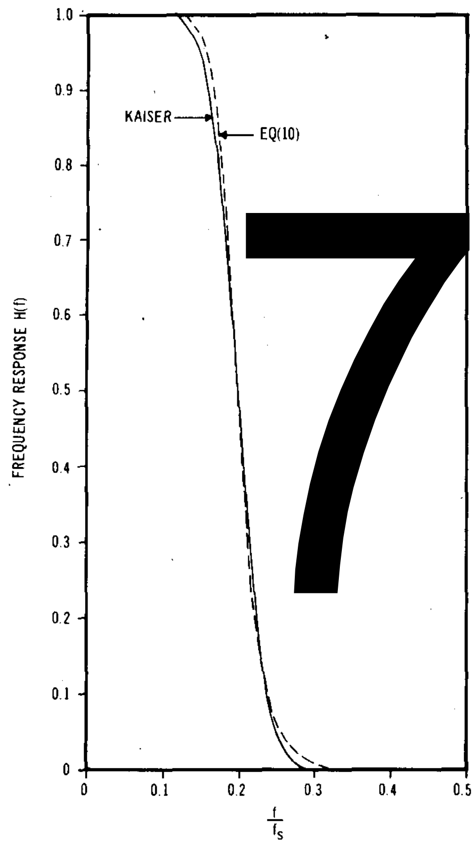

Fig. 1. Comparison of frequency response functions.

variation with N in the measured values of Δ. All were within 1% of the specified value. The point of the comparison, however, is not to claim that the proposed design is better, but that it combines respectable performance with both formal simplicity and computational economy.

References

[1] A. V. Oppenheim and R. W. Schafer, *Digital Signal Processing*. Englewood Cliffs, NJ: Prentice-Hall, 1975.

[2] X. Z. Sun and N. Venetsanopoulos, "On the evaluation of the Fourier series coefficients in the design of digital FIR filters," *IEEE Trans. Circuits Syst.*, vol. CAS-32, pp. 1304–1308, Dec. 1985.

[3] A. Erdelyi *et al.*, *Tables of Integral Transforms*, vol. 1. New York: McGraw-Hill, 1954.

[4] J. F. Kaiser, "Design subroutine (MXFLAT) for symmetric FIR lowpass digital filters with maximally-flat pass and stop bands," in *Programs for Digital Signal Processing*. New York: IEEE Press, 1979.

State-Space Simulation of *LC* Ladder Filters

D. A. JOHNS AND A. S. SEDRA

***Abstract*—A new design method to obtain a state-space system which simulates the operation of an *LC* ladder prototype is introduced. The state-space system is relatively sparse and can be found using simple algebraic manipulations. Through the use of an example, it is shown that the resulting filter maintains the low-sensitivity properties of the ladder prototype.**

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I. Introduction

Properly designed doubly-terminated LC ladder networks are known to have low element sensitivities in their filter passband [1], [2]. Because of this fact, many papers have been written proposing methods for the design of filters based on the operational simulation of LC ladder prototypes [3]–[6]. In the case of all-pole filters, almost all these methods (including the one presented in this letter) yield the well-known leap-frog filter structure. However, structures obtained from different methods vary significantly when the filter is derived from a noncanonic LC ladder which has finite transmission zeros. This letter proposes a simple method to obtain a canonic state-space system which maintains the low-sensitivity properties of a canonic or noncanonic LC ladder.

II. Design Procedure

A Nth-order state-space system is described by the equations

$$sx = Ax + bu$$
$$y = c^T x + du \tag{1}$$

where u is the input signal, x is a vector of N states, which in fact are the integrator outputs, y is the output signal, and A, b, c, and d are coefficients relating these variables.

Given an LC ladder prototype we wish to simulate, we first choose a set of N states from the ladder. With a canonic ladder, we choose all the capacitor voltages and inductor currents as the states. In the case of a noncanonic ladder with N_f finite loss poles, we choose N linearly independent states from the set of $N + N_f$ capacitor voltages and inductor currents. Requiring that the N states be linearly independent implies we must not choose all the capacitor voltages or inductor currents in a reactive cutset or tieset. However, even with this constraint satisfied, there are still many ways to choose N linearly independent states from a noncanonic ladder. Since it has been shown that different selections for the states from the LC ladder can lead to structures with dramatically different performance [7], care must be exercised in this selection. It has been found that good filter realizations are obtained if the element in each resonant tank which is part of a cutset or tieset is not chosen as one of the states to be simulated. In the eighth-order LC ladder of Fig. 1, V_{C_3} and V_{C_5} are the element voltages which should not be simulated. This simple method of choosing states to simulate has given good results on all filters simulated to date.

Once a choice of states from the LC ladder has been made, we proceed to find the state-space system which will simulate these states. For each capacitor whose voltage has been chosen as a state, we write a node equation expressing the current through the capacitor in terms of the other state variables. Similarly, for each chosen inductor current, we write a loop equation expressing the voltage across the inductor in terms of the other state variables. Using cutset and tieset dependencies, all equations can be written containing only element voltages or currents which are states. These equations can be put in the following form:

$$Msx = Nx + RV_i \tag{2}$$

where x is, as before, an $N \times 1$ vector of states, V_i is the input voltage, M and N are $N \times N$ real matrices, and R is an $N \times 1$

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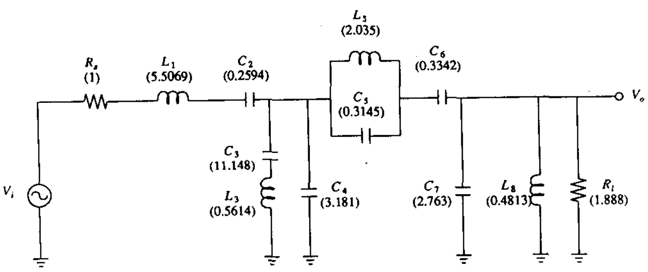

Fig. 1. An eighth-order bandpass LC ladder to be simulated.

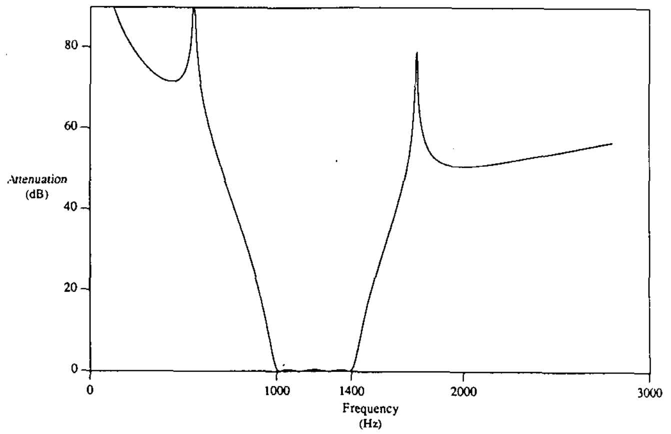

Fig. 2. Transfer function of eighth-order design example.

real matrix. Multiplying the above equation by M^{-1} results in a canonic state-space equation where A and b are given by

$$A = M^{-1}N \tag{3}$$

$$b = M^{-1}R. \tag{4}$$

The c vector and d scalar of the state-space system can then be easily obtained by writing the final output voltage as a sum of states and the input.

$$A = \begin{bmatrix} -0.1816 & -0.1816 & 0 & -0.1816 & 0 & 0 & 0 & 0 \\ 3.855 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -0.0414 & 0 & 2.2895 & 0 & 0.0534 & 0 & 0 \\ 0.2998 & 0 & -0.2998 & 0 & -0.1463 & 0 & -0.0089 & -0.0165 \\ 0 & 0 & 0 & 0.4914 & 0 & -0.4914 & -0.4914 & 0 \\ 0.1374 & 0 & -0.1374 & 0 & 1.3893 & 0 & 0.0838 & 0.1582 \\ 0.0166 & 0 & -0.0166 & 0 & 0.1680 & 0 & -0.1816 & -0.3428 \\ 0 & 0 & 0 & 0 & 0 & 0 & 2.0777 & 0 \end{bmatrix} \quad b = \begin{bmatrix} 0.1816 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}.$$

$$c^T = [0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 1 \ 0] \quad d = 0.$$

Before leaving this section, it is interesting to note that (2) describes a noncanonic system which simulates an LC ladder. The system would consist of N integrators with resistive and capacitive feed-ins to the integrators. For low-pass filters, the structures obtained would be equivalent to those derived by the capacitor-splitting technique described in [5]. Also of interest is the fact that for most LC ladder simulations, matrix M is close to being diagonal and thus keeping the left- and right-hand sides of (2) separated, the standard state-space system description can be obtained using simple algebraic manipulations.

III. Design Example and Sensitivity Comparison

In this section, an eighth-order canonic state-space simulation of the ladder shown in Fig. 1 will be designed and the simulation's integrator sensitivities compared with the ladder's element sensitivities. The denormalized transfer function of the ladder is shown in Fig. 2.

Using the guidelines of the previous section, we choose not to simulate V_{C_3} and V_{C_5}, and use the following equivalents found from cutset and tieset dependencies[1]:

$$V_{C_3} = \frac{C_2}{C_3} V_{C_2} - \frac{C_4}{C_3} V_{C_4} - \frac{C_6}{C_3} V_{C_6} \tag{5}$$

$$V_{C_5} = V_{C_4} - V_{C_6} - V_{C_7}. \tag{6}$$

Eight independent equations are then written involving the eight chosen states as follows:

$$sL_1 I_{L_1} + R_s I_{L_1} + V_{C_2} + V_{C_4} = V_i \tag{7}$$

$$sC_2 V_{C_2} - I_{L_1} = 0 \tag{8}$$

$$sL_3 I_{L_3} + \left(\frac{C_2}{C_3} V_{C_2} - \frac{C_4}{C_3} V_{C_4} - \frac{C_6}{C_3} V_{C_6} \right) - V_{C_4} = 0 \tag{9}$$

$$sC_4 V_{C_4} + I_{L_3} - I_{L_1} + I_{L_5} + sC_5 \left(V_{C_4} - V_{C_6} - V_{C_7} \right) = 0 \tag{10}$$

$$sL_5 I_{L_5} - V_{C_4} + V_{C_6} + V_{C_7} = 0 \tag{11}$$

$$sC_6 V_{C_6} - I_{L_5} - sC_5 \left(V_{C_4} - V_{C_6} - V_{C_7} \right) = 0 \tag{12}$$

$$sC_6 V_{C_6} - sC_7 V_{C_7} - I_{L_8} - \frac{V_{C_7}}{R_l} = 0 \tag{13}$$

$$sL_8 I_{L_8} - V_{C_7} = 0. \tag{14}$$

Substituting the values of the capacitors, inductors, and resistors in the above equations and using the approach in Section II to obtain the state-space system, we find the following matrices:

In order to compare the sensitivity properties of the state-space simulation with the original LC ladder, we define maximum integrator sensitivity, max S, as

$$\max S = \max_i |S^{|T(s)|}_{\text{integ}_i}| \tag{15}$$

[1]In the capacitive cutset formed by V_{C_2}, V_{C_3}, V_{C_4}, and V_{C_6}, the capacitor currents add to zero and thus the capacitor voltages weighted by their element values add to a constant. This constant can be forced to zero since the ladder being simulated has at least one loss pole at dc.

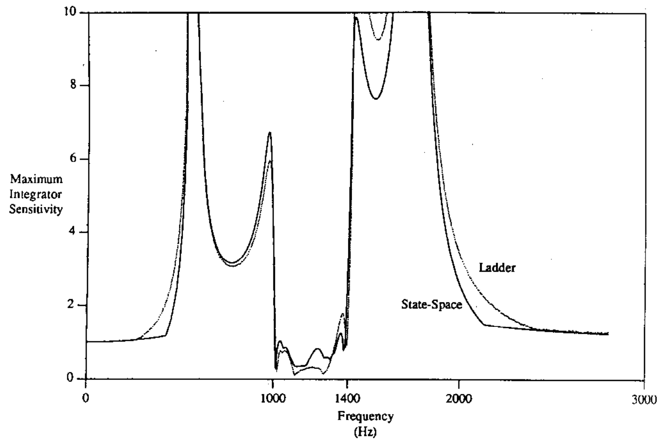

Fig. 3. Maximum integrator sensitivity: *LC* ladder versus state-space simulation.

where integ_i is the ith integrator of either the ladder or state-space system. For the ladder, each of the reactive elements are considered integrators and thus maximum integrator sensitivity is equivalent to maximum element sensitivity.

Shown in Fig. 3 are the maximum integrator sensitivities of the ladder and state-space simulation for the eighth-order design example. The ladder's sensitivities were found using SPICE and post-processing the SPICE output, whereas analytic formulas presented in [7] were used to calculate the state-space system's sensitivities.

IV. Conclusions

This letter presented a simple, systematic method to obtain a state-space system which simulates the operation of an *LC* ladder prototype. The states of the system are chosen from an *LC* ladder and then node and loop equations are written around elements of the ladder using only the chosen states. The system matrices $\boldsymbol{A}$, $\boldsymbol{b}$, $\boldsymbol{c}$, and d are then found using simple algebraic manipulations. An example illustrated that the sensitivity properties of filters obtained from this design method closely approximate that of the original *LC* ladder prototype.

References

[1] H. J. Orchard, "Inductorless filters," *Electron Lett.*, vol. 2, pp. 224–225, June 1966.

[2] H. J. Orchard, G. C. Temes, and T. Cataltepe, "Sensitivity formulas for terminated lossless two-ports," *IEEE Trans. Circuits Syst.*, vol. CAS-32, pp. 459–466, May 1985.

[3] P. O. Brackett and A. S. Sedra, "Direct SFG simulation of *LC* ladder networks with applications to active filter design," *IEEE Trans. Circuits Syst.*, vol. CAS-23, pp. 61–67, Feb. 1976.

[4] L. T. Bruton, "Multiple-amplifier *RC*-active filter design with emphasis on GIC realizations," *IEEE Trans. Circuits Syst.*, vol. CAS-25, pp. 830–845, Oct. 1978.

[5] G. M. Jacobs, D. J. Allstot, R. W. Broderson, and P. R. Gray, "Design techniques for MOS switched-capacitor ladder filters," *IEEE Trans. Circuits Syst.*, vol. CAS-25, pp. 1014–1021, Dec. 1978.

[6] E. Hayahara, "Design of active *RC* filters simulating node equations of *LC* filters," *IEEE Trans. Circuits Syst.*, vol. CAS-31, pp. 394–398, Apr. 1984.

[7] W. M. Snelgrove and A. S. Sedra, "Synthesis and analysis of state-space active filters using intermediate transfer functions," *IEEE Trans. Circuits Syst.*, vol. CAS-33, pp. 287–301, Mar. 1986.

Some Results on Two-Dimensional Pseudoquadrature Mirror Filters

A. TABATABAI

***Abstract* — One-dimensional (1-D) pseudoquadrature mirror filters (PQMF's) have been introduced as a generalization of quadrature mirror filter (QMF) concepts. Use of such filters allows one to split directly the spectrum of an input signal into N equally spaced subbands which can be decimated by N:1, then interpolated and recombined to reproduce the original signal. In this paper, a detailed derivation of conditions specifying 2-D PQMF design constraints for a distortion-free (i.e., aliasing and amplitude distortions) reconstruction of the original signal are obtained. Potential applications for such filters include efficient subband coding of digital pictures.**

I. Introduction

Since its introduction [1], conventional quadrature mirror filter (QMF) banks have received considerable attention for subband coding of speech [2], [3]. In this technique, by parallel application of a low-pass and a high-pass filter, the input signal spectrum is split into two overlapping subbands where each band is then decimated by a factor two, and coded separately. For reconstruction, decimated signals are decoded, interpolated, and filtered by a similar set of filters before being added to reproduce the original signal. Note that, in the absence of channel and quantization noise, QMF design requirements allow a near-perfect reconstruction of the input signal. Decomposition of the input signal into more than two bands can similarly be accomplished by repeating the above process in a tree-type filter structure. This approach, however, demands N to be a power of two if N equally spaced subbands are desired.

As an alternative, Nussbaumer [4] and Rothweiler [5] have presented a parallel bandpass filter structure that can be substituted directly for the binary tree QMF structure. This technique is based upon using N equally spaced adjacent bandpass filters to split the input signal into N equally spaced bands, and subsampling each band at $1/N$ the original rate, where N may not necessarily be a power of 2. The reconstruction is done by inserting $N-1$ zero-valued samples between successive samples of individual bands. The resulting sequences are then passed through a set of parallel bandpass filters, and added to reproduce the original signal. For N equally spaced subbands, the parallel bandpass filters are formed by 1) designing a low-pass prototype filter satisfying certain design requirements, and 2) modulating the low-pass prototype filter by a sinusoid whose center frequencies are at odd multiples of $\pi/2N$. Subsequent papers [6]–[8] have provided detailed derivations and computationally efficient methods for realizing such filters. In this paper, we shall refer to such parallel bandpass filters as pseudoquadrature mirror filters (PQMF's).

Vetterli [9] and Wackersreuther [10] have extended the PQMF concept to multidimensional case. They have not, however, provided detailed derivations of conditions leading to a distortion-free reconstruction of the input signal in a back-to-back arrangement of analysis and synthesis filter banks. In this paper, depend-

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