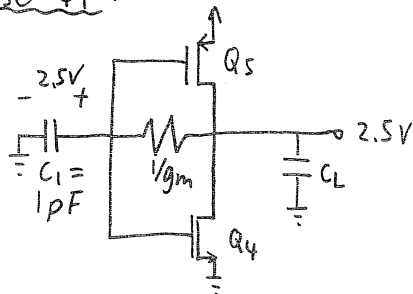


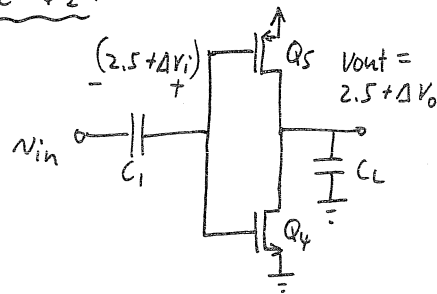
Chapter 7 - Problems

7.1) Find offset due to charge injection

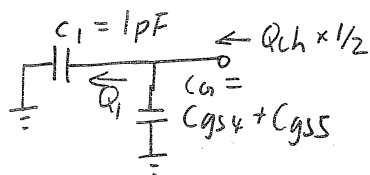
Phase Φ_1 :



Phase Φ_2 :



Channel charge stored in Q_1 is partially* injected into C_1 at the end of Φ_{1a} .



* Assume half is injected into the node attached to the source of Q_1

$$Q_{ch} = V_{eff,1} C_{ox} W_1 L_1$$

$$V_{eff,1} = V_{GS1} - V_{tn}$$

$$\begin{aligned} V_{tn} &= V_{tn0} + \gamma (\sqrt{V_{SB} + 2\phi_F} - \sqrt{2\phi_F}) \\ &= 0.8 + 0.5 (\sqrt{2.5 + 0.7} - \sqrt{0.7}) \\ &= \underline{1.28 V} \end{aligned}$$

$$\therefore V_{eff,1} = (5 - 2.5) - 1.28 = 1.22 V$$

$$\therefore Q_{ch} = 1.22 V \times 1.9 \times 10^{-3} \frac{PF}{\mu m^2} \times 5 \mu m \times 0.8 \mu m = 9.27 \times 10^{-15} C$$

$$\begin{aligned} \text{Now } C_{gs5} &= \frac{2}{3} C_{ox} W_5 L_5 \quad (\text{neglect overlap capacitance}) \\ &= \frac{2}{3} \times 1.9 \times 10^{-3} \times 92 \times 0.8 \\ &= 0.093 pF \end{aligned}$$

$$\begin{aligned} C_{gs4} &= \frac{2}{3} C_{ox} W_4 L_4 \\ &= 0.030 pF \end{aligned}$$

$$\therefore C_G = C_{gs5} + C_{gs4} = 0.123 pF$$

(Cont.)

7.1 (cont.)

$$\therefore Q = -CV$$

$$\therefore \Delta V_i = \frac{-\Delta Q}{C} = \frac{-Q_{ch}}{2(C_1 + C_2)} = \frac{-9.27 \times 10^{-15}}{2(1\text{pF} + 0.123\text{pF})} = -4.1\text{mV}$$

Since the inverters gain is given as -24

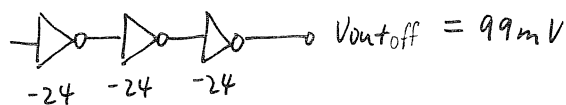
$$\Delta V_o = -24 \times \Delta V_i$$

$$\therefore \Delta V_o = -24 \times \Delta V_i = \underline{+99\text{mV}}$$

\therefore V_{out} shifts up by 99mV due to charge injection.

7.2) Find the input-referred offset voltage.

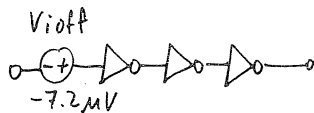
From Problem 7.1, we found the output offset voltage to be $+99\text{mV}$.



To refer this back to the input, we divide by the total gain

$$\text{i.e., } V_{ioff} = \frac{V_{outoff}}{(-24)^3}$$

$$\underline{V_{ioff} = -7.2\text{mV}}$$

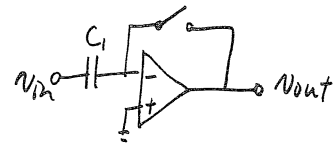
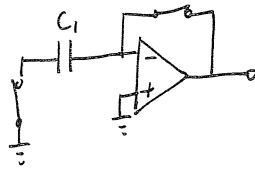


7.3) Estimate time constants :

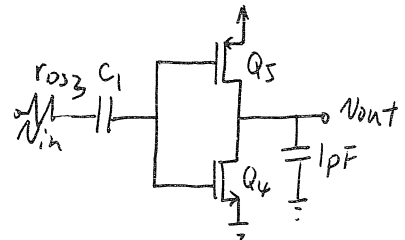
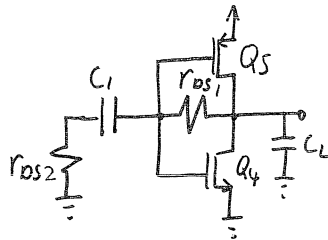
Reset Phase (ϕ_1)

Comparison Phase (ϕ_2)

CONCEPT

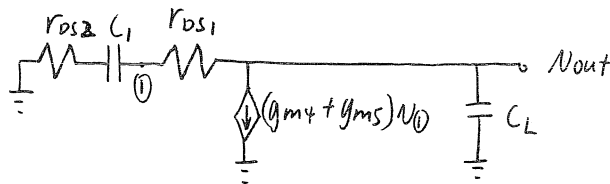


CIRCUIT

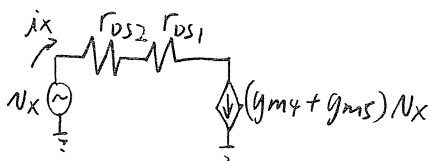


Reset Phase (ϕ_1) :

To find system time constants, calculate the resistances associated with each capacitor.



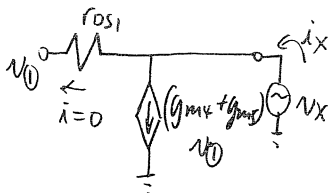
for C_1 :



$$i_x = (g_{m4} + g_{m5}) N_x$$

$$\therefore R_1 = 1 / (g_{m4} + g_{m5})$$

for C_2 :



Again,

$$i_x = (g_{m4} + g_{m5}) N_0 = (g_{m4} + g_{m5}) N_x$$

$$\therefore R_2 = N_x / i_x = 1 / (g_{m4} + g_{m5})$$

$$\therefore \tau_{\phi_1} = \tau_1 + \tau_2 = R_1 C_1 + R_2 C_2$$

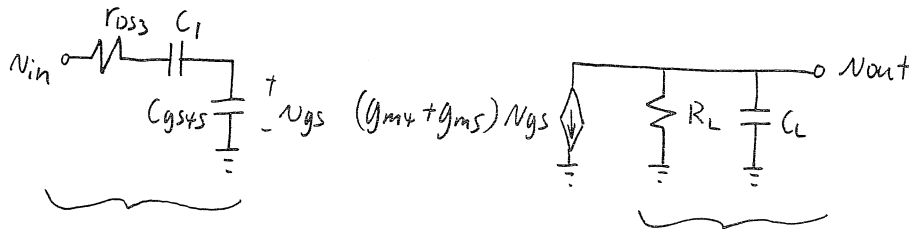
$$\tau_{\phi_1} = \frac{(C_1 + C_2)}{g_{m4} + g_{m5}}$$

for the reset phase

(cont.)

7.3) (cont.)

Comparison Phase (Φ_2):



$$\tau_{in} = \left(C_1 \text{ in series w/ } \right) \times r_{DS3}$$

$$\approx C_{gs45} r_{DS3}$$

$$\tau_{out} = R_L C_L$$

$$= (r_{DS4} // r_{DS5}) C_L$$

$$\tau_{\Phi_2} = \tau_{in} + \tau_{out} = r_{DS3} C_{gs45} + r_{DS4} // r_{DS5} \times C_L \approx r_{DS4} // r_{DS5} \times C_L$$

Calculating values:

$$g_{m4} = \mu_n C_{ox} \frac{W}{L} V_{eff4} = 92 \times 10^{-6} \times 30 / 0.8 \times (2.5 - 0.8)$$

$$= 5.9 \text{ mA/V}$$

$$g_{m5} = \mu_p C_{ox} \frac{W}{L} V_{eff5} = 30 \times 10^{-6} \times 92 / 0.8 \times (2.5 - 0.8)$$

$$= 5.9 \text{ mA/V}$$

$$r_{DS1} = \frac{1}{\mu_n C_{ox} \frac{W}{L} V_{eff1}} = \frac{1}{92 \times 10^{-6} \times 5 / 0.8 \times (2.5 - V_{tn1})} \quad \text{where}$$

$$V_{tn1} = V_{tn0} + \gamma (\sqrt{V_{SB1} + 2\phi_F} - \sqrt{2\phi_F})$$

$$= 0.8 + 0.5 (\sqrt{2.5 + 0.7} - \sqrt{0.7}) = 1.28 \text{ V}$$

$$\therefore r_{DS1} = 1.4 \text{ k}\Omega$$

$$r_{DS3} = \frac{1}{92 \times 10^{-6} \times 5 / 0.8 \times (2.5 - 0.8)} = 1 \text{ k}\Omega$$

$$\therefore \tau_{\Phi_1} = \frac{C_1 + C_2}{g_{m4} + g_{m5}} = \frac{2 \text{ pF}}{2 \times 5.9 \text{ mA/V}} = \underline{0.17 \text{ nsec}}$$

$$\tau_{\Phi_2} = r_{DS4} // r_{DS5} \times C_L \quad \text{where}$$

$$I_{D4} = I_{D5} = \frac{\mu_n C_{ox}}{2} \frac{W}{L} V_{eff}^2 = \frac{92 \times 10^{-6}}{2} \times \frac{30}{0.8} (2.5 - 0.8)^2$$

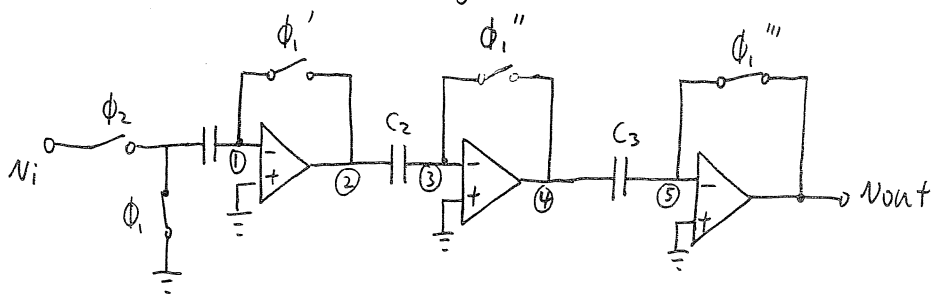
$$= 5 \text{ mA}$$

$$\therefore r_{DS4} = \frac{8000 \times 0.8}{5 \text{ mA}} = 1.3 \text{ k}\Omega, \quad r_{DS5} = \frac{12000 \times 0.8}{5 \text{ mA}} = 1.9 \text{ k}\Omega$$

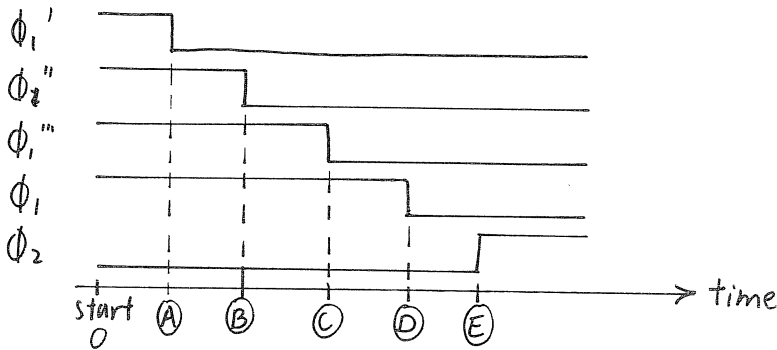
$$\therefore \tau_{\Phi_2} = 1.3 \text{ k}\Omega // 1.9 \text{ k}\Omega \times 1 \text{ pF} = \underline{0.77 \text{ nsec}}$$

\therefore The time constants during the reset and comparison phases are 0.17 nsec and 0.77 nsec respectively.

7.4) Show clock feedthrough is $|V_{err-3}|/(A_1 A_2)$



Clcking diagram :



* switches are closed with waveforms high *

Time	$V_{(1)}$	$V_{(2)}$	$V_{(3)}$	$V_{(4)}$	$V_{(5)}$	V_{out}
0	0V	0V	0V	0V	0V	0V
(A)	V_{err1}	$-A_1 V_{err1}$	0V	0V	0V	0V
(B)	V_{err1}	$-A_1 V_{err1}$	V_{err2}	$-A_2 V_{err2}$	0V	0V
(C)	V_{err1}	$-A_1 V_{err1}$	V_{err2}	$-A_2 V_{err2}$	V_{err3}	$-A_3 V_{err3}$
(D)	"	"	"	"	"	"
(E)	$V_i + V_{err1}$	$-A_1 (V_i + V_{err1})$	$-A_1 V_i + V_{err2}$	$(A_1 A_2 V_i - A_2 V_{err2})$	$(-A_1 A_2 A_3 V_i - A_3 V_{err3})$	$(A_1 A_2 V_i + V_{err3})$

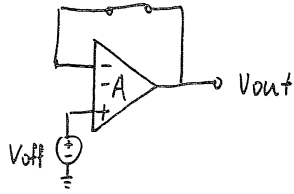
∴ at the end of ϕ_2 ,

$$V_{out} = -(A_1 A_2 A_3 V_i + A_3 V_{err3})$$

$$= -A_1 A_2 A_3 (V_i + V_{i\text{off}}) \text{ where } V_{i\text{off}} = \frac{V_{err3}}{A_1 A_2}$$

∴ the input-referred offset is $\frac{V_{err3}}{A_1 A_2}$. Q.E.D.

7.5) Repeat Problem 7.4, but now each opamp has input offset, V_{offi} .



$$V_{out} = -A(V_{off} - V_{out})$$

$$\therefore V_{out} = \frac{-A}{1-A} V_{off}$$

Time	V_D	$V_{(2)}$	$V_{(3)}$	$V_{(4)}$	$V_{(5)}$	V_{out}
0	$\frac{-A_1}{1-A_1} V_{off1}$	$\frac{-A_1}{1-A_1} V_{off1}$	$\frac{-A_2}{1-A_2} V_{off2}$	$\frac{-A_2}{1-A_2} V_{off2}$	$\frac{-A_3}{1-A_3} V_{off3}$	$\frac{-A_3}{1-A_3} V_{off3}$
	$\equiv K_1$		$\equiv K_2$		$\equiv K_3$	
(A)	$K_1 + V_{err1}$	$K_1 - A_1 V_{err1}$	K_2	K_2	K_3	K_3
(B)	"	"	$K_2 + V_{err2}$	$K_2 - A_2 V_{err2}$	K_3	K_3
(C)	"	"	"	"	$K_3 + V_{err3}$	$K_3 - A_3 V_{err3}$
(D)	"	"	"	"	"	"
(E)	$(V_i + V_{err1} + K_1)$	$(-A_1 V_i + V_{err2} + K_2)$	$(A_1 A_2 V_i + V_{err3} + K_3)$			
	$(-A_1(V_i + V_{err1}) - K_1)$	$(A_1 A_2 V_i - A_2 V_{err2} + K_2)$				

$$V_{out} = -A_1 A_2 A_3 V_i - A_3 V_{err3} + K_3$$

$$\therefore V_{out} = -(A_1 A_2 A_3 V_i + A_3 V_{err3} + \frac{A_3}{1-A_3} V_{off3})$$

$$= -A_1 A_2 A_3 (V_i + V_{ioff})$$

$$\text{where } V_{ioff} = \frac{V_{err3}}{A_1 A_2} + \frac{1}{A_1 A_2 (1-A_3)} V_{off3}$$

\therefore the input-referred offset voltage is

$$V_{ioff} = (V_{err3} + \frac{1}{1-A_3} V_{off3}) / (A_1 A_2)$$

7.6)

$$\omega_u = g_m / C_{gs}$$

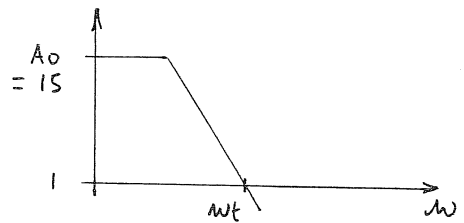
$$\omega_t = \omega_u / 2 = g_m / 2C_{gs}$$

$$\text{But } C_{gs} = \frac{2}{3} C_{ox} WL$$

$$\text{and } g_m = \mu_n C_{ox} W/L V_{eff}$$

$$\therefore \omega_t = \frac{\mu_n C_{ox} W/L V_{eff}}{\frac{4}{3} C_{ox} WL}$$

$$\omega_t = \frac{3 \mu_n V_{eff}}{4 L^2}$$



The time constant, τ , for a single stage is

$$\tau = \frac{1}{\omega_{p1}}$$

The gain-bandwidth product = $\omega_t = A_0 \omega_{p1}$

$$\therefore \omega_{p1} = \frac{\omega_t}{A_0} \text{ and } \tau = A_0 / \omega_t$$

$$\tau_{\text{comparator}} = 3 \times \tau = 3 A_0 / \omega_t$$

$$= 3 A_0 \times \frac{4 L^2}{3 \mu_n V_{eff}}$$

$$= \frac{4 A_0 L^2}{\mu_n V_{eff}}$$

Assuming $V_{eff} = 0.25V$, $\mu_n = 0.05 \text{ m}^2/\text{v}\cdot\text{s}$ and a settling time of $3\tau_{\text{comparator}}$,

$$\tau_{\text{comparator}} = \frac{4(15)(0.8 \times 10^{-6})^2}{0.05 \times 0.25} = 3.07 \text{ nsec}$$

$$\therefore f_{\text{max}} = \frac{1}{6 \tau_{\text{comparator}}} = \frac{1}{6 \times 3.07 \times 10^{-9}}$$

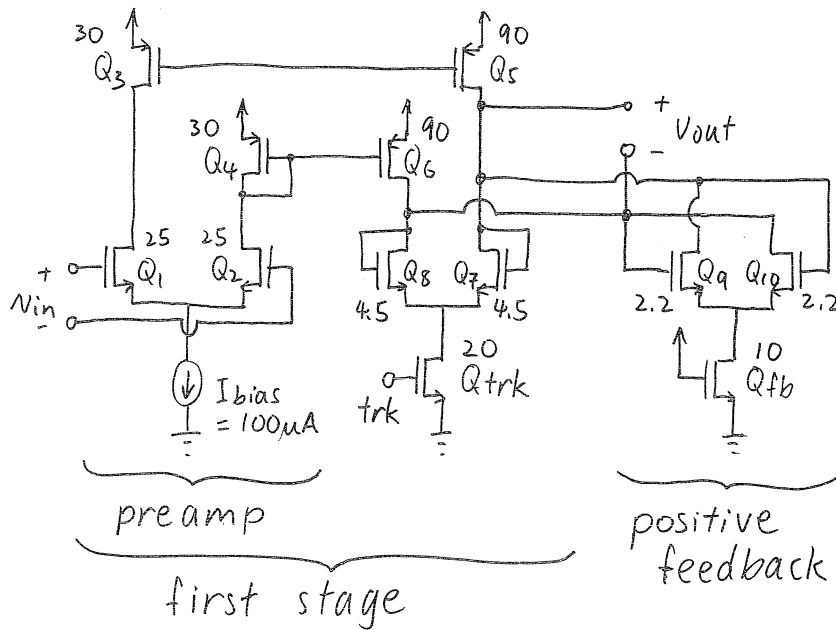
$\therefore \underline{f_{\text{max}} = 54 \text{ MHz}}$ is the maximum frequency of operation

$$\text{resolution} = A_0^3 = 15^3 = 3375$$

$$= \underline{71 \text{ dB}}$$

7.7) Determine the transistor sizes.

* Devices are all of length $L = 0.8 \mu\text{m}$ *

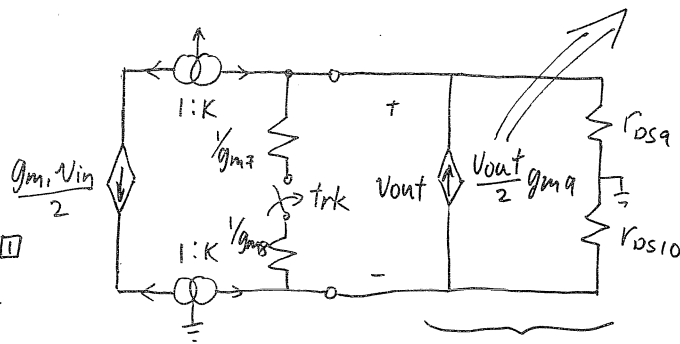


ASIDE:
by source
absorption

$$R = \frac{-2}{g_{m9}}$$

Simplified
small signal
model

$$K = \frac{W_{5,6}}{W_{3,4}} = \frac{2 I_{D5,6}}{I_{bias}} \quad \square$$



positive feedback

negligible

KCL at V_{out+} :

$$K g_{m1} \frac{V_{in}}{2} + \frac{V_{out}}{2} g_{m9} - V_{out} / \left(\frac{1}{g_{m7}} + \frac{1}{g_{m8}} \right) - \cancel{V_{out} / (r_{O5a} + r_{O5b})} = 0$$

$$\therefore V_{in} \frac{K g_{m1}}{2} + V_{out} \left(\frac{g_{m9}}{2} - \frac{g_{m7} g_{m8}}{g_{m7} + g_{m8}} \right) \approx 0$$

$$\therefore \frac{V_{out}}{V_{in}} \approx \frac{-K g_{m1} / 2 \times (2 (g_{m7} + g_{m8}))}{g_{m9} (g_{m7} + g_{m8}) - 2 g_{m7} g_{m8}} = \frac{-K g_{m1} (g_{m7} + g_{m8})}{g_{m9} (g_{m7} + g_{m8}) - 2 g_{m7} g_{m8}}$$

Assuming Q_7 and Q_8 are matched so that $g_{m7} = g_{m8}$

$$\therefore \frac{V_{out}}{V_{in}} = \frac{K g_{m1}}{g_{m7} - g_{m9}} \quad \square \quad \text{Tracking gain}$$

(cont.)

7.7 (cont.)

The gain of the first stage is determined by ignoring the positive feedback circuit.

$$\therefore \text{Let } g_{mq} \equiv 0$$

$$\text{and } \frac{v_{out}}{v_{in}} = K \frac{g_{m1}}{g_{m7}} \equiv 5$$

$$\therefore \underline{g_{m7} = K/5 \times g_{m1}} \quad \text{[B]}$$

Setting the tracking gain to 10 gives

$$\frac{K g_{m1}}{g_{m7} - g_{mq}} \equiv 10 \quad \text{[C]}$$

$$\text{sub [B]} \rightarrow \text{[C]} \quad \frac{K g_{m1}}{K/5 \times g_{m1} - g_{mq}} = 10$$

$$\underline{g_{mq} = \frac{K g_{m1}}{10}} \quad \text{[D]} \quad \text{or} \quad \underline{g_{mq} = 1/2 g_{m7}} \quad \text{[E]}$$

Under ideal bias conditions, $v_{out+} = v_{out-}$.

Thus, even though Q_9 and Q_{10} are cross-coupled unlike Q_7 and Q_8 , their terminals are biased at the same voltages as Q_9 and Q_{10} .

\therefore To satisfy equation [E], we simply scale device widths such that

$$\underline{w_{9,10} = 1/2 w_{7,8}} \quad \text{[F]}$$

Note that the drain currents are also scaled

$$\text{i.e., } \underline{I_{D9,10} = 1/2 I_{D7,8}} \quad \text{[G]}$$

Now find $w_{7,8}$ in terms of $w_{1,2} = 25 \mu\text{m}$:

From [B], [I]

$$g_{m7} = K/5 g_{m1} = \frac{2}{5} \times \frac{I_{D5,6}}{I_{bias}} \times g_{m1}$$

$$\text{where } I_{D5,6} = I_{D7,8} + I_{D9,10} \quad \text{[H]}$$

$$\text{[G]} \rightarrow \text{[H]} \quad = 3/2 I_{D7,8}$$

(cont.)

7.7) (cont.)

$$\therefore g_{m7}^2 = \left(\frac{8}{5} \times \frac{3}{2} \frac{I_{D7,8}}{I_{bias}} \times g_{m1} \right)^2$$

$$\therefore \frac{2 \mu_n C_{ox}}{L} W_{7,8} I_{D7,8} = \frac{9}{25} \times \frac{I_{D7,8}}{I_{bias}} \times \frac{2 \mu_n C_{ox}}{L} W_{1,2} \frac{I_{bias}}{2}$$
$$\frac{W_{7,8}}{W_{1,2}} = \frac{9}{50} \frac{I_{D7,8}}{I_{bias}} \quad \text{for } W_{1,2} = 25 \mu\text{m}$$

$$\therefore W_{7,8} = \frac{9}{2} \frac{I_{D7,8}}{I_{bias}} \mu\text{m}$$

If we let $I_{D7,8} = I_{bias}$,

$$\therefore \underline{W_{7,8} = \frac{9}{2} \mu\text{m} = 4.5 \mu\text{m}}$$

$$\therefore \underline{W_{9,10} = \frac{1}{2} W_{7,8} = 2.2 \mu\text{m}}$$

$$\therefore K = \frac{2 I_{D5,6}}{I_{bias}} = \frac{8 \left(\frac{3}{2} I_{D7,8} \right)}{I_{bias}} = \underline{3} = \frac{W_{5,6}}{W_{3,4}}$$

If we let $W_{3,4} = 30 \mu\text{m}$

then $W_{5,6} = 90 \mu\text{m}$

Since Q_{trk} and Q_{fb} operate as switches in the triode region, their size is not critical.

If we set

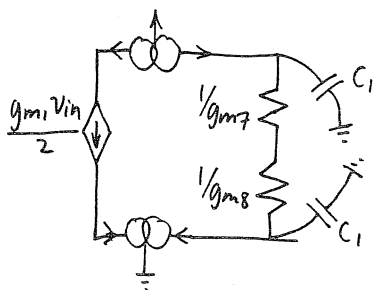
$$\underline{W_{trk} = 20 \mu\text{m}},$$

then to match voltage levels as closely as possible choose

$$\underline{W_{fb} = \frac{1}{2} W_{trk} = 10 \mu\text{m}}$$

Final device widths are marked on the original schematic diagram.

7.8) Estimate time constant in track mode, $\tau_{trk} \approx \tau_{gain} + \tau_{pre}$



Gain stage

$$\tau_{gain} = \frac{C_1}{g_{m7}}$$

where $C_1 = C_{db5} + C_{db7} + C_{gs10} + C_{db9}$
 $\approx C_{db5}$ (device Q_5 is much larger than devices Q_7, Q_9, Q_{10})

$$= A_{d5} C_{d5} + P_{dsw5} \times C_{ov}$$

$$= 30 \times 4 \times 0.8 \times 4.5 \times 10^{-4} \text{ pF}/\mu\text{m}^2$$

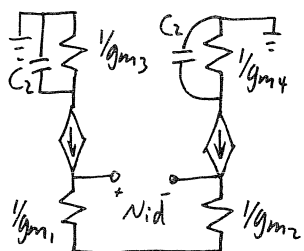
$$+ 2 \times 4 \times 0.8 \times 2.5 \times 10^{-4} \text{ pF}/\mu\text{m}^2$$

$$C_1 \approx 52 \text{ fF}$$

and $g_{m7} = 0.107 \text{ mA/V}$ from Problem 7.7.

$$\therefore \tau_{gain} = \frac{52 \times 10^{-15} \text{ F}}{0.107 \times 10^{-3} \text{ A/V}} = 0.47 \text{ nsec}$$

Now look at time constant associated with the preamp, τ_{pre} .



Preamp stage

$$\tau_{pre} = \frac{C_2}{g_{m3}} \text{ where}$$

$$C_2 = C_{db3} + C_{gs3} + C_{gs5} + C_{db1} \quad \text{negligible } \because Q_1 \text{ small}$$

$$C_{db3} = A_{d3} C_{d3} + P_{dsw3} \times C_{ov}$$

$$= 100 \times 0.8 \times 4 \times 4.5 \times 10^{-4} +$$

$$(2 \times 4 \times 0.8 + 100) \times 2.5 \times 10^{-4}$$

$$= 170 \text{ fF}$$

$$C_{gs3} = \frac{2}{3} W L C_{ox} + W C_{ov}$$

$$= \frac{2}{3} \times 100 \times 0.8 \times 1.9 \times 10^{-3} + 100 \times 2 \times 10^{-4}$$

$$= 120 \text{ fF}$$

$$C_{gs5} = \frac{2}{3} W L C_{ox} + W C_{ov}$$

$$= \frac{2}{3} \times 30 \times 0.8 \times 1.9 \times 10^{-3}$$

$$+ 30 \times 2 \times 10^{-4}$$

$$= 40 \text{ fF}$$

$$\therefore C_2 = 170 + 120 + 40 \text{ fF} = 330 \text{ fF}$$

$$\text{and } g_{m3} = \sqrt{2 \mu_n C_{ox} W/L I_{D3}} = \sqrt{2 \times 30 \times 10^{-6} \times 100 \times 0.8 \times 50 \times 10^{-6}}$$

$$= 0.612 \text{ mA/V}$$

$$\therefore \tau_{pre} = \frac{330 \times 10^{-15}}{0.612 \times 10^{-3}} = 0.54 \text{ nsec}$$

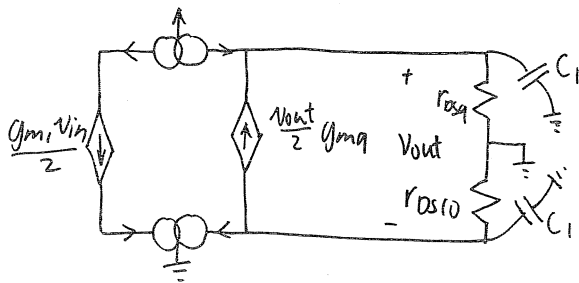
$$\therefore \tau_{trk} \approx \tau_{gain} + \tau_{pre} = 1.01 \text{ nsec}$$

(cont.)

7.8 (cont.)

Assuming it takes three time constants to settle from a step change of 50 mV, it would take 3 nsec.

7.9) Determine time constant in latch mode.



From Problem 7.8,

$$C_1 = 52 \text{ fF}$$

$$\begin{aligned} g_{m9} &= \sqrt{2 \mu_n C_{ox} W/L I_{D9}} \\ &= \sqrt{2 \times 92 \times 10^{-6} \times 2.4/0.8 \times 5 \times 10^{-6}} \\ &= 53 \text{ mA/V} \end{aligned}$$

KCL at V_{out+} :

$$\frac{g_{m1} V_{in}}{2} + \frac{V_{out}}{2} g_{m9} - \frac{V_{out}}{r_{os9} + r_{os10}} - V_{out} s C_1 = 0$$

$$\begin{aligned} \therefore \frac{V_{out}}{V_{in}} &\approx \frac{-g_{m1}/2}{g_{m9}/2 - \frac{1}{2r_{os9}} - s C_1/2} \\ &= \frac{g_{m1}}{C_1 (s - g_{m9}/C_1 + \frac{1}{C_1 r_{os9}})} \approx \frac{g_{m1}}{C_1} \times \frac{1}{s - g_{m9}/C_1} \end{aligned}$$

\therefore there is an unstable pole at

$$\omega = g_{m9}/C_1$$

and the associated time constant is

$$\tau = C_1/g_{m9} = 52 \text{ fF}/0.53 \text{ mA/V}$$

$$\tau = 0.99 \text{ nsec}$$

For a differential output of 2 V,

$$V_{out}(t) = V_{out0} e^{t/\tau}$$

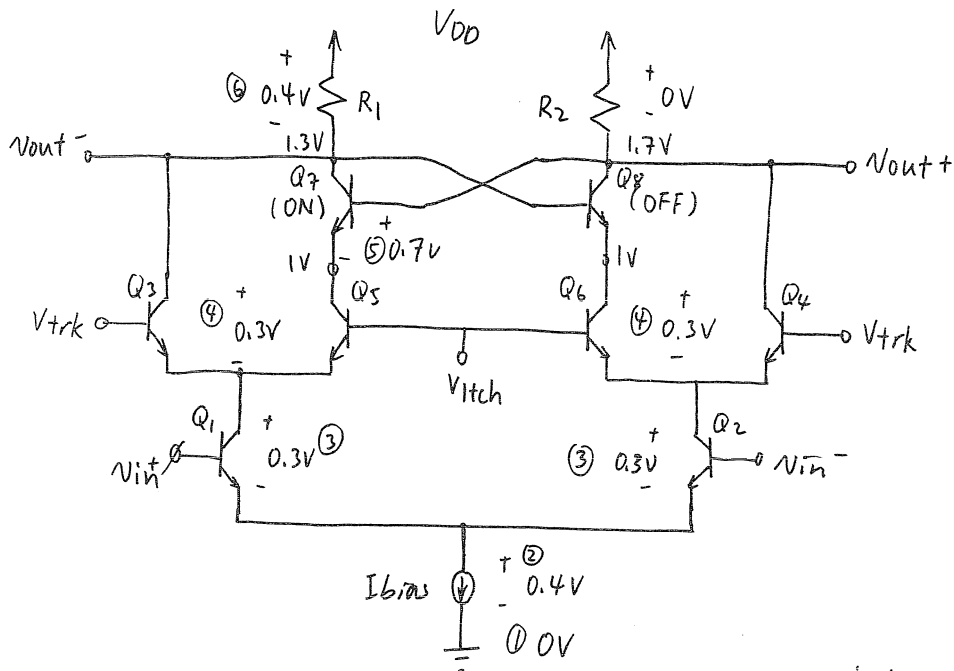
$$\therefore 2 \text{ V} = 0.05 e^{t/0.99 \times 10^{-9}}$$

$$t = 0.99 \times 10^{-9} \ln 40$$

$$= \underline{3.65 \text{ nsec}}$$

\therefore it takes 3.65 nsec.

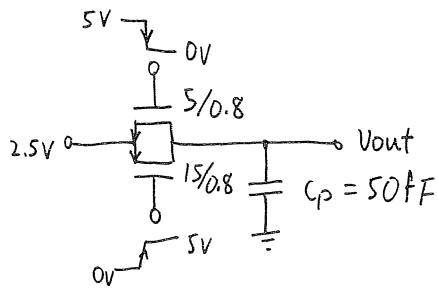
7.10) Find ^{the} minimum power supply voltage.



From the above diagram we see that the ^{minimum} voltage drops from R_2 through devices Q_7 , Q_5 , Q_1 , & I_{bias} to ground add up to 1.7V. Assuming $V_{DD} = 1.7V$, we see that V_{out}^- is at $1.7V - 0.4V = 1.3V$.

- ∴ we have correctly assumed that Q_8 is off.
- ∴ this circuit requires a minimum supply voltage of 1.7V.

7.11)



Determine the voltage change due to each device.

N-MOS:

$$\Delta V_N = - \frac{V_{effn} C_{ox} W_n L_n}{2C_p} \quad \text{where}$$

$$V_{effn} = 5V - 2.5V - V_{tn}$$

$$V_{tn} = V_{tn0} + \gamma (\sqrt{V_{sB} + 2\phi_F} - \sqrt{2\phi_F})$$

$$= 0.8 + 0.5 (\sqrt{2.5 + 0.7} - \sqrt{0.7})$$

$$V_{tn} = 1.28V$$

$$\therefore V_{effn} = 1.22V$$

$$\therefore \Delta V_N = - \frac{1.22V \times 1.9 \times 10^{-3} \text{ PF}/\mu\text{m}^2 \times 5 \times 0.8}{2 \times 50 \times 10^{-15}} = \underline{\underline{-93 \text{ mV}}}$$

PMOS: $\Delta V_p = \frac{V_{effp} C_{ox} W_p L_p}{2C_p}$ where

$$V_{effp} = 2.5V - 0 - V_{tp}$$

$$V_{tp} = V_{tp0} + \gamma (\sqrt{V_{bS} - 2\phi_F} - \sqrt{2\phi_F})$$

To determine ϕ_F , use

$$\gamma = \frac{\sqrt{2qK_s \epsilon_0 N_A}}{C_{ox}}$$

$$\therefore 0.8 = \frac{\sqrt{2 \times 1.602 \times 10^{-19} \times 8.85 \times 10^{-12} \times 11.8 \times N_A}}{1.9 \times 10^{-15}}$$

$$\therefore \underline{N_A = 6.9 \times 10^{22} \text{ m}^{-3}}$$

$$\Rightarrow \phi_F = \frac{kT}{q} \ln \left(\frac{N_A}{n_i} \right) = \frac{1.38 \times 10^{-23} \times 300}{1.602 \times 10^{-19}} \ln \left(\frac{6.9 \times 10^{22}}{1.1 \times 10^{16}} \right)$$

$$= 0.40V$$

(cont.)

7.11 (cont.)

$$\begin{aligned} \therefore V_{tp} &= -0.9 - 0.8(\sqrt{2.5+0.8} - \sqrt{0.8}) \\ &= 1.64V \end{aligned}$$

$$\therefore V_{effp} = 2.5 - 1.64V = 0.86V$$

$$\therefore \Delta V_p = \frac{0.86V \times 1.9 \times 10^{-3} \text{ pF} \times 15 \times 0.8}{2 \times 50 \times 10^{-3} \text{ pF}} = +196 \text{ mV}$$

$$\begin{aligned} \therefore \Delta V_{out} &= \Delta V_n + \Delta V_p = -93 \text{ mV} + 196 \text{ mV} \\ &= \underline{+103 \text{ mV}} \end{aligned}$$

\therefore the final output voltage is $2.5V + 103 \text{ mV} \approx 2.6V$