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*a reprint from Applied Optics
volume 22, number 1, January 1, 1983*

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Photoelectron counting distributions are obtained for sources which obey compound Poisson statistics. Various cases are considered in which the sources (semiconductor lasers) emit coherent light and their intensity fluctuates in accordance with a Gaussian distribution of operating temperatures. The lasers are otherwise assumed to be ideal, and the quantum efficiency of the detector is assumed to be unity. This paper represents an ideal situation where the source is the only concern in the calculation of the photoelectron counting distributions. It is found that for large temperature fluctuations ($\sigma > 10$ K), a substantial downward shift of the peak of the photon probability density function is observed. The function becomes more asymmetric and the mean value decreases as the standard deviation of the temperature increases.

I. Introduction

Statistical photon counting distributions for coherent sources of radiation have been discussed by many authors (see, for example, Refs. 1-4). However, in their determination of these distributions, these studies usually considered sources either with constant amplitudes or with specific types of modulation (i.e., pulsed, triangular, sine wave⁴). In this paper, on the other hand, the modulation is in the form of a stationary random process which arises from fluctuations in operating temperature.

Since the advent of the laser and specifically the semiconductor laser, development of the photon/photoelectron distributions and of the experimental determination of these distributions has been of great interest. Intrinsic fluctuations of the output intensity of semiconductor lasers have been dealt with by Paoli.^{5,6} These intensity fluctuations are usually attributed to noise in the lasers. Modal, shot noise, and noise due to spontaneous emission in the laser can all account for stochastic fluctuations in the output intensity. Fluctuations in the operating temperature of the device will also give rise to intensity fluctuations.^{7,8} Given that the temperature of a certain environmental condition is stochastic, the intensity fluctuations due solely to changes in the temperature will be stochastic.

II. Theory

We consider first a monochromatic source that is illuminating an ideal detector, whose internal quantum efficiency is unity. Since the nature of the source is being explored, we assume that the area of the detector is much smaller than the coherence area of the source. The detected photons will have a statistical distribution about a mean value with a characteristic coherence time. This is simply due to the statistical nature of the arrival

rate of the photons. There will then be a particular probability associated with the detection of n photons within a certain time interval Δ . This distribution is given by the Mandel formula⁹ (simple Poisson statistics) in the absence of intensity fluctuations (i.e., constant value)³

$$p(n/\lambda_0) = \frac{\lambda_0^n \exp(-\lambda_0)}{n!}, \quad (1)$$

where $\lambda_0 = \langle n \rangle$ is the time average of the distribution over the interval Δ (i.e., the average intensity),

$$\lambda_0 = \int_{\tau}^{\tau+\Delta} I(\tau') d\tau', \quad (2)$$

and where I = the intensity of the laser.

Now consider the situation where the time-averaged value λ_0 is not constant over the time interval of interest Δ . We assume here that the time-averaged value is described by a stochastic process. This could be analogous to the random intensity fluctuations that are inherent in semiconductor lasers.

It has been found⁷ that the output intensity I and subsequently the time-averaged intensity λ of a semiconductor laser (well above threshold) vary linearly with the laser diode current density J . We have

$$\lambda = k_1 J, \quad (3)$$

where k_1 = a constant of proportionality,
 λ = time-averaged laser intensity, and
 J = laser diode current density.

The threshold current of the source varies exponentially with temperature.⁸

$$J_{th} = k_2 \exp(T/T_0), \quad (4)$$

where J_{th} = threshold current density of the laser,
 k_2 = a constant of proportionality,
 T = temperature, and
 T_0 = constant dependent upon the laser type

Over a wide range of temperatures the variation in threshold current due to fluctuations in temperature will produce a corresponding change in the operating current of the source. The relationship between the variation in threshold current and the operating current

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Received 29 July 1982.

0003-6935/83/010149-03\$01.00/0.

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was assumed to be linear over the range of interest. From the data of Tsang *et al.*,¹⁰ for GaAs-Al_xGa_{1-x}As buried stripe lasers, the range of operating temperatures that would produce this linear variational relationship was found to be 25–100°C. Therefore, within this range, the operating current and subsequently the output intensity of the laser will vary exponentially with temperature. By letting $J = k_3 J_{th}$ and by substituting Eq. (4) into Eq. (3) we obtain

$$\lambda = k \exp(T/T_0), \quad (5)$$

where λ = time-averaged laser intensity,
 T = temperature, and
 k = a constant of proportionality.

The value of the parameter T_0 plays an important role in the final determination of the photon distribution. It describes the temperature sensitivity of the laser source.¹¹ The current available literature offers no concise theory of its origin. It is dependent on a number of variables that makes its theoretical determination almost impossible. Therefore, a phenomenological/experimental approach has been taken.

Experimentally, the value of T_0 has been found to be 60–70 K for $T > 250$ K and 110 K for $T < 250$ K for InGaAsP–InP double-heterostructure (DH) lasers.^{12–17} For GaAs–Al_xGa_{1-x}As DH lasers $T_0 \approx 150$ –180 K for the 100 K $< T < 350$ K temperature range.^{12,18} The value of T_0 in this paper was chosen to be within the experimental range.

If a Gaussian probability density function $p_T(t)$ is assumed, as defined by the following relation,¹⁹

$$\begin{aligned} P_T(t) &= 0 & t < 0, \\ &= \frac{1}{2} \operatorname{erfc} \left(\frac{t_0}{\sqrt{2}\sigma} \right) \delta(t) & t = 0, \\ &= \frac{1}{\sqrt{2\pi}\sigma} \exp \left[-\frac{(t-t_0)^2}{2\sigma^2} \right] & t > 0, \end{aligned} \quad (6)$$

where σ = the standard deviation of temperature fluctuations (K) and t_0 = mean temperature or operating heat sink temperature (K). The probability density function of the output intensity $p_I(\lambda)$ will then be given by

$$\begin{aligned} p_I(\lambda) &= 0 & \lambda < k, \\ &= \frac{T_0}{2k} \operatorname{erfc} \left(\frac{t_0}{\sqrt{2}\sigma} \right) \delta(\lambda - k) & \lambda = k, \\ &= \frac{T_0}{\sqrt{2\pi}\sigma} \frac{1}{\lambda} \exp \left\{ -\frac{\left[T_0 \ln \left(\frac{\lambda}{k} - t_0 \right) \right]^2}{2\sigma^2} \right\} & \lambda > k, \end{aligned} \quad (7)$$

where $\delta(\lambda - k)$ = Dirac delta function. Equation (6), for the case of $t = 0$, indicates a truncated Gaussian probability density function.

Note that the expected value of λ is

$$\bar{\lambda} = \lambda_0 = E(\lambda) = \int_{-\infty}^{\infty} \lambda p_I(\lambda) d\lambda = k \exp \left(\frac{t_0}{T_0} \right), \quad (8)$$

and the constant of proportionality k becomes

$$k = \lambda_0 \exp \left(\frac{t_0}{T_0} \right). \quad (9)$$

Now Eq. (7) may be rewritten in the following form:

$$\begin{aligned} p_I(\lambda) &= 0 & \lambda < k, \\ &= \frac{T_0}{2\lambda_0} \exp \left(\frac{t_0}{T_0} \right) \operatorname{erfc} \left(\frac{t_0}{\sqrt{2}\sigma} \right) \delta(\lambda - k) & \lambda = k, \\ &= \frac{1}{\sqrt{2\pi}\sigma_1} \frac{1}{\lambda} \exp \left\{ -\frac{[\ln(\lambda/\lambda_0)]^2}{2\sigma_1^2} \right\} & \lambda > k, \end{aligned} \quad (10)$$

where $\sigma_1 = \sigma/T_0$ and λ = the random variable. The photoelectron probability density function is then given by

$$p_I(n) = \int_0^{\infty} p(n|\lambda) p_I(\lambda) d\lambda. \quad (11)$$

This function satisfies the relation for probability distributions that

$$\int_0^{\infty} p_I(n) dn = 1. \quad (12)$$

Given that the probability density function of the average value of the intensity is as described above, the actual counting probability density function is given by

$$p_I(n) = \int_0^{\infty} \frac{\lambda^n \exp(-\lambda)}{n!} \frac{1}{\sqrt{2\pi}\sigma_1} \frac{1}{\lambda} \exp \left[-\frac{\left(\ln \frac{\lambda}{\lambda_0} \right)^2}{2\sigma_1^2} \right] d\lambda. \quad (13)$$

The calculation of this integral was performed using an algorithm based upon Simpson's rule over the 10^{-5} –50.0 interval with a step size h of 0.167. The truncation error is of the order h^5 for Simpson's rule.

For very small values of λ , the logarithmic singularity may be addressed by more appropriate Gauss-Quadrature schemes.²⁰

III. Results and Discussion

Figure 1 shows the results of the calculation of the photon distributions for four values of standard deviation of the temperature fluctuations. The first represents the simple Poisson distribution with a constant mean value (simple Poisson process). The other curves show the density function that will result from Gaussian temperature fluctuations (complex Poisson process) of specified standard deviation.

Figure 1 shows the case for the smallest standard deviation ($\sigma = 1$ K) considered. The complex Poisson distribution approaches the simple Poisson distribution as one would expect. As the standard deviation of the temperature goes to zero, the modulation function [Eq. (7)] approaches a Dirac delta function, and the integration [Eq. (13)] reduces to the simple Poisson distribution [Eq. (1)].

The maximum standard deviation considered in Fig. 1 is $\sigma = 40$ K. Note that the distribution broadens markedly. The mean of the density function is also shifted to a lower value. A comparison of the curves

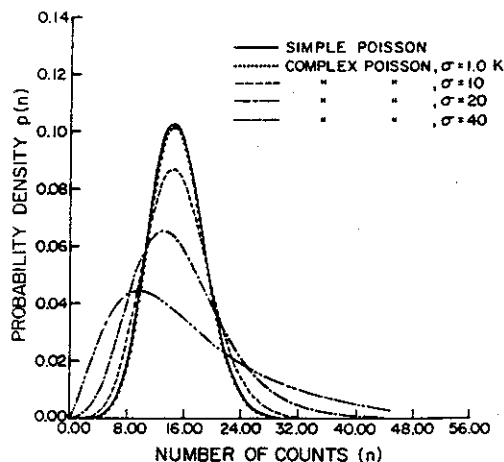


Fig. 1. Photon counting probability density function for Gaussian temperature fluctuations with indicated standard deviation.

show that as the standard deviation of the Gaussian distribution increases the downward shift of the mean also increases. It should also be noted that the density function becomes much more asymmetric with increasing temperature fluctuations.

The broadening of the curves is directly related to two parameters T_0 and σ . From Eq. (10), it can be seen that both the broadening and the shift depend on the value of σ_1 (for constant λ). As T_0 decreases, the sensitivity of the laser to fluctuations in temperature increases. As the sensitivity increases the effect on the photon distribution will also increase for constant temperature fluctuations. Similarly if the sensitivity is constant and the size of the temperature fluctuations increases, again the effect on the distribution will increase. The trend here shows that an increase in σ or a decrease in T_0 (the latter of which is dependent upon the type of laser) will cause a downward shift in the mean and an overall asymmetric broadening of the probability density function.

At present there is no experimental data to support or refute this phenomenon. It seems, however, that only for relatively large values of the standard deviation will this effect be seen. Under most circumstances the fluctuations in the temperature will be small, thus giving a relatively small standard deviation ($\sigma < 1.0$ K) for the fluctuations in the intensity. One might then assume that there will usually be other factors (i.e., receiver noise, amplifier noise, laser modal noise) involved in a system utilizing semiconductor lasers that will predominate over this small effect. In an unstable environment where large temperature fluctuations are possible the decreasing shift in the mean will most readily be observed.

It is important to recognize that photocounting statistics will only be employed for weak optical signals, so that count intervals will in many cases be over extended time periods. Even long-term temperature fluctuations will, therefore, be important in determination of the photocounting statistics.

It is also important to note that, for the special case of temperature fluctuations which are slow in compar-

ison with the count intervals, the photocounting distribution reduces to a lognormal distribution, as described in Diamant and Teich.²

IV. Conclusions

For Gaussian temperature fluctuations the output intensity of a semiconductor laser will fluctuate. Usually these temperature fluctuations will be small, and accordingly the standard deviation of the temperature distribution will be small. This assumes, of course, that the operating temperature of the heat sink onto which the laser is mounted remains relatively constant. If this is the case, the downward shift of the mean value will be unobservable. On the other hand, for rather large temperature fluctuations (induced by unstable thermal environments or catastrophic events) a substantial suppression of the mean count rate and a broadening of the photoelectron probability density function will be observed.

The authors wish to thank H. C. Card (Department of Electrical Engineering, University of Manitoba, Canada) and G. Troup (Department of Physics, Monash University, Australia) for their informative discussions. This work was supported in part by the Natural Sciences and Engineering Research Council of Canada under grant A1330.

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