)] + R_2(C_{gd1} + C_2)$$
$$b = R_{in}R_2(C_{gd1}C_{gs1} + C_{gs1}C_2 + C_{gd1}C_2)$$

Q.E.D.

3.5) * Assume minimum feature size, $\lambda = 0.8\,\mu m$ *

A sample layout of a transistor:



Estimating parasitic capacitances found in the layout above:

$$C_{gs1} = \frac{2}{3} W L C_{ox} + C_{gs\text{-}ov} W$$
$$= \frac{2}{3} \times 75 \times 1.6 \times 1.9 \times 10^{-15} + 0.2 \times 10^{-15} \times 75$$
$$= \underline{167\,fF}$$

$$C_{gd1} = W\,C_{gd\text{-}ov} = 75 \times 0.2 \times 10^{-15}$$
$$= \underline{15\,fF}$$

For C_{db1} & C_{db2}, we need to first calculate C_{jd} and C_{jsw}.
(assume $V_{SB} = V_{DB} = 5V - 2.5V = 2.5V$)

$$C_{jd1} = \frac{C_{jd0}}{\sqrt{1 + V_{DB}/\Phi_0}} = \frac{2.4 \times 10^{-4}}{\sqrt{1 + 2.5/0.9}} = 1.23 \times 10^{-4}\ pF/\mu m^2$$

$$C_{jsw1} = \frac{C_{jsw0}}{\sqrt{1 + V_{SB}/\Phi_{sw}}} = \frac{2.0 \times 10^{-4}}{\sqrt{1 + 2.5/0.9}} = 1.03 \times 10^{-4}\ pF/\mu m$$

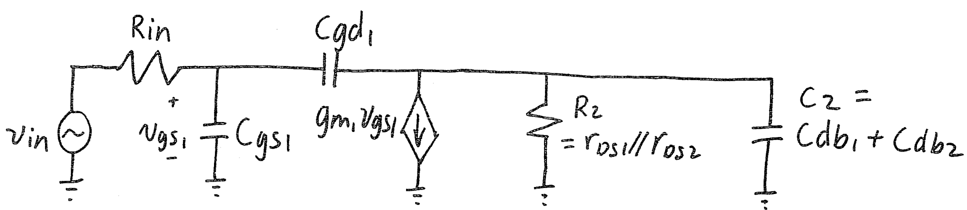
(cont.)

3.5) (cont.) Similarly, $C_{jd2} = 2.3 \times 10^{-4}\ pf/\mu m^2$ and $C_{jsw2} = 1.29 \times 10^{-4}\ pf/\mu m$

$$\therefore\ C_{db1} = A_{d1} C_{jd1} + P_d C_{jsw1}$$
$$= 2 \times 25 \times 4 \times 0.8 \times 1.23 \times 10^{-4} + (25 + 4 \times 4 \times 0.8) \times 1.03 \times 10^{-4}$$
$$= \underline{24 fF}$$

$$C_{db2} = A_{d2} C_{jd2} + P_{d2} C_{jsw2}$$
$$= 2 \times 25 \times 4 \times 0.8 \times 2.3 \times 10^{-4} + (25 + 4 \times 4 \times 0.8) \times 1.29 \times 10^{-4}$$
$$= \underline{42 fF}$$

Small-signal model:



$$\omega_{3dB} \cong \frac{1}{R_{in}[C_{gs1} + C_{gd1}(1 + g_{m1} R_2)] + R_2 (C_{gd1} + C_2)}$$

where

$$R_2 = r_{os1} // r_{os2} = \frac{4800 L}{I_{bias}} \text{ from Problem 3.2}$$
$$= \frac{4800 \times 1.6}{75 \mu A}$$
$$= \underline{102\ k\Omega}$$

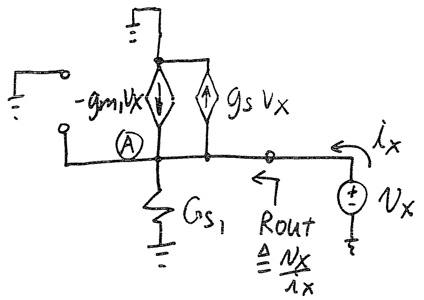
$$g_{m1} = \sqrt{2 \mu_n C_{ox} W/L\ I_{bias}} = \sqrt{2 \times 92 \times 10^{-6} \times 75/1.6 \times 75 \times 10^{-6}}$$
$$= 0.804\ mA/V$$

If we assume $R_{in} \approx R_2 = 102 k\Omega$,

$$\omega_{3dB} \cong \frac{1}{102 \times 10^3 [167 + 15(1 + 0.804 \times 10^{-3} \times 102 \times 10^3)] \times 10^{-15} + 102 \times 10^3 \times 10^{-15} \times (15 + 24 + 42)}$$

$$= 6.57 \times 10^6 \text{ rads/sec} = \underline{2\pi \times 1.05 \text{ MHz}}$$

3.6)



Summing currents at node A :

$$i_x - g_{m1} v_x - g_s v_x - v_x G_{S1} = 0$$

$$\therefore G_{out} = 1/R_{out} = i_x/v_x$$
$$= g_{m1} + g_s + G_{S1}$$

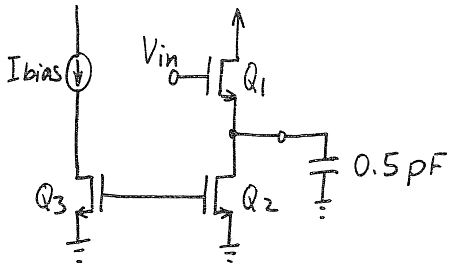
where $G_{S1} = g_{DS1} + g_{DS2}$

or $R_{out} = [g_{m1} + g_s + \overbrace{g_{DS1} + g_{DS2}}^{G_{S1}}]^{-1} \cong \frac{1}{g_{m1} + g_s}$

3.7) Following the same analysis as in Problem 3.6, this time ignoring R_{S1} (i.e., G_{S1}), the impedance looking into the source is

$$R_{out} = [g_{m1} + g_s]^{-1}$$

3.8)



Find ω_o, Q and zero frequency.

a) $g_{m1} = \sqrt{2 \mu_n C_{ox} \frac{W}{L} I_{bias}} = \sqrt{2 \times 92 \times 10^{-6} \times \frac{100}{1.6} \times 100 \times 10^{-6}} = 1.06 \frac{mA}{V}$

$r_{DS1} = r_{DS2} = \frac{8000 L}{I_{bias}} = \frac{8000 (1.6 \times 10^{-6})}{100 \times 10^{-6}} k\Omega = 128 k\Omega$

$g_{s1} = \frac{\gamma g_m}{2\sqrt{V_{SB} + |2\Phi_F|}} \approx \frac{0.5 g_m}{2\sqrt{2+0.7}} = 0.16 \ mA/V$

$G_{in} = [180 \times 10^3]^{-1} = 5.56 \times 10^{-6} \ \mho$

$G_{s1} = g_{s1} + g_{DS1} + g_{DS2} = 0.176 \ mA/V$

$C_s = C_L + C_{sb1} = 0.5 + 0.04 \ pF = 0.54 pF$

$C_{in}' = C_{in} + C_{gd1} = 45 fF$

$\therefore \omega_o = \sqrt{\frac{G_{in}(g_{m1} + G_{s1})}{C_{gs1} C_s + C_{in}'(C_{gs1} + C_s)}}$ where $C_{gs1} = 0.2pF$

$= 2.21 \times 10^9$ rads/sec $= 2\pi \times 35$ MHz

$Q = \frac{\sqrt{G_{in}(g_{m1} + G_{s1})[C_{gs1}C_s + C_{in}'(C_{gs1} + C_s)]}}{G_{in} C_s + C_{in}'(g_{m1} + G_{s1}) + C_{gs1} G_{s1}}$

$= \frac{3.1149 \times 10^{-17}}{9.382 \times 10^{-17}} = 0.332$

$\omega_z = -\frac{g_{m1}}{C_{gs1}} = 5.3 \times 10^9$ rads/sec $= 2\pi \times 844$ MHz

(cont.)

3.8) (cont.)

b) Source is connected to the substrate.
Assume C_{gs}, C_{gd1}, and C_{sb1} remain unchanged, and that the only difference is the elimination of transconductance, g_{s1}.

$\therefore$ All parameters unchanged except

$$G_{S1} = g_{DS1} + g_{DS2} = 15.6\ \mu A/V$$

$$\therefore\ \omega_0 = \sqrt{\frac{5.976\times10^{-9}}{1.413\times10^{-25}}} = 2.06\times10^{8}\ \frac{rads}{sec} = \underline{2\pi\times 33\,MHz}$$

$$Q = \frac{2.906\times10^{-17}}{5.140\times10^{-17}} = \underline{0.57}$$

$$\omega_z = \underline{2\pi\times 844\,MHz}$$ (unchanged from part a))

As expected, the elimination of the body effect increases Q.

3.9) a) $$C_1 = \frac{g_{m1} C_{gs1} C_s}{(g_{m1}+G_{s1})(C_{gs1}+C_s)} = \frac{(1.06\times10^{-3})(0.2\times10^{-12})(0.54\times10^{-12})}{(1.06\times10^{-3}+0.176\times10^{-3})(0.2+0.54)\times10^{-12}}$$

$$= \frac{1.145\times10^{-28}}{9.146\times10^{-16}} = \underline{0.125\,pF}$$

$$R_1 = \frac{(C_{gs1}+C_s)^2}{C_{gs1} C_s g_{m1}} = \frac{(0.74\times10^{-12})^2}{0.2\times0.54\times10^{-24}\times1.06\times10^{-3}}$$

$$= \frac{5.476\times10^{-25}}{1.145\times10^{-28}} = \underline{4780\,\Omega}$$

$$P_1 \cong \frac{G_{in}}{C_{gs1}+C_{in}} = \underline{2\pi\times3.61\,MHz}$$ (unchanged from Ex. 3.10)

$$p_2 = \frac{g_{m1}+G_{s1}}{C_{gs}+C_L} = \frac{1.236\times10^{-3}}{0.7\times10^{-12}} = \underline{2\pi\times281\,MHz}$$

(cont.)

3.9) b) Now $G_{S1} = g_{DS1} + g_{DS2} = 15.6\ \mu A/V$

With all other values unchanged,

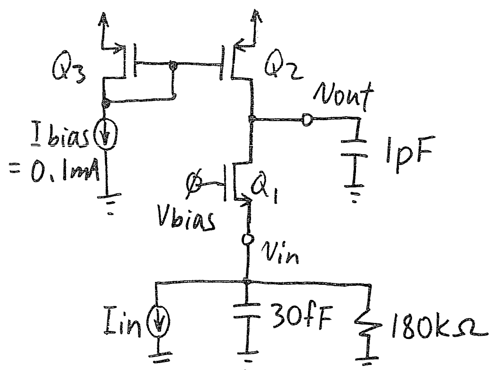
$$C_1 = \frac{1.145\times10^{-28}}{1.075\times10^{-3}\times0.74\times10^{-12}} = \underline{0.144\ pF}$$

$R_1 = \underline{4780\ \Omega}$ (unchanged from part a))

$p_1 = \underline{2\pi \times 3.61\ MHz}$ (same as in part a))

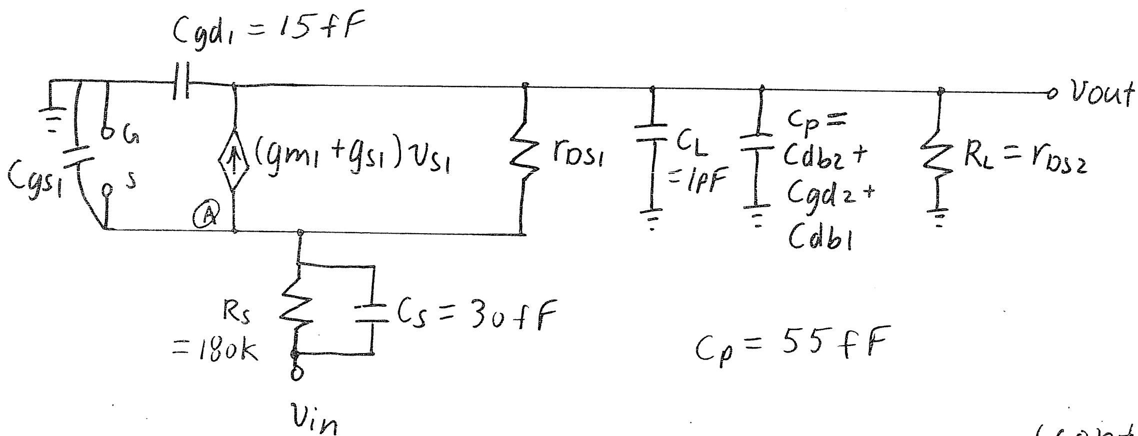
$$p_2 = \frac{1.075\times10^{-3}}{0.7\times10^{-12}} = \underline{244\ MHz \times 2\pi}$$

3.10)



Find the -3dB frequency.

Small-signal model:



$C_p = 55fF$

(cont.)

$$r_{ds1} = \frac{8000\times1.6}{0.1} = 128\ k\Omega \qquad r_{ds2} = \frac{12{,}000\times1.6}{0.1} = 192\ k\Omega$$

3.10) (cont.)

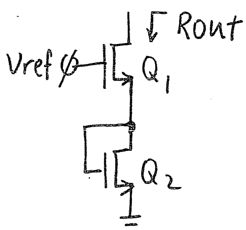
using the open-circuit time-constant approach, the output node dominates.

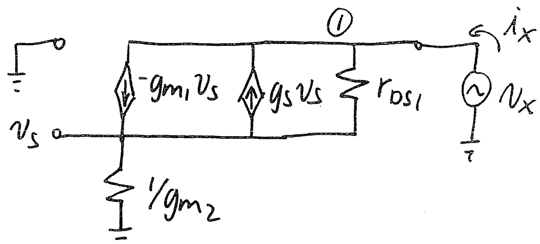
$$\omega_{3dB} \cong \frac{1}{[r_{ds2} \,//\, (r_{ds1} + R_S)][C_L + C_p + C_{gd1}]}$$

$$= \frac{1}{(118.3k\Omega)(1.07pF)} = 2\pi \times 1.26\,MHz$$

Roughly $f_{3dB} = \underline{1.3\,MHz}$

3.11)





Use test voltage, v_x, to find Rout.

K.C.L. at node ① :

$$-g_{m1} v_s - g_{s1} v_s - i_x + v_x g_{os1} - v_s g_{os1} = 0$$

$$-v_s(g_{m1} + g_{s1} + g_{os1}) - i_x + v_x g_{os1} = 0 \quad Ⓐ$$

K.C.L. at v_s :

$$v_s g_{m2} + (g_{s1} + g_{m1}) v_s + (v_s - v_x) g_{os1} = 0$$

$$v_s(g_{m1} + g_{m2} + g_{s1} + g_{os1}) - v_x g_{os1} = 0$$

$$\therefore \quad v_s = \frac{g_{os1}}{g_{m1} + g_{m2} + g_{s1} + g_{os1}} v_x \quad Ⓑ$$

(cont.)

3.11 (cont.)

sub Ⓑ → Ⓐ

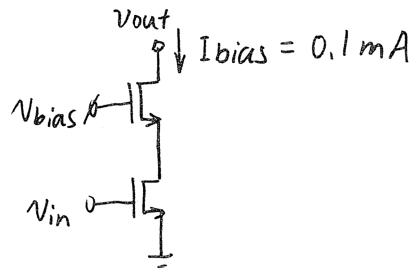
$$\left[-\frac{g_{DS1}(g_{m1}+g_{s1}+g_{DS1})}{(g_{m1}+g_{s1}+g_{DS1}+g_{m2})}+g_{DS1}\right]v_x = i_x$$

$$\therefore\ R_{out} \triangleq \frac{v_x}{i_x} = \underset{\sim\sim\sim\sim\sim\sim\sim\sim}{\frac{g_{m1}+g_{s1}+g_{DS1}+g_{m2}}{g_{DS1}\,g_{m2}}}$$

$$= r_{DS1} + \frac{g_{m1}+g_{s1}}{g_{m2}} \times r_{DS2} + 1/g_{m2}$$

$\therefore$ $\underset{\sim\sim\sim\sim\sim}{R_{out} \approx 2\, r_{DS1}}$ which is consistent with the value obtained using the source degeneration formula.

3.12)



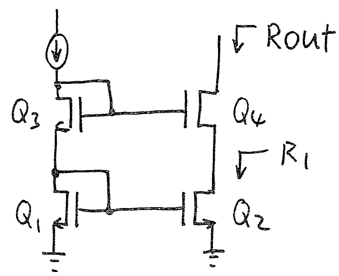
From (3.40),

$$V_{out} > V_{DS2} + V_{eff} = 2\,V_{eff1} + V_{tn1}$$

$$\text{where } V_{eff1} = \sqrt{\frac{2\,I_{bias}}{\mu_n C_{ox} W/L}} = 0.264\,V$$

$$\therefore\ V_{out} \geq 2 \times 0.264 + 0.8\,V = \underset{\sim\sim\sim}{1.33V}$$

3.13)

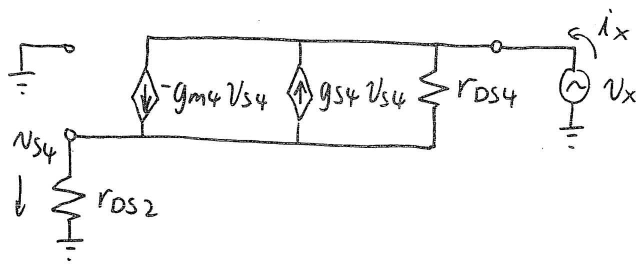


Find Rout,

From section 3.1,

$$R_1 = r_{DS2}$$

Giving us the resulting small signal model :



KCL at output :

$$-(g_{m4} + g_{s4}) v_{s4} + g_{DS4} v_x - g_{DS4} v_{s4} - i_x = 0$$

$$\therefore \quad (g_{m4} + g_{s4} + g_{DS4}) v_{s4} + i_x = g_{DS4} v_x$$

But $\quad v_{s4} = i_x / g_{DS2}$

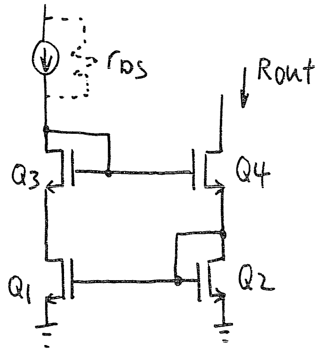
$$\therefore \quad i_x \left[(g_{m4} + g_{s4} + g_{DS4}) \times 1/g_{DS2} + 1 \right] = g_{DS4} v_x$$

$$\therefore \quad R_{out} \triangleq \frac{v_x}{i_x} = \frac{1}{g_{DS4}} \left[1 + \frac{1}{g_{DS2}} (g_{m4} + g_{s4} + g_{DS4}) \right]$$

$$\underline{R_{out} = r_{DS4} \left[1 + r_{DS2} (g_{m4} + g_{s4} + g_{DS4}) \right]}$$

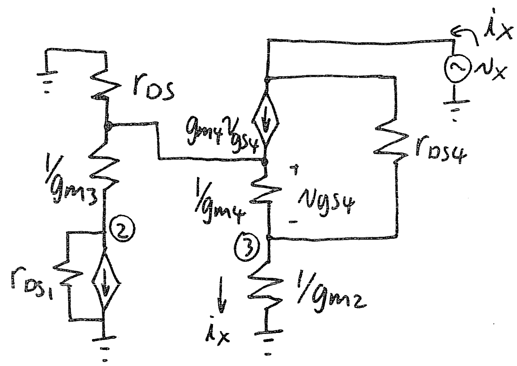
This result is consistent with that obtained using source degeneration equation (3.29).

3.14)



Find Rout

Method 1 : Nodal Analysis



$$v_{gs4} = v_{g4} - v_{s4} = v_{②} \times \frac{r_{DS}}{1/g_{m3} + r_{DS}} - v_{③}$$

But $r_{DS} >> 1/g_{m3}$

$$\therefore \quad \frac{r_{DS}}{1/g_{m3} + r_{DS}} \approx 1 \quad \text{and}$$

$$v_{gs4} \approx v_{②} - v_{③}$$

KCL at output :

$$-i_x + g_{m4} v_{gs4} + g_{DS4} (v_x - v_{③}) = 0$$

$$-i_x + g_{m4} (v_{②} - v_{③}) + g_{DS4} (v_x - v_{③}) = 0$$

$$\therefore \; -i_x - v_{③} (g_{m4} + g_{DS4}) + g_{m4} v_{②} + g_{DS4} v_x = 0 \quad Ⓐ$$

But $v_{③} = i_x / g_{m2}$ and

$$v_{②} = - g_{m1} v_{③} \times [r_{DS1} // (1/g_{m3} + r_{DS})]$$

$$\approx - \frac{g_{m1} r_{DS1}}{2 g_{m2}} \times i_x \quad Ⓑ$$

(cont.)

3.14 (cont.)

sub (B) $\rightarrow$ (A)

$$-i_x - \frac{1}{g_{m2}} \times (g_{m4} + g_{os4})\, i_x - \frac{g_{m1} g_{m4} r_{DS1}}{2 g_{m2}}\, i_x + g_{os4} v_x = 0$$

$$g_{os4} v_x = i_x \left(1 + \frac{g_{m4} + g_{os4}}{g_{m2}} + \frac{g_{m1} g_{m4}}{2 g_{m2}} \times r_{DS1}\right)$$

$$\therefore\ R_{out} \triangleq \frac{v_x}{i_x} \approx r_{os4} \times \left(2 + \frac{g_{m1}}{2} r_{os1}\right)$$

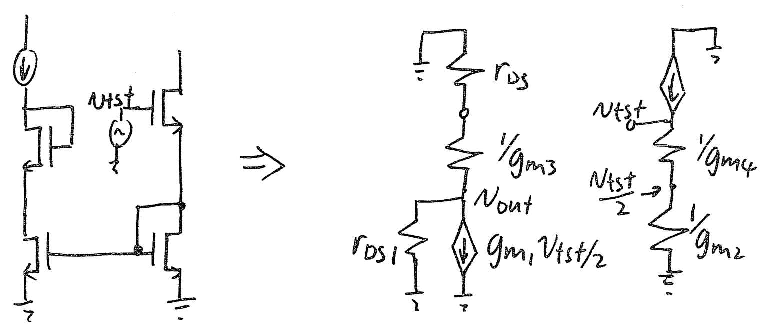
$$\because\ \frac{g_{m1} r_{DS1}}{2} \gg 2$$

$$\therefore\ \underbrace{R_{out} \approx r_{os4}\left(\frac{g_{m1} r_{DS1}}{2}\right)}\qquad \text{Q.E.D.}$$

Method 2: Feedback Analysis

Rout = (1 − Aβ) Rout (open loop)

where Aβ is the loop gain. To determine Aβ, break the feedback loop and apply a test voltage as follows:



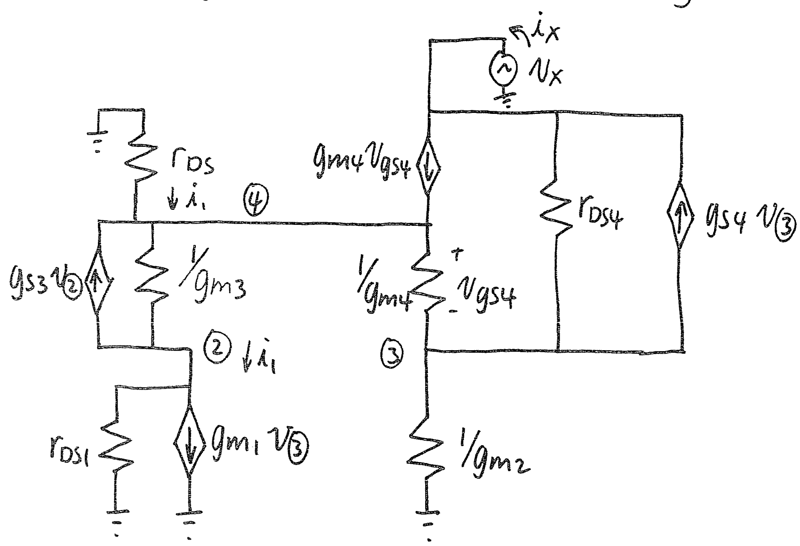
$$v_{out} = -g_{m1} \times \frac{v_{tst}}{2} \times (r_{DS1} \,//\, r_{DS})$$

$$\therefore\ \text{Loop gain} = A\beta = -\frac{g_{m1} r_{DS1}}{4}$$

$$\therefore\ R_{out} = \left(1 + \frac{g_{m1} r_{DS1}}{4}\right) \times 2 r_{DS4}$$

$$\therefore\ \underbrace{R_{out} \approx \left(\frac{g_{m1} r_{DS1}}{2}\right) \times r_{DS4}}\qquad \text{Q.E.D.}$$

3.15) Small signal model incorporating the body effect :



$$v_{gs4} = v_4 - v_3$$

$$v_4 = v_2 + (i_1 + g_{s3}v_2)\,1/g_{m3} \qquad \boxed{A}$$

$$i_1 = -g_{os}v_4 \qquad \boxed{B}$$

$\boxed{B} \rightarrow \boxed{A}$

$$\therefore\ v_4\left(1+\frac{g_{os}}{g_{m3}}\right) = \left(1+\frac{g_{s3}}{g_{m3}}\right)v_2$$

$$v_4 = \frac{g_{m3}+g_{s3}}{g_{m3}+g_{os}}\,v_2$$

$$v_4 \approx \left(1+\frac{g_{s3}}{g_{m3}}\right)v_2$$

$$\therefore\ v_{gs4} \cong \left(1+\frac{g_{s3}}{g_{m3}}\right)v_2 - v_3$$

new term due to body effect.

KCL at output :

$$-i_x + g_{m4}v_{gs4} + g_{os4}(v_x - v_3) - g_{s4}v_3 = 0$$

$$-i_x + g_{m4}\left(\left(1+\frac{g_{s3}}{g_{m3}}\right)v_2 - v_3\right) + g_{os4}v_x - v_3(g_{os4}+g_{s4}) \approx 0$$

$$-i_x - v_3(g_{m4}+g_{os4}+g_{s4}) + g_{m4}\left(1+\frac{g_{s3}}{g_{m3}}\right)v_2 + g_{os}v_x \approx 0 \quad \boxed{C}$$

new terms

(cont.)

3.15 (cont.)

But $v_3 = i_x/g_{m2}$ $\boxed{D}$

$v_2 = (i_1 - g_{m1} v_3)\, r_{DS1}$

$= -(v_4\, g_{DS} + g_{m1} v_3)\, r_{DS1}$

$= -[(1 + g_{S3}/g_{m3})\, g_{DS}\, v_2 + g_{m1} v_3]\, r_{DS1}$

$\therefore\ v_2 = -\dfrac{g_{m1} r_{DS1}}{2 + g_{S3}/g_{m3}}\, v_3$ $\boxed{E}$

$\boxed{D}, \boxed{E} \rightarrow \boxed{C}$

$$-i_x - \frac{i_x}{g_{m2}}\left(g_{m4}\left(1+\left(1+\frac{g_{S3}}{g_{m3}}\right)\frac{g_{m1} r_{DS1}}{2+g_{S3}/g_{m3}}\right) + g_{DS4} + g_{S4}\right) + g_{DS4} v_x \approx 0$$

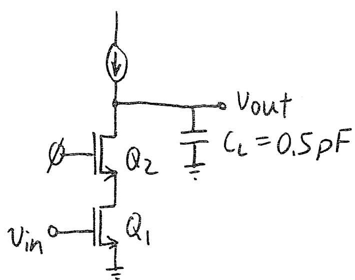
$$\therefore\ R_{out} \triangleq \frac{v_x}{i_x} \cong r_{DS4}\left[1 + \frac{g_{m4}}{g_{m2}}\left(1+\left(1+\frac{g_{S3}}{g_{m3}}\right)\frac{g_{m1} r_{DS1}}{2+g_{S3}/g_{m3}}\right) + \frac{g_{S4}}{g_{m2}} + \frac{g_{DS4}}{g_{m2}}\right]$$

$$\approx r_{DS4}\left[2 + \left(1+\frac{g_{S3}}{g_{m3}}\right)\frac{g_{m1} r_{DS1}}{2+g_{S3}/g_{m3}} + \frac{g_{S4}}{g_{m2}}\right]$$

$$R_{out} \approx r_{DS4}\left(1+\frac{g_{S3}}{g_{m3}}\right) \times \frac{g_{m1} r_{DS1}}{2+g_{S3}/g_{m3}} \rightarrow r_{DS4}\left(\frac{g_{m1} r_{DS1}}{2}\right) \text{ as } g_{S3} \to 0$$

Q.E.D.

3.16)



$C_{S2} = 0.26\ pF$

$C_{d2} = C_{gd2} + C_{db2} + C_L + C_{bias}$

$= 15 + 20 + 500 + 20\ fF$

$= 555\ fF$

$\therefore\ \tau_{cgs1} = 36\,nsec$

$\tau_{cgd1} = 75\,nsec$ } unchanged

$\tau_{cs2} = 13\,nsec$

$\tau_{cd2} = C_{d2} \dfrac{g_m r_{ds}^2}{2} = 555 \times 10^{-15} \times \dfrac{10^{-3} \times 10^{10}}{2} = 2.8\,\mu sec$

$\because\ \tau_{cd2} \gg \tau_{cgd1}, \tau_{cgs1}, \text{ and } \tau_{cs2}$

$\therefore\ \omega_{-3dB} \approx \dfrac{1}{\tau_{cd2}} = 2\pi \times 57\,kHz$

3.17) Derive ω_{0dB}

From (3.161)

$$|A(s)| \approx \frac{g_{m1}}{sC_L} \quad \text{for } s = j\omega,\ \omega \gg \omega_{-3dB}$$

for the unity gain frequency

$$|A(s)|\Big|_{s=j\omega_{0dB}} \approx \frac{g_{m1}}{\omega_{0dB} C_L} \equiv 1$$

$$\therefore\ \omega_{0dB} = \frac{g_{m1}}{C_L}$$

Assuming $C_L = 2\,pF$,

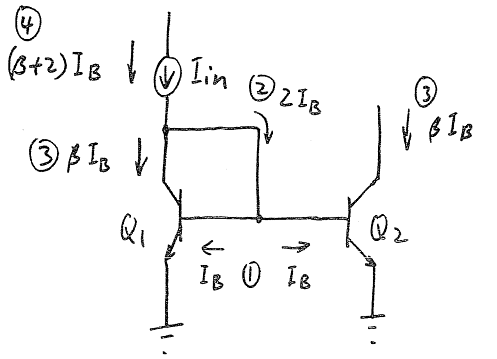
$$\omega_{0dB} = \frac{1\,mA/V}{2\,pF} = 5 \times 10^8\ rads/sec = 2\pi \times 80\,MHz$$

In order to use (3.161), we need to verify that $\omega_{0dB} \ll \omega_{p2}$. From (3.165),

$$\omega_{p2} \approx \frac{g_{m2}}{C_{s2}} = \frac{1\,mA/V}{0.26\,pF} = 2\pi \times 610\,MHz \gg \omega_{0dB}\ ✓$$

$\therefore$ assumption is valid.

3.18) Derive current gain. For $\beta >> 1$, show $\frac{I_{out}}{I_{in}} \approx 1 - 2/\beta$



* Numbers indicate process of reasoning or analysis *.

① Because $V_{be1} = V_{be2}$

$\therefore \quad I_{B1} = I_{B2} \triangleq I_B$

From above analysis,

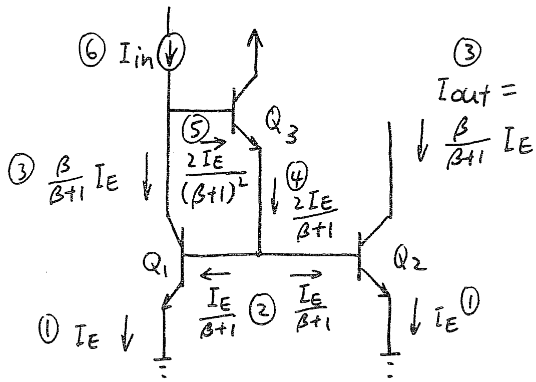
$$I_{out} = \beta I_B$$
$$I_{in} = (\beta+2) I_B$$

$$\therefore \frac{I_{out}}{I_{in}} = \frac{\beta \cancel{I_B}}{(\beta+2)\cancel{I_B}} = \frac{\beta+2}{\beta+2} - \frac{2}{\beta+2}$$

$$\therefore \frac{I_{out}}{I_{in}} \approx 1 - \frac{2}{\beta} \quad \text{if } \beta >> 1$$

Q.E.D.

3.19) Show $\frac{I_{out}}{I_{in}} \approx 1 - 2/\beta^2$ for $\beta \gg 1$



* Numbers indicate process of analysis *

① $\because V_{be1} = V_{be2}$

$\therefore I_{E1} = I_{E2} \triangleq I_E$

③ $I_{out} = \frac{\beta}{\beta+1} I_E$

⑥ $I_{in} = \frac{\beta}{\beta+1} I_E + \frac{2}{(\beta+1)^2} I_E$

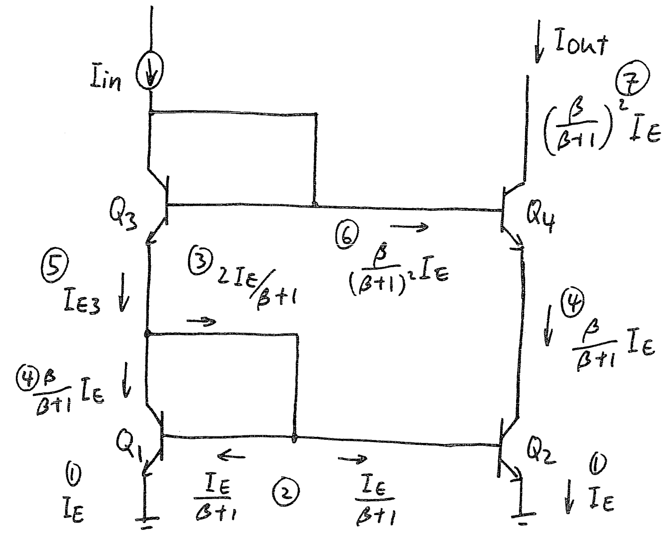
$$\therefore \frac{I_{out}}{I_{in}} = \frac{\beta/\beta+1}{\frac{\beta}{\beta+1} + 2/(\beta+1)^2} = \frac{\beta(\beta+1)}{\beta(\beta+1)+2}$$

$$= \frac{\beta(\beta+1)+2}{\beta(\beta+1)+2} - \frac{2}{\beta(\beta+1)+2} = 1 - \frac{2}{\beta^2+\beta+2}$$

$$\therefore \underbrace{\frac{I_{out}}{I_{in}} \approx 1 - \frac{2}{\beta^2}}\ \text{for } \beta \gg 1$$

Q.E.D.

3.20) Show $\frac{I_{out}}{I_{in}} \simeq 1 - 4/\beta$ for $\beta >> 1$



① $\because V_{be1} = V_{be2}$,

$\therefore I_{E1} = I_{E2} \triangleq I_E$

⑦ $I_{out} = \left(\frac{\beta}{\beta+1}\right)^2 I_E$

⑤ $I_{E3} = \frac{\beta}{\beta+1} I_E + \frac{2}{\beta+1} I_E = \left(\frac{\beta+2}{\beta+1}\right) I_E$

$\therefore I_{B3} = \frac{\beta+2}{(\beta+1)^2} I_E \quad , \quad I_{C3} = \frac{\beta(\beta+2)}{(\beta+1)^2} I_E$

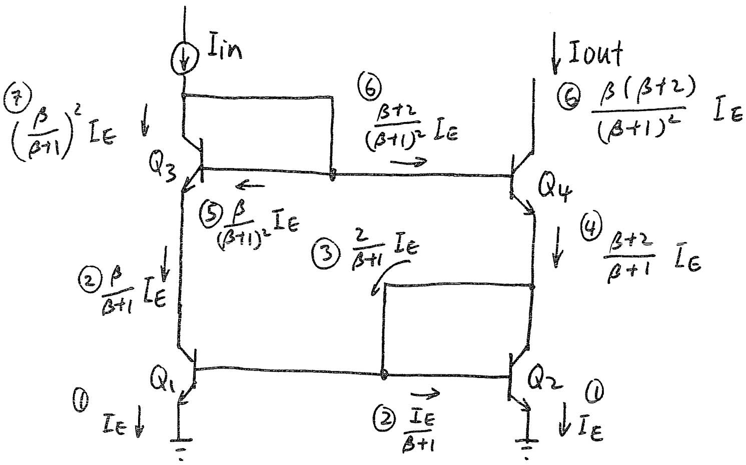
$$I_{in} = I_{C3} + I_{B3} + I_{B4}$$
$$= \left[\frac{(\beta+2)\beta}{(\beta+1)^2} + \frac{\beta+2}{(\beta+1)^2} + \frac{\beta}{(\beta+1)^2}\right] I_E$$
$$= \frac{1}{(\beta+1)^2}\left[\beta^2 + 4\beta + 2\right] I_E$$

$$\therefore \frac{I_{out}}{I_{in}} = \frac{\beta^2}{\beta^2+4\beta+2} = \frac{\beta^2+4\beta+2}{\beta^2+4\beta+2} - \frac{4\beta+2}{\beta^2+4\beta+2}$$

$$\therefore \frac{I_{out}}{I_{in}} \approxeq 1 - \frac{4\beta}{\beta^2} = 1 - 4/\beta$$

Q.E.D.

3.21) Show $\frac{I_{out}}{I_{in}} = 1 - \frac{2}{\beta^2 + 2\beta + 2}$



① $\because V_{be1} = V_{be2}$

$\therefore I_{E1} = I_{E2} \triangleq I_E$

From the above schematic analysis,

$$I_{in} = I_{C3} + I_{B3} + I_{B4} = \left[\left(\frac{\beta}{\beta+1}\right)^2 + \frac{\beta}{(\beta+1)^2} + \frac{\beta+2}{(\beta+1)^2}\right] I_E$$

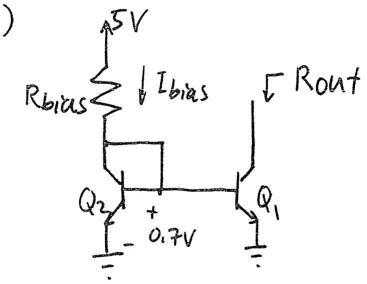
$$I_{out} = \frac{\beta(\beta+2)}{(\beta+1)^2} I_E$$

$$\therefore \frac{I_{out}}{I_{in}} = \frac{\beta(\beta+2)}{\beta^2+\beta+\beta+2} = \frac{\beta^2+2\beta}{\beta^2+2\beta+2}$$

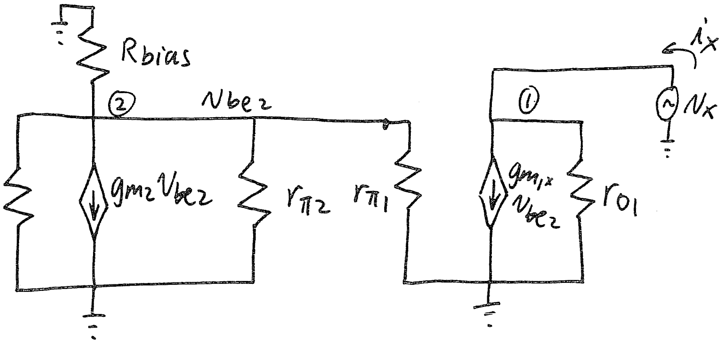
$$\therefore \frac{I_{out}}{I_{in}} = \frac{\beta^2+2\beta+2}{\beta^2+2\beta+2} - \frac{2}{\beta^2+2\beta+2} = 1 - \frac{2}{\beta^2+2\beta+2}$$

Q.E.D.

3.22)



$\Rightarrow$



KCL at ①:

$$-i_x + g_{m1} v_{be2} + v_x / r_{o1} = 0 \qquad \boxed{A}$$

KCL at ②: Let $R_T \triangleq R_{bias} // r_{o2} // r_{\pi 2} // r_{\pi 1}$

$$\frac{\cancel{v_{be2}}}{R_T} + g_{m2} \cancel{v_{be2}} = 0 \quad \text{which implies}$$

$$\underline{R_T = -1/g_{m2}} \quad \text{or} \quad \underline{\underline{v_{be2} = 0}} \checkmark$$

X (impossible as $R_T \geq 0\,\Omega$)

$\therefore \boxed{A}$ $\quad -i_x + v_x / r_{o1} = 0$

$$\Rightarrow \underline{R_{out} \triangleq \frac{v_x}{i_x} = r_{o1}}$$

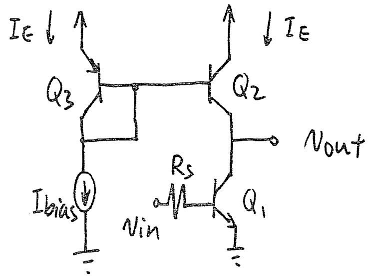
For $I_{bias} = 0.2\,mA$, assuming $V_{be2} \approx 0.7V$

$$R_{bias} = \frac{5 - 0.7V}{0.2mA} = \underline{21.5k\Omega}$$

And

$$R_o = r_{o1} = \frac{V_A}{I_C} = \frac{80V}{0.2mA} = \underline{400k\Omega}$$

3.23)



Find A_v

$$I_{bias} = \frac{\beta+2}{\beta+1} I_E \Rightarrow I_E = \frac{\beta+1}{\beta+2} I_{bias}$$

$$I_{C1} = \frac{\beta}{\beta+1} I_E = \frac{\beta}{\beta+2} I_{bias}$$

$$\underbrace{g_{m1} = \frac{I_{C1}}{V_T} = \frac{\beta}{\beta+2}\frac{I_{bias}}{V_T}}$$

$$A_v \triangleq \frac{v_{out}}{v_{in}} = -g_{m1}(r_{o1} \,//\, r_{o2}) \times \frac{r_{\pi 1}}{r_{\pi 1}+R_S}$$

where $r_{\pi 1} = \beta / g_{m1}$

$$= (\beta+2)\, V_T / I_{bias}$$

$$r_{o1} \approx r_{o2} = \frac{V_A}{I_{C2}} = \frac{V_A}{\left(\frac{\beta}{\beta+2}\right) I_{bias}} = \frac{\beta+2}{\beta}\frac{V_A}{I_{bias}}$$

$$\therefore r_{o1} \,//\, r_{o2} = \frac{\beta+2}{2\beta}\frac{V_A}{I_{bias}}$$

$$\therefore A_v = -g_{m1}(r_{o1} \,//\, r_{o2}) \frac{r_{\pi 1}}{r_{\pi 1}+R_S}$$

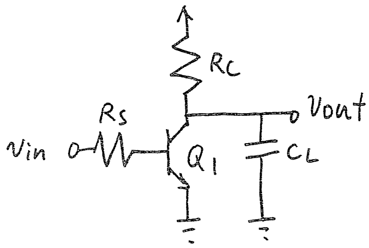
$$= -\frac{\cancel{\beta}}{\cancel{\beta+2}} \times \frac{\cancel{I_{bias}}}{V_T} \times \frac{\cancel{\beta+2}}{2\cancel{\beta}} \times \frac{V_A}{\cancel{I_{bias}}} \times \frac{(\beta+2)V_T/I_{bias}}{R_S+(\beta+2)V_T/I_{bias}}$$

$$\underbrace{A_v = -\frac{V_A}{2V_T} \times \frac{1}{1+\frac{I_{bias}}{(\beta+2)V_T} \times R_S}}$$

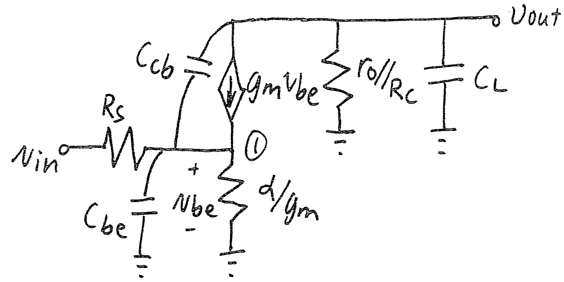
IF $R_S \to 0$ then $A_v = -\dfrac{V_A}{2V_T}$

AND is independent of I_{Bias}.

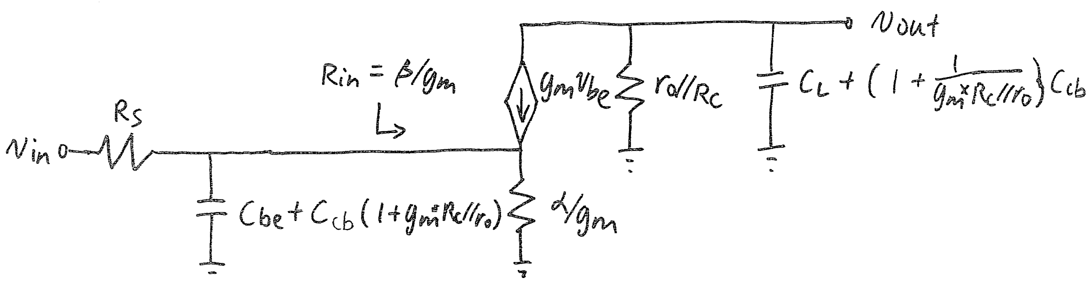
3.24)



⇒



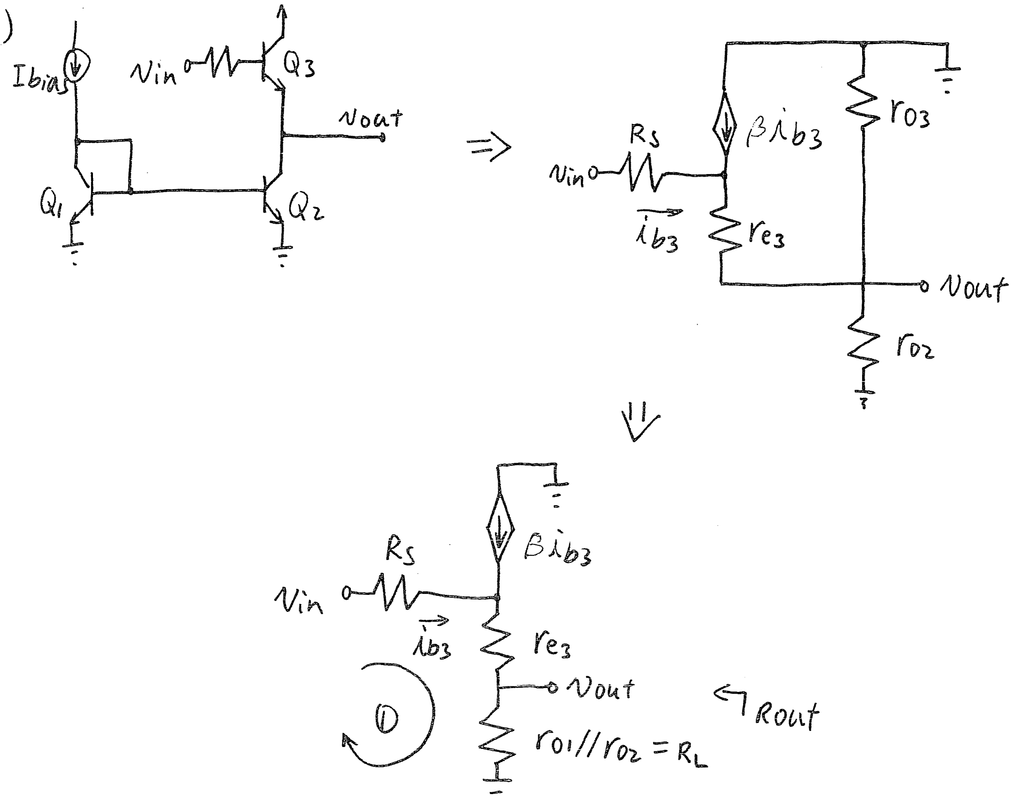
⇙ Using Miller's Theorem



Using the method of adding time constants,

$$\left\{ \begin{aligned} \omega_{-3dB} &\approx \frac{1}{\tau_{①}} + \frac{1}{\tau_{out}} \quad \text{where} \\ \tau_{①} &= [R_s // \beta/g_m][C_{be} + C_{cb}(1 + g_m^*(R_C//r_o))] \\ \tau_{out} &= [r_o // R_C][C_L + (1 + \frac{1}{g_m^*(R_C//r_o)})C_{cb}] \\ &\approx [r_o // R_C]C_L \end{aligned} \right\}$$

3.25)



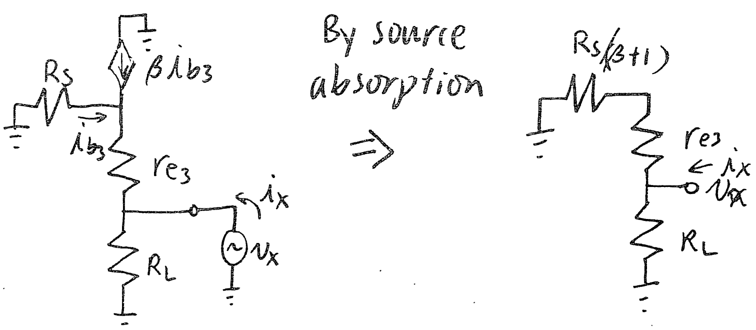
DC gain:

KVL ①: $v_{in} = R_s i_{b3} + (\beta+1) i_{b3} (r_{e3} + R_L)$

and $v_{out} = (\beta+1) i_{b3} R_L$

$$\therefore \frac{v_{out}}{v_{in}} = \frac{(\beta+1) R_L}{(\beta+1)(r_{e3}+R_L) + R_s} = \underline{\frac{R_L}{r_{e3} + R_L + R_s/(\beta+1)}}$$

Calculating Rout:



$R_{out} \triangleq \frac{v_x}{i_x}$

$\underline{R_{out} = R_L // (r_{e3} + \frac{R_s}{\beta+1})}$

(cont.)

3.25 (cont.)

When $I_{bias} = 0.5\,mA$, and assuming $\frac{R_S}{\beta+1} \ll r_{e3}$,

$$I_{C3} \approx I_{C2} = \frac{\beta}{\beta+2} I_{bias} = \frac{100}{102} \times 0.5\,mA = 0.49\,mA$$

and $r_{o3} \approx r_{o2} = \frac{V_A}{I_{C2}} = \frac{80V}{0.49mA} = 163\,k\Omega$

$\therefore R_L = r_{o1} // r_{o2} = 81\,k\Omega$, $\quad r_{e3} = \frac{\beta}{(\beta+1) g_{m3}} = \frac{\beta}{(\beta+1)\frac{I_{C3}}{V_T}}$

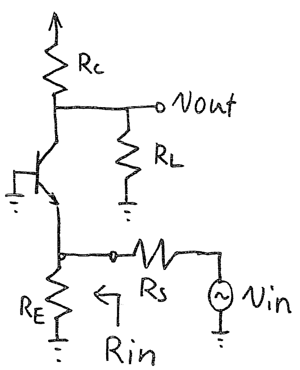
$$= \frac{100}{101 \times \frac{0.49mA}{26mV}} = \underline{53\,\Omega}$$

and $\frac{v_{out}}{v_{in}} \cong \frac{81\,k\Omega}{81\,k\Omega + 53\,\Omega}$

$= \underline{0.9993}$

$$R_{out} = R_L // (r_{e3} + R_S/(\beta+1))$$
$$\approx R_L // r_{e3}$$
$$= 81\,k\Omega // 53\,\Omega$$
$$= \underline{52\,\Omega}$$

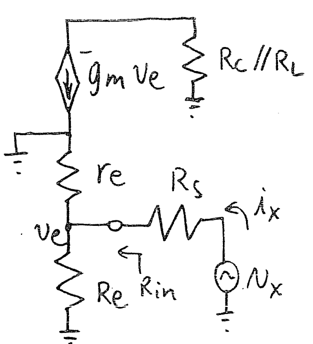
3.26)



Find $\frac{V_{out}}{V_{in}}$ and R_{in}

From the small signal diagram it is clear that $\underline{R_{in} = r_e // R_e}$.

Also, $v_e = \frac{r_e // R_e}{R_s + r_e // R_e} v_{in}$ (voltage divider)



$v_{out} = + g_m v_e \times R_C // R_L$ where

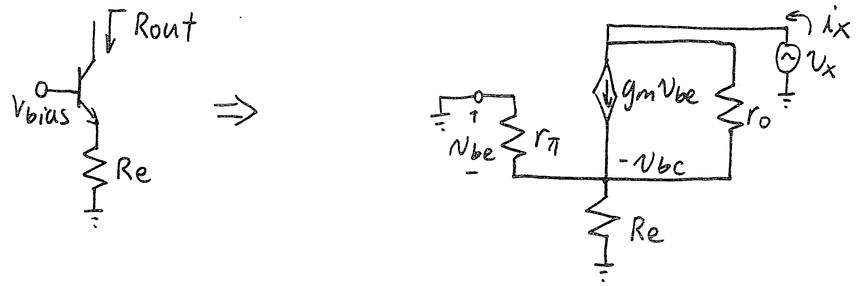
$g_m = \alpha / r_e = \frac{\beta}{(\beta+1) r_e}$

$\therefore v_{out} = + \frac{\beta}{(\beta+1) r_e} \times \frac{r_e // R_e}{R_s + r_e // R_e} \times R_C // R_L \, v_{in}$

$$\frac{v_{out}}{v_{in}} = \frac{\beta}{(\beta+1)\cancel{r_e}} \times \frac{\cancel{r_e} R_e}{r_e + R_e} \times \frac{1}{R_s + \left(\frac{r_e R_e}{r_e + R_e}\right)} \times R_C // R_L$$

$$\underline{\frac{v_{out}}{v_{in}} = \frac{\beta}{\beta+1} \times \frac{R_e}{r_e R_s + R_e R_s + r_e R_e} \times R_C // R_L}$$

3.27)



KCL at the output :

$$-i_x + g_m v_{be} + \frac{v_x + v_{be}}{r_o} = 0 \qquad \boxed{1}$$

But $i_x = \frac{-v_{be}}{r_\pi // R_e} \Rightarrow v_{be} = -i_x \times (r_\pi // R_e)$ $\boxed{2}$

$\boxed{2} \rightarrow \boxed{1}$

$$+i_x + g_m i_x \times r_\pi // R_e + \frac{i_x}{r_o}(r_\pi // R_e) = v_x / r_o$$

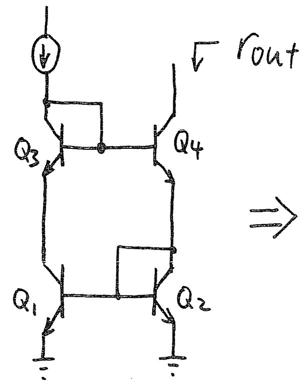
$\therefore R_{out} \triangleq \frac{v_x}{i_x} = r_o(1 + (g_m + \frac{1}{r_o}) r_\pi // R_e)$

$\because g_m \gg 1/r_o$

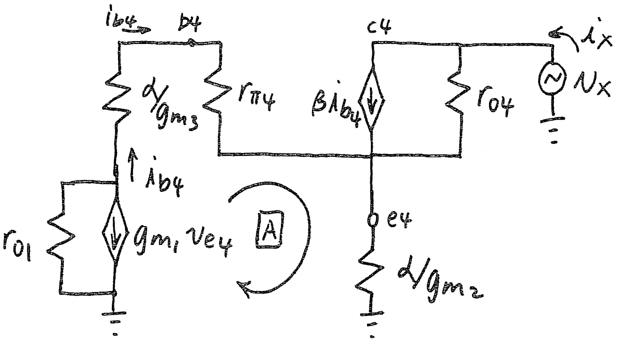
$\therefore$ $R_{out} \approx r_o(1 + g_m \times (R_e // r_\pi))$ Q.E.D.

3.28) * Refer to problems 3.20 and 3.21. *

3.29) Show $r_{out} \approx \frac{\beta r_o}{2}$



$\Rightarrow$



KCL at output: $\quad i_x - \beta i_{b4} - (v_x - v_{e4})/r_{o4} = 0 \quad \boxed{1}$

KCL at e4 $\quad : \quad i_x + i_{b4} - v_{e4}\, g_{m2}/\alpha = 0 \quad \boxed{2}$

Now express i_{b4} and v_{e4} in terms of i_x and v_x.

$\boxed{1} \times r_{o4} \qquad (i_x - \beta i_{b4})\, r_{o4} - v_x + v_{e4} = 0$

$$v_{e4} = v_x + (\beta i_{b4} - i_x)\, r_{o4} \quad \boxed{3}$$

$\boxed{3} \rightarrow \boxed{2} \qquad i_x + i_{b4} - [\, v_x + (\beta i_{b4} - i_x)\, r_{o4}\,]\, g_{m2}/\alpha = 0$

$$i_x + i_{b4} + (i_x - \beta i_{b4})\, r_{o4} \times g_{m2}/\alpha = \frac{g_{m2}}{\alpha} v_x$$

$$i_x \Big(1 + \underbrace{\frac{r_{o4}\, g_{m2}}{\alpha}}_{\gg 1}\Big) + i_{b4}\big(1 - \underbrace{(\beta+1)\, r_{o4}\, g_{m2}}_{\gg 1}\big) = \frac{g_{m2}}{\alpha} v_x$$

$$\therefore \quad r_{o4}\, g_{m2}\, i_x - i_{b4}(\beta+1)\, r_{o4}\, g_{m2} \approx g_{m2}\, v_x$$

$$\Rightarrow \quad i_{b4} \approx \frac{1}{(\beta+1)\, r_{o4}\, g_{m2}} \left[r_{o4}\, g_{m2}\, i_x - g_{m2}\, v_x \right]$$

$$i_{b4} \approx \frac{i_x}{\beta+1} - \frac{v_x}{(\beta+1)\, r_{o4}} \quad \boxed{4}$$

$\boxed{4} \rightarrow \boxed{2} \qquad i_x + \frac{i_x}{\beta+1} - \frac{v_x}{(\beta+1)\, r_{o4}} - v_{e4} \frac{g_{m2}}{\alpha} = 0$

$$i_x \Big(1 + \underbrace{\frac{1}{\beta+1}}_{\ll 1}\Big) - \frac{v_x}{(\beta+1)\, r_{o4}} = v_{e4} \frac{g_{m2}}{\alpha}$$

$$v_{e4} \approx \frac{\alpha}{g_{m2}} \left[i_x - \frac{v_x}{(\beta+1)\, r_{o4}} \right] \quad \boxed{5}$$

(cont.)

3.29 (cont.)

KVL around bias network loop $\boxed{A}$:

$$(g_m v_{e4} + i_{b4}) r_{o1} + (\alpha/g_{m3} + r_{\pi 4}) i_{b4} + v_{e4} = 0$$

$$v_{e4}(\underbrace{g_{m1} r_{o1}+1}_{\gg 1}) + i_{b4}(r_{o1} + \underbrace{r_{\pi 4} + \alpha/g_{m3}}_{\ll r_{o1}}) = 0 \quad \boxed{6}$$

$\boxed{4}$
$\boxed{5} \rightarrow \boxed{6}$

$$\frac{\alpha}{\cancel{g_{m2}}}\left[i_x - \frac{v_x}{(\beta+1) r_{o4}}\right] \cancel{g_{m1}} r_{o1} + \left(\frac{i_x}{\beta+1} - \frac{v_x}{(\beta+1) r_{o4}}\right) r_{o1} \approx 0$$

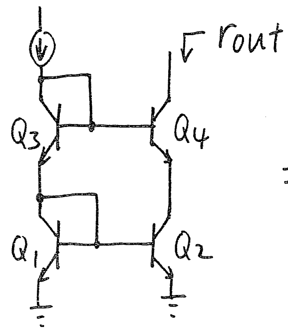
$$r_{o1}\left[i_x - \frac{v_x}{(\beta+1) r_{o4}}\right] + \frac{i_x r_{o1}}{\beta+1} - \frac{v_x}{\beta+1} \approx 0$$

$$i_x r_{o1}(1 + \underbrace{\frac{1}{\beta+1}}_{\ll 1}) - v_x \times \frac{2}{\beta+1} \approx 0$$

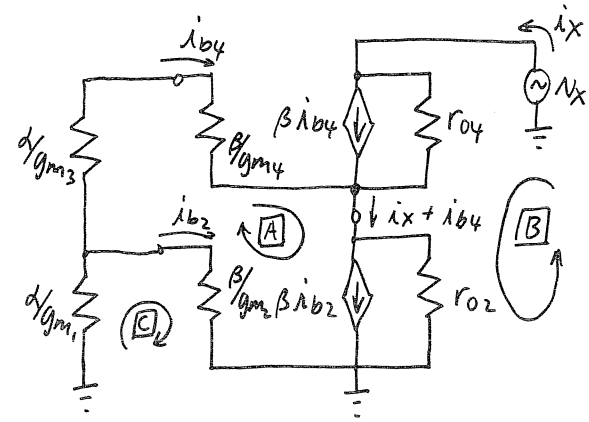
$$\therefore r_{out} \triangleq \frac{v_x}{i_x} \approx \frac{\beta+1}{2} r_{o1} \approx \frac{\beta}{2} r_{o1}$$

$$\therefore \underline{r_{out} \approx \frac{\beta}{2} r_{o1}} \quad \text{Q.E.D.}$$

3.30) Show $r_{out} \approx \beta r_o / 2$



$\Rightarrow$



KVL about loop $\boxed{B}$:

$$-v_x + (i_x - \beta i_{b4}) r_o + (i_x + i_{b4} - \beta i_{b2}) r_o = 0$$

$$2 r_o i_x + (1-\beta) r_o i_{b4} - \beta r_o i_{b2} = v_x \quad \boxed{1}$$

KVL about loop $\boxed{A}$:

$$(\alpha/g_{m3} + \beta/g_{m4}) i_{b4} + (i_x + i_{b4} - \beta i_{b2}) r_o - i_{b2}\, \beta/g_{m2} = 0$$

$$(\alpha/g_{m3} + \beta/g_{m4} + r_o) i_{b4} + r_o i_x - (\beta r_o + \beta/g_{m2}) i_{b2} = 0 \quad \boxed{2}$$

KVL about loop $\boxed{C}$:

$$(i_{b4} + i_{b2})\, \alpha/g_{m1} + i_{b2}\, \beta/g_{m2} = 0$$

$$\alpha/g_{m1} \times i_{b4} + (\beta/g_{m2} + \alpha/g_{m1}) i_{b2} = 0$$

$$i_{b4} = -\frac{g_{m1}}{\alpha}\left(\frac{\beta}{g_{m2}} + \frac{\alpha}{g_{m1}}\right) i_{b2}$$

$$= -\left((\beta+1)\frac{g_{m1}}{g_{m2}} + 1\right) i_{b2}$$

$$\underline{i_{b4} \approx -(\beta+2) i_{b2}} \quad \boxed{3}$$

$\boxed{3} \to \boxed{1}$

$$2 r_o i_x + (\beta-1)(\beta+2) r_o i_{b2} - \beta r_o i_{b2} = v_x$$

$$\therefore \quad i_{b2} = \frac{2 r_o i_x - v_x}{r_o(-\beta^2 - 2\beta + 2)} \quad \boxed{4}$$

$\boxed{3}, \boxed{4} \to \boxed{2}$

$$\underbrace{(\alpha/g_{m3} + \beta/g_{m4} + r_o)}_{\approx \beta/g_{m4} + r_o} \times -\underbrace{(\beta+2)}_{\approx \beta}\frac{2 r_o i_x - v_x}{r_o(-\beta^2 - 2\beta + 2)} + i_x r_o - \underbrace{(\beta r_o + \beta/g_{m2})}_{\approx \beta r_o}\frac{2 r_o i_x - v_x}{r_o(-\beta^2 - 2\beta + 2)} = 0$$

(cont.)

3.30 (cont.)

$$\frac{2 r_o i_x - v_x}{r_o(-\beta^2-\beta+2)}\left[(\beta/g_{m4}+r_o)(-\beta) - \beta r_o\right] + i_x r_o \approx 0$$

$$\frac{2 r_o i_x - v_x}{r_o(+\beta^2)}\left[2\beta r_o + \beta^2/g_{m4}\right] + i_x r_o \approx 0$$

$$i_x\Big(\underbrace{4r_o/\beta + 2/g_{m4} + r_o}_{\approx r_o}\Big) - v_x\Big[\underbrace{2/\beta + 1/(r_o g_m)}_{2/\beta}\Big] \approx 0$$

$\therefore$ $\underline{r_{out} \triangleq \frac{v_x}{i_x} \approx \frac{\beta}{2} r_o}$ Q.E.D.

3.31) Find DC gain and ω_{-3dB}

DC gain $\triangleq \frac{v_{out}}{v_{in}} = g_{m1} \times r_{DS2} // r_{DS4}$

$$g_{m1} = \sqrt{2 \mu_n C_{ox} W/L \times I_{bias}/2} = \sqrt{2 \times 92 \times 10^{-6} \times \frac{100}{1.6} \times 0.05 \times 10^{-3}}$$

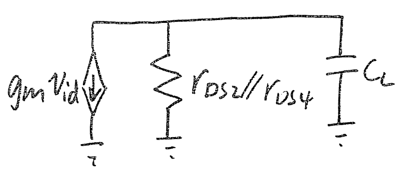
$$= \underline{0.758 \text{ mA/V}}$$

$$r_{DS2} = \frac{8000 L}{I_D} = \frac{8000 \times 1.6}{0.05 mA} = 256 k\Omega$$

$$r_{DS4} = \frac{12000 L}{I_D} = \frac{12000 \times 1.6}{0.05 mA} = 384 k\Omega$$

$\Big\}$ $r_{DS2} // r_{DS4} = 154 k\Omega$

$\therefore$ DC gain $= 0.758 \times 10^{-3} \times 154 \times 10^{3} = \underline{116}$



$$\omega_{-3dB} = \frac{1}{r_{DS2}//r_{DS4} \times C_L}$$

$$= \frac{1}{154 \times 10^{3} \times 100 \times 10^{-12}}$$

$$= \underline{2\pi \times 10 \text{ kHz}}$$