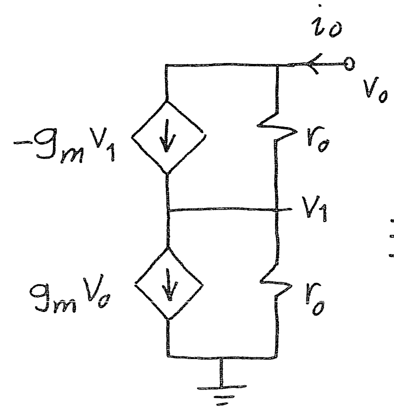
Chapter 6 – Problems

6.1) KVL:

$$\begin{cases} V_o = r_o i_o + g_m r_o V_1 + i_o r_o + (-g_m r_o V_o) \\ V_1 = r_o i_o - g_m r_o V_o \end{cases}$$



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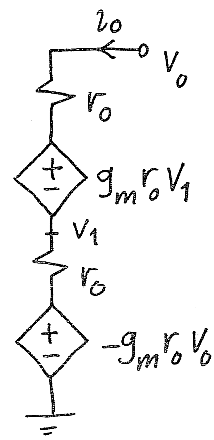


$$\therefore \; V_o = 2 r_o i_o + g_m r_o^2 i_o - g_m^2 r_o^2 V_o - g_m r_o V_o$$

$$\Rightarrow \; r_{out} = \frac{V_o}{i_o} = \frac{r_o (2 + g_m r_o)}{g_m^2 r_o^2 + g_m r_o + 1}$$

Assuming $g_m r_o \gg 1 \;\Rightarrow\; g_m^2 r_o^2 \gg g_m r_o$

$$\Rightarrow \; r_{out} \approx \frac{r_o (g_m r_o)}{g_m^2 r_o^2} \simeq \frac{1}{g_m}$$

6.2) For transistors Q_1 to Q_4:

$$50\,\mu A = \frac{\mu_n C_{ox}}{2} \left(\frac{W}{L}\right)_i (0.2)^2 \quad \text{for } i = 1 \text{ to } 4$$

$$\Rightarrow \; \left(\frac{W}{L}\right)_i = 27.2 \quad \text{for } i = 1 \text{ to } 4$$

Also, $V_{GS5} = V_{GS4} + V_{DS3} = V_{eff4} + V_{tn} + V_{eff3} + 0.15 = 1.35\,V$

$$\Rightarrow V_{eff5} = V_{GS5} - V_{tn} = 0.55\,V$$

Using $I_{BIAS} = 50\,\mu A \Rightarrow$

$$50\,\mu A = \frac{\mu_n C_{ox}}{2} \left(\frac{W}{L}\right)_5 (0.55)^2 \Rightarrow \left(\frac{W}{L}\right)_5 = 3.6$$

6.3) Reducing L by 1.6 will decrease V_{eff} by a factor of $\sqrt{1.6}$. Therefore,

$$V_{DS3} = V_{GS5} - V_{GS4} = 1.35 - (V_{tn} + V_{eff})$$
$$= 1.35 - (0.8 + 0.16) = \underline{0.39\ V}$$

$$V_{DS4} = V_{D4} - V_{DS3} = V_{GS3} - V_{DS3} = (V_{tn} + V_{eff}) - 0.39$$
$$= (0.8 + 0.16) - 0.39 = \underline{0.57\ V}$$

6.4) $I_{D3} = I_{D2} \Rightarrow \left(\frac{W}{L}\right)_2 V_{eff2}^2 = \left(\frac{W}{L}\right)_3 V_{eff3}^2 \Rightarrow \underline{V_{eff3} = 2V_{eff2}}$

Also, $V_{GS3} = V_{GS2} + R_B I$

$$\Rightarrow V_{eff3} = \frac{V_{eff3}}{2} + R_B \frac{\mu_n C_{ox}}{2} \left(\frac{W}{L}\right)_3 V_{eff3}^2$$

$$\Rightarrow \underline{10\,\mu_n C_{ox} R_B V_{eff3} = 1}$$

Using $V_{eff3} = 0.2 \Rightarrow \underline{R_B = 5.43\ K\Omega}$

6.5) Using the result of Problem 6.4, we have:

$$10\,\mu_n C_{ox} R_B V_{eff3} = 1$$

Also, $\dfrac{\mu_n(100^\circ C)}{\mu_n(20^\circ C)} = \left(\dfrac{373}{293}\right)^{-1.5} = \underline{0.7}$

Therefore, V_{eff3} will be increased by $\frac{1}{0.7}$.

Equivalently, $V_{eff3} = \frac{1}{0.7} \times 0.2 = \underline{0.29\ V}$

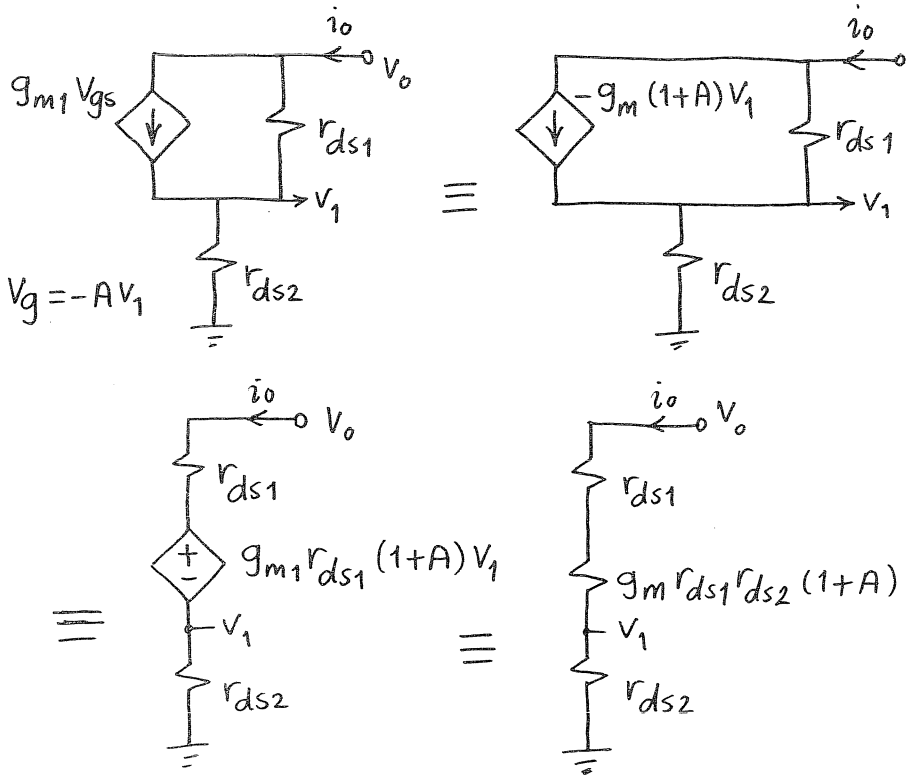
6.6) $R_B(100\ °C) = \left(1 + 80 \times \frac{0.3}{100}\right) R_B(20°C) = \underset{\sim\sim\sim\sim}{6.73\ K\Omega}$

Using the result of Problem 6.5, $\mu_n R_B$ is decreased at 100 °C by a factor of $0.7 \times 1.24 = 0.868$.

Therefore, V_{eff3} will increase by $\frac{1}{0.868}$.

$\therefore V_{eff3} = \frac{1}{0.868} \times 0.2 = \underset{\sim\sim\sim\sim}{0.23\ V}$

6.7) The equivalent circuit of Fig. 6.3 can be simplified:

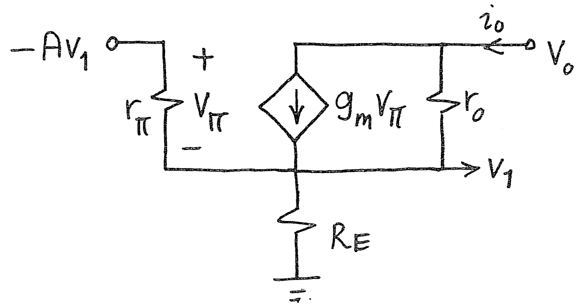


$$\Rightarrow r_o = \frac{V_o}{i_o} = r_{ds1} + r_{ds2} + (1+A) g_{m1} r_{ds1} r_{ds2}$$

ignoring the first two terms:

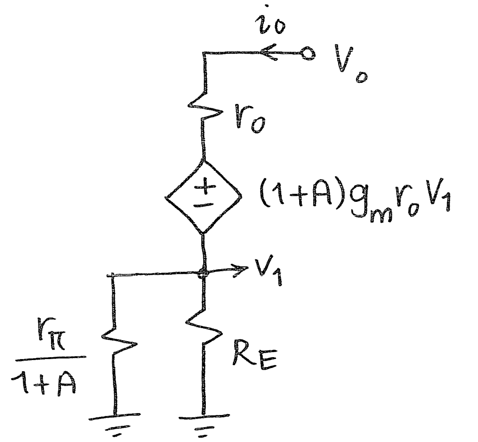
$$\underset{\sim\sim\sim\sim\sim\sim\sim\sim\sim}{r_o \approx (1+A) g_{m1} r_{ds1} r_{ds2}}$$

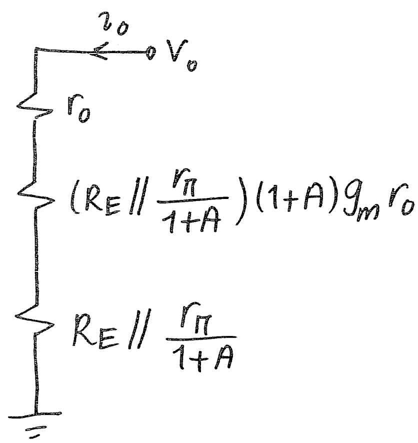
6.8) The equivalent circuit of Fig. P6.8 can be simplified as:



Noting that $-AV_1 = V_\pi + V_1 \Rightarrow V_\pi = -(1+A)V_1$

The circuit can be simplified further as:





$$\Rightarrow R_{out} = \frac{V_o}{i_o} = r_o + \left(R_E // \frac{r_\pi}{1+A}\right)\left(1 + (1+A)g_m r_o\right)$$

Assuming $R_E \gg r_\pi \Rightarrow R_E // \frac{r_\pi}{1+A} \simeq \frac{r_\pi}{1+A}$

$$\Rightarrow R_{out} \simeq r_o + \frac{r_\pi}{1+A}(1+A)g_m r_o = r_o(1 + r_\pi g_m) = r_o(1+\beta)$$

This result is independent of A !

6.9) Using (6.10): $R_{out} = g_{m1} r_{ds1} r_{ds2} (1+A)$

where $A = g_{m3} r_{ds3}$ for the circuit of Fig. 6.6.

$\therefore \; R_{out} = g_{m1} r_{ds1} r_{ds2} (1 + g_{m3} r_{ds3})$

$I_{BIAS} = 50\,\mu A \Rightarrow I_{D1} = I_{D2} = 350\,\mu A \;\&\; I_{D3} = 200\,\mu A$

$\Rightarrow r_{ds1} = r_{ds2} = \dfrac{8K \times 1.6}{0.35} = 36.57\,k\Omega$

$r_{ds3} = \dfrac{8K \times 1.6}{0.2} = 64\,k\Omega$

$g_{m1} = \sqrt{2 I_{D1} \mu_n C_{ox} \frac{70}{1.6}} = 1.68\,mA/V$

$g_{m3} = \sqrt{2 I_{D3} \mu_n C_{ox} \frac{10}{1.6}} = 0.48\,mA/V$

$\Rightarrow R_{out} = 2.27\,M\Omega\,(1 + 30.72) = 7.13 \times 10^{7}\,\Omega$

the enhanced output impedance is 32 times larger than that of a wide-swing cascode current mirror given by $R'_{out} = g_{m1} r_{ds1} r_{ds2} = 2.25\,M\Omega$

6.10) $2 I_{D3} = \dfrac{1mW}{4V} = 250\,\mu A \Rightarrow I_{D3} = I_{D4} = 125\,\mu A$.

Also, $5 I_{D5} = 125\,\mu A \Rightarrow I_{D5} = I_{D6} = 25\,\mu A$

$\& \; I_{D1} = I_{D2} = 100\,\mu A$

$g_{m1} = \sqrt{2 I_{D1} \mu_n C_{ox} (W/L)_1} = 1.9\,mA/V$

$\omega_t = g_{m1}/C_L = \dfrac{1.9\,mA/V}{10\,pF} = 1.9 \times 10^{8}\,rad/s \Rightarrow f_t = 30.2\,MHz$

(cont.)

6.10) (cont.) the slew rate without the clamp transistors:

$$SR = \frac{I_{D4}}{C_L} = 12.5 \ V/\mu s$$

With the clamp transistors:

$$I_{D11} = \frac{I_{bias2} + \frac{125\mu}{30}}{31} = \frac{200\mu + \frac{125\mu}{30}}{31} = 6.6 \mu A$$

$$\Rightarrow I_{D3} = 30 \, I_{D11} = 198 \ \mu A$$

$$\Rightarrow \ SR = \frac{198 \mu A}{10 \, pF} = 19.8 \ V/\mu s$$

6.11) Ignoring the junction capacitances, the total capacitance at the drain of Q_2 can be calculated as:
(also ignore Q_{13} since it is small)

$$C_{p2} = C_{dq2} + C_{dq4} + C_{sq5} = C_{gd(overlap)} (W_2 + W_4 + W_5) + \frac{2}{3} W_5 L_5 C_{ox}$$

$$= 0.2 \, fF/\mu m \, (300 + 300 + 60) + \frac{2}{3} \times 60 \times 1.6 \times 1.9 fF = 254 \ fF$$

the total conductance at this node is dominated by g_{m5}. Using the result of problem 6.10, we have:

$$g_{m5} = \sqrt{2 I_{D5} \mu_p C_{ox} (W/L)_5} \doteq 0.237 \ m\mho$$

The second pole (half-circuit concept):

$$\omega_2 = \frac{g_{m5}}{C_{p2}} = 9.33 \times 10^8 \ rad/s$$

$$\Rightarrow \ f_2 = \frac{\omega_2}{2\pi} = 148.5 \ MHz$$

(cont.)

6.11) (cont.) Using (5.52), for a 70° phase margin, we must have:

$$\frac{f_t'}{f_2} = \tan 20^\circ \Rightarrow \underline{f_t' = 54.05\text{ MHz}}$$

Using (6.30): $C_L' = \dfrac{g_{m1}}{\omega_t'} = \dfrac{1.9m}{2\pi \times f_t'} = \underline{5.59\text{ pF}}$

Finally, using (16.32):

$$SR' = \frac{I_{D4}}{C_L'} = \frac{125\mu}{5.59p} = \underline{22.3\text{ V}/\mu s}$$

With the clamp transistors: $SR' = \dfrac{198\mu}{5.59p} = \underline{35.4\text{ V}/\mu s}$

6.12) This is equivalent to a 40° phase margin in the original design. Using (5.52), we have:

$$\frac{f_t'}{f_{p2}} = \tan 50^\circ \Rightarrow f_t' = 1.19 f_2 \simeq \underline{176.7\text{ MHz}}$$

$$f_z = 1.2 f_t' = \underline{212\text{ MHz}}$$

The final unity-gain frequency: $f_t = 1.2 f_t' = \underline{212\text{MHz}}$

$$\Rightarrow C_L = \frac{g_{m1}}{\omega_t} = \underline{1.43\text{ pF}}$$

$\omega_z = \dfrac{1}{R_c C_L} \Rightarrow R_c = \dfrac{1}{\omega_z C_L} = \underline{525\,\Omega}$, $SR = \dfrac{I_{D4}}{C_L} = \underline{87\text{ V}/\mu s}$

With the clamp xtors: $SR = \dfrac{198\mu}{1.43p} = \underline{138\text{ V}/\mu s}$

6.13) $I_{D1} = I_{D2} = K I_{D5} = K I_{D6}$

$$I_{total} = 2(I_{D1} + I_{D5}) = 2(1+K) I_{D5}$$

$$\Rightarrow I_{D5} = \frac{I_{total}}{2(K+1)} \quad , \quad I_{D1} = \frac{K}{K+1} \cdot \frac{I_{total}}{2}$$

$$g_{m1} = \frac{2 I_{D1}}{V_{eff1}} = \underline{\frac{K}{K+1} \frac{I_{total}}{V_{eff1}}} \quad , \quad \omega_t = \underline{\frac{K}{K+1} \frac{I_{total}}{V_{eff1} \cdot C_L}}$$

Assuming a constant I_{total} & V_{eff1}, both g_{m1} & ω_t increase with increasing K.

6.14) The dc gain is given by $A_{V_0} = g_{m1} r_{out}$

where, approximately : $r_{out} \simeq g_{m8} \frac{r_{ds8}^2}{2}$

and $g_{m8} = \frac{2 I_{D8}}{V_{eff8}}$, $r_{ds8} = \frac{\alpha L}{I_{D8}}$, α is constant!

$$\Rightarrow r_{out} \simeq \frac{\alpha^2 L^2}{V_{eff8} I_{D8}}$$

$$\Rightarrow A_{V_0} = \frac{2 I_{D1}}{V_{eff1}} \frac{\alpha^2 L^2}{V_{eff8} I_{D8}} = \left(\frac{I_{D1}}{I_{D8}}\right)\left(\frac{2 \alpha^2 L^2}{V_{eff1} V_{eff8}}\right)$$

Noting $I_{D8} = I_{D5}$ and $I_{D1} = K I_{D5}$ results in:

$$\underline{A_{V_0} = K \left(\frac{2 \alpha^2 L^2}{V_{eff1} V_{eff8}}\right)}$$

A_{V_0} increases with K !

6.15) For the folded-cascode amplifier of Fig. 6.9, we have:

$$\omega_t = \frac{g_{m1}}{C_L} = \frac{\sqrt{2 I_{D1} \mu_n C_{ox} (W/L)_1}}{C_L}$$

Also, $2I_{D1} + \frac{2}{K} I_{D1} = I_{total} \Rightarrow I_{D1} = \frac{K}{2(K+1)} I_{total}$

$$\Rightarrow \omega_t \text{(folded-cascode)} = \frac{1}{C_L} \sqrt{\frac{\mu_n C_{ox} I_{total} (W/L)_1 K}{K+1}}$$

For the current-mirror opamp, ω_t is given by (6.48).

Therefore,

$$\frac{\omega_t \text{(folded-cas.)}}{\omega_t \text{(current-mir.)}} = \sqrt{\frac{K+3}{2K(K+1)}}$$

K	1	2	4
ω_t Ratio	1	0.645	0.418

6.16) From example 6.4 : $C_p = 0.46\ pF$

$$C_L = 5 + 5 + \frac{5(1+0.46)}{1+0.46+5} = 11.13\ pF$$

$$\omega_t = \frac{K g_{m1}}{C_L} = \frac{2 \times 1.7\ mA/V}{11.13\ pF} = 3.05 \times 10^8\ rad/s \quad \text{or} \quad f_t = 48.6\ MHz$$

$$\beta = \frac{C_2}{C_1 + C_p + C_2} = \frac{5}{1+0.46+5} = 0.77$$

$$\Rightarrow \tau = \frac{1}{\beta \omega_t} = 4.24\ nsec$$

For a 1% accuracy, we need 4.6τ or 19.5 nsec.

6.17) Using (6.53) & (6.68): $SR = 25.4\ V/\mu s$.

Assuming a high-gain opamp:

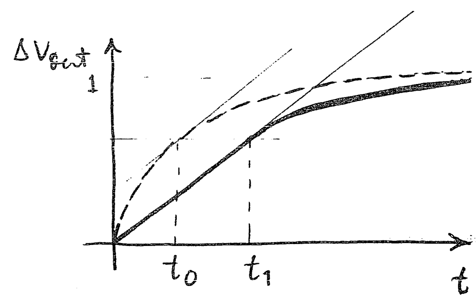
$$A_V = \frac{\Delta V_o}{\Delta V_{in}} = \frac{-C_1}{C_2} = \frac{-5pF}{5pF} = -1 \Rightarrow \Delta V_o = -1V.$$

$\Rightarrow$ the output voltage rate of change would be

$$\left.\frac{dv_o}{dt}\right|_{max} = \frac{\Delta V_o}{\tau} = \frac{1}{7.8\,ns} = 128\ V/\mu s > 25.4\ V/\mu s$$

Therefore, the output will be limited by the slew rate!

the output voltage will ramp up with 25.4 V/μs until the exponential-curve derivative is equal to the slew rate:



At time t_0, we have: $\frac{1}{\tau} e^{-t_0/\tau} = 25.4\ V/\mu s$

$\Rightarrow$ $t_0 = 12.63\ ns$ & $(1-e^{-t_0/\tau}) = 0.8\ V$

$\Rightarrow$ at time $t=t_1$, the output has reached to 80% of its final value. ($t_1 = 31.5\ ns$)

For $t \geq t_1$: $\Delta V_o(t) = 0.8 + 0.2(-e^{-(t-t_1)/\tau} + 1)$

For $\Delta V_o(t) = 0.99 \Rightarrow t - t_1 = 23.37\ ns$

This is the time required after t_1 for the output to settle to 1% of its final value.

6.18) Fully dif. folded-cas.:

$$\text{positive } SR = \frac{I_3 - I_q}{C_L} = \frac{(K+1)I_q - I_q}{C_L} = \frac{K I_q}{C_L}$$

$$\text{negative } SR = \frac{I_q}{C_L}$$

$\therefore$ for $K=2$ and $I_q = 40\,\mu A$:

$$\underbrace{\text{positive } SR = 8\ V/\mu s} \quad \underbrace{\text{negative } SR = 4\ V/\mu s}$$

Fully dif. current-mir.:

$$\text{positive } SR = \text{negative } SR = \frac{K I_{BIAS}}{2C_L}$$

$\therefore$ for $K=2$ & $I_{BIAS} = 160\,\mu A$, we have:

$$\text{positive } SR = \text{negative } SR = \underbrace{16\ V/\mu s}$$

6.19) $$\left.\frac{dV_{out+}}{dt}\right|_{max} = \left.\frac{dV_{out-}}{dt}\right|_{max} = \underbrace{\frac{K I_{BIAS}}{C_L}}$$

Note that both maximums occur simultaneously.

6.20) $$\omega_t = \frac{K g_{m4}}{C_L} \quad \text{where } g_{m4} = \sqrt{2 I_{D4}\, \mu_n C_{ox} (W/L)_4}$$

Also, $I_{total} = (2+K) I_{D4} \Longrightarrow I_{D4} = \dfrac{I_{total}}{2+K}$

$$\therefore \quad \underbrace{\omega_t = \frac{K}{C_L}\sqrt{\frac{2}{2+K} I_{total}\, \mu_n C_{ox} (W/L)_4}}$$

6.21) The maximum value of V_{eff} without the I_B-transistor going into the triode region is $2 - |V_{TP}| = 1.1\,V$. At this voltage, all the bias current will flow through either Q_1 or Q_2.

Therefore, $I = K(1.1)^2$

For the bias condition ($V_{out+} = 0$), we have:

$$I/2 = K V_{eff}^2 \Rightarrow V_{eff} = \frac{1.1}{\sqrt{2}} = 0.78\ V$$

Transistor Q_1 will shut off when $V_{SG1} = |V_{TP}| = 0.9\,V$. At this point, $V_{S1} = 2\,V$ and I_B flows through Q_2.

$$\therefore V_{out+}\Big|_{max} = 2 - 0.9 = 1.1\ V$$

When V_{out+} goes below zero, V_{S1} starts to fall off from its bias value (i.e. 1.68 V) until it reaches 0.9 V. At this point, Q_2 shuts off and I_B will pass through Q_1.

$$\therefore V_{out+}\Big|_{min} = 0.9 - 2 = -1.1\ V$$

Note that a V_{eff} (bias) that is higher than the optimum value (i.e. 0.78 V) will cause I_B-transistor to enter the triode region at a lower V_{out+} voltage. Also, a V_{eff} (bias) that is lower than the optimum will shut off Q_1 at a lower V_{out+} voltage. In both cases, the voltage range of V_{out+} for linear operation is reduced.

6.22) $|V_{tp}| = |V_{tpo}| + \gamma\left(\sqrt{|V_{SB}| + |2\phi_F|} - \sqrt{|2\phi_F|}\right)$

For $V_S = 2\text{ V}$ & $V_B = 2.5\text{ V}$, $|2\phi_F| \simeq 0.7\text{ V}$

$\Rightarrow |V_{tp}| = 1.1\text{ V} \Rightarrow I = K(2 - 1.1)^2.$

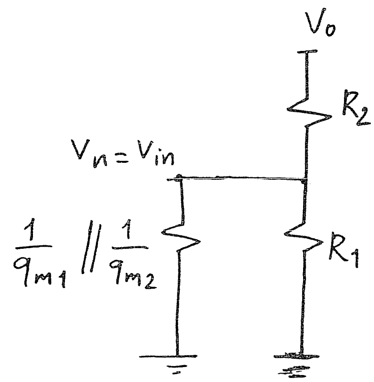
For the bias condition ($V_{out+} = 0$), we have

$$I/2 = K(V_{eff})^2 \Rightarrow V_{eff} = \frac{0.9}{\sqrt{2}} = 0.63\text{ V}$$

$$V_{out+}\big|_{max} = 2 - 1.1 = 0.9\text{ V}$$

$$V_{out+}\big|_{min} = |V_{tp}| - V_{SG1} = |V_{tp}| - (|V_{tp}| + 0.9) = -0.9\text{ V}$$

6.23) $$V_{in} = V_o \frac{R_1 \| \frac{1}{g_{m1}} \| \frac{1}{g_{m2}}}{R_2 + R_1 \| \frac{1}{g_{m1}} \| \frac{1}{g_{m2}}}$$



$$\Rightarrow \frac{V_o}{V_{in}} = \frac{R_1 + R_2}{R_1} + R_2(g_{m1} + g_{m2})$$

The first term can be recognized as the voltage gain when $g_{m1} = g_{m2} = 0$

6.24) Using Eq. (6.62), (6.64), (6.65), and (6.66):

$$\tau = \frac{1}{\omega_{3dB}} = \frac{1}{\beta\omega_t} \quad \text{where } \beta = \frac{C_1/M}{C_1/M + C_1 + C_p} \ \& \ \omega_t = \frac{Kg_m}{C_o + \frac{C_1/M\,(C_1+C_p)}{C_1/M + C_1 + C_p}}$$

$$\Rightarrow \tau = \frac{1}{Kg_m}\left[(M+1)C_o + C_p + \frac{MC_oC_p}{C_1} + C_1\right]$$

$$\frac{\partial\tau}{\partial C_1} = 0 \Rightarrow -\frac{C_oC_p}{C_1^2} + \frac{1}{M} = 0 \Rightarrow C_{1,opt} = \sqrt{MC_oC_p}$$

6.25) $$\tau = \frac{1}{Kg_m}\left[2C_o + C_p + \frac{C_oC_p}{C_1} + C_1\right] = \frac{1}{Kg_m}\left[2.05 + \frac{0.05}{C_1} + C_1\right] p$$

where C_1 must be expressed in pF unit.

$$C_{1,opt} = \sqrt{C_oC_p} = 0.22\ pF$$

$$\Rightarrow Kg_m\tau\Big|_{min} = 2.5\ ps$$

The following table shows $Kg_m\tau$ for some other values of C_1

C_1 (pF)	0.1	0.3	0.5	0.7	0.9	1
$Kg_m\tau$ [ps]	2.65	2.52	2.65	2.82	3	3.1