

An Improved Approximation for the Isolated Transition in Saturated Magnetic Recording

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Abstract—An improved analytic approximation for the isolated transition in saturated magnetic recording is developed. This, in turn, has led to a determination of the analytic form of the magnetization pattern which appears to be a more general representation of the traditional arctangent form.

INTRODUCTION

IT IS APPARENT that an accurate description for the characteristic pulse shape derived from a single magnetic transition region is needed to be able to successfully model the high density behavior of the magnetic medium through the use of linear superposition. The validity of linear superposition is a prerequisite for the successful application of linear post-equalization (receiver equalization) necessary to combat the effects of intersymbol interference and noise.

Morrison and Speliotis [1] report this range of validity to go up to 60 000 bit/in (27 000 bit/cm), a packing density beyond reach of current technology. Other authors have suggested that this figure is too high, but the alternative to superposition is the Dynamic Hysteretic Model [2]–[4]. This method, though more accurate than superposition, particularly at very high packing densities, involves a much greater computational effort and therefore was not considered in this study.

Within the context of linear superposition, it is demonstrated that the readback voltage of the isolated transition can be approximated by a quartic function which in turn leads to a more general representation of the traditional arctangent magnetization pattern.

EXPERIMENTAL RESULTS

A Burroughs-type head/disk interface was selected as the test vehicle for the investigation of the magnetization distribution in the transition region between oppositely magnetized states. The magnetic head and disk interface is similar to those which conventionally provide a density of 2389 bit/cm (6060 bit/in). The principal head parameters are 0.052-mm track width, 1.52- μ m gap length, and 0.75- μ m flying height at 3600 r/min. The recording medium is standard oriented gamma (γ) Fe₂O₃ suspended in a polymer binder. This re-

movable media device is typical of the high performance rotating memories available on the market today.

Several expressions have previously been chosen to represent the basic pulse analytically. Sierra [5] originally proposed the Gaussian expression e^{-t^2} . Kusters and Speliotis [6] alternately proposed a Lorentzian representation. The latter is of the form

$$e(t) = \frac{1}{1 + \left[\frac{t}{t_0}\right]^2} \quad (1)$$

where $2t_0$ is the width of the basic pulse at 50 percent of the maximum amplitude, universally indicated by PW_{50} . The justification for this analytic form is based on the fact that it is the derivative of an arctangent function which has widely been assumed as a good representation of the magnetization distribution in an isolated transition. Determined to resolve any further discussion on this topic MacIntosh [7] compared isolated pulses obtained from several currently available rotating disks memories with nine potential analytic expressions.

After a thorough comparison of the experimentally obtained pulse shapes to the mathematical models in [7], a quartic function,

$$e(t) = \frac{K}{1 + at^2 + bt^4} \quad (2)$$

was determined to be an excellent representation of the actual pulse shape. This was established by taking a photograph, from a Tektronix 466 storage oscilloscope, of isolated pulses coming directly from the read head damped with a 400- Ω resistor. These waveforms were then digitized and analysed on an Amdahl 470/V7 computer. The asymmetry of the experimental data in Fig. 1 has been attributed to the magnetization component normal to the disk coating [8], [9], to asymmetry of the transition zone [10], [11], and to read/write electronics and time lags caused by head inductance and eddy current effects [12].

A least squares fit to the experimental data was performed using the Gaussian, Lorentzian, and the quartic analytic expressions. Table I indicates the results. Most analytic expressions have little problem defining an accurate fit of the pulse above the PW_{50} amplitude level. It is the "tails" of the isolated pulse that are most important in bit error rate calculations yet they are the most difficult to fit. Hence, Table I indicates the least squares error below the PW_{50} amplitude level (i.e., the tails) in addition to the least squares error for the en-

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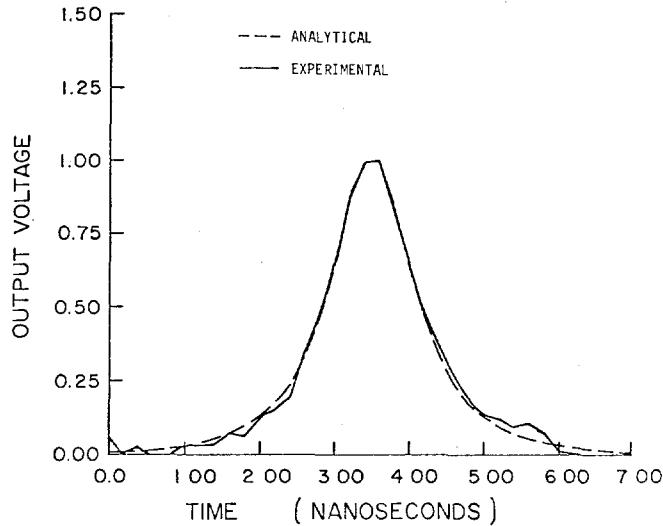


Fig. 1. Least squares fit to experimentally observed isolated pulse using quartic analytic expressions.

TABLE I
LEAST SQUARES ERROR BETWEEN ANALYTIC EXPRESSIONS AND EXPERIMENTAL DATA FOR AN ISOLATED READBACK PULSE

	Least Squares Error	Least Squares Error $e(t) < PW_{50}$
Ke^{-at^2}	0.0864	0.06710
$\frac{K}{1+at^2}$	0.0369	0.03287
$\frac{K}{1+at^2+bt^4}$	0.0210	0.01887

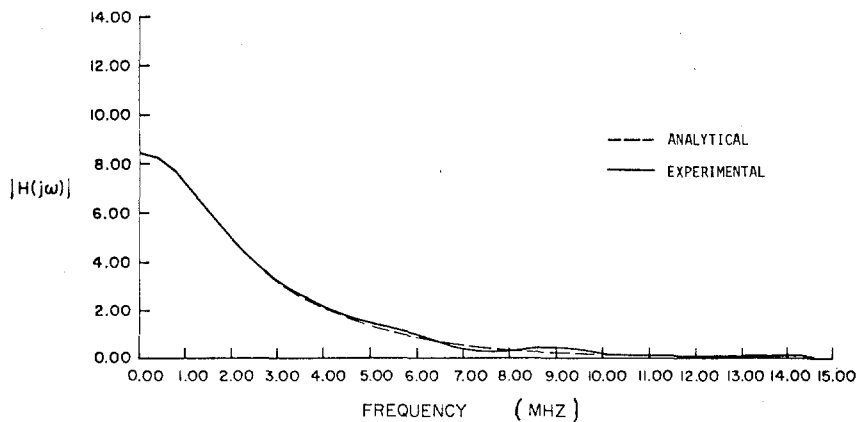


Fig. 2. Magnitude spectrum of quartic analytic expression and experimentally observed isolated pulse.

tire waveform. Either of these criterion indicates that the quartic function provides the best fit to the experimental data. In addition to least squares criterion, the magnitude and phase spectra of the experimental and the three analytic expressions were calculated. These amplitude spectra, which were later compared to experimental spectrum analyser results (Fig. 2), again indicated that the quartic function provides the best fit to the experimental data.

DISCUSSION

In the readback process, the relationship between the head output voltage $e(t)$ and the magnetization pattern $m(t)$ is given

by

$$e(t) = \left[\frac{d}{dt} m(t) \right] * h(t) \tag{3}$$

where the asterisk means convolution and $h(t)$ represents the magnetic head field distribution characterized by the response due to a unit step function in $m(t)$. When the head field function $h(t)$ is an impulse (for the ideal head) the magnetization pattern is simply the integral of $e(t)$. This was the premise by which the magnetization patterns displayed in Fig. 3 were developed. The arctangent transition [13],

$$m(t) = M_1 \tan^{-1} C_1 t \tag{4}$$

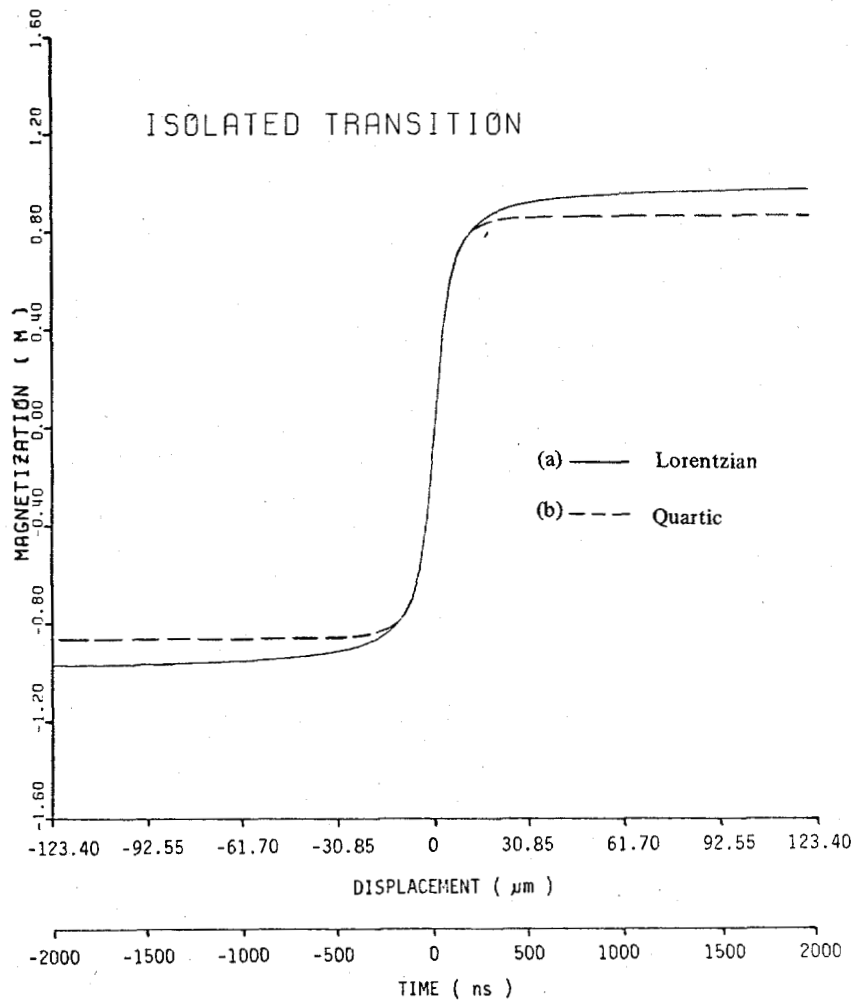


Fig. 3. Computed magnetization patterns for the Lorentzian and Quartic isolated pulses. (a) Lorentzian—general form: $M_1 \tan^{-1} C_1 t$. (b) Quartic—general form: $M_2 \tan^{-1} C_2 t - M_3 \tan^{-1} C_3 t$ where M_2, M_3, C_2, C_3 are nonlinear functions of K, a, b in the equation $K/(1 + at^2 + bt^4)$ (see Appendix). Note: $M(x) = M(\nu t)$, $x = \text{displacement}$, $t = \text{time}$, $\nu = 61.7 \text{ m/s}$.

is the integral of the Lorentzian pulse, a widely accepted analytic form of the isolated pulse. The step function transition appears as a special case of the arctangent distribution when C_1 approaches infinity.

Integrating the quartic function [14], which more accurately fitted our experimental data, yields

$$m(t) = M_2 \tan^{-1} C_2 t - M_3 \tan^{-1} C_3 t. \tag{5}$$

This appears to be an even more general representation of the traditional arctangent form. The smaller asymptotic value indicates that there is less energy concentrated in the tails, hence, adjacent bit interaction is smaller than the Lorentzian transition region would indicate.

A note of caution must be injected here. These results were interpreted on the assumption that the head field distribution was an impulse function. This assumption, though a good first approximation, is not absolutely true. Rather, the head field distribution is responsible for spatial filtering of the true isolated transition of which our experimental data are representative. The impulse approximation to the field distribution function $h(t)$ becomes more accurate as the ratio of the flying height to the gap length becomes smaller. In our case the ratio is 0.5. The Karquist relation [15] indicates that the head field distribution drops to about 10 percent at a distance of twice the gap length. With a pulse width $PW_{50} \cong 6 \mu\text{m}$ and

a gap length of approximately $1.5 \mu\text{m}$ the head field distribution is a window function consisting of a narrow (in comparison to the transition length) lobe which contains most of the energy. Hence, the impulse approximation is a good one.

CONCLUSION

In spite of the complex processes involved, the analysis of digital recording and reproduction has led to some remarkably accurate approximations. It has been found that the readback voltage of the isolated transition can be accurately approximated by a quartic function:

$$\frac{K}{1 + at^2 + bt^4}$$

This, in turn, has led to a determination of the analytic form of the magnetization pattern which appears to be a more general representation of the traditional arctangent form.

APPENDIX

Relationship between the parameters

$$K, a, b \text{ of } \frac{K}{1 + at^2 + bt^4}$$

and 1) PW_{50} of the isolated pulse.

Solve for the t at which the pulse amplitude is $K/2$:

$$\frac{K}{2} = \frac{K}{1 + at^2 + bt^4}$$

$$bt^4 + at^2 - 1 = 0.$$

Therefore,

$$t = \pm \left[\frac{1}{2} \sqrt{\frac{-2a}{b} + 2 \sqrt{\left(\frac{a}{b}\right)^2 + \frac{4}{b}}} \right]$$

$$\begin{aligned} PW_{50} &= \sqrt{-2 \left(\frac{a}{b}\right) + 2 \sqrt{\left(\frac{a}{b}\right)^2 + \frac{4}{b}}} \\ &= \sqrt{\frac{-2a + 2\sqrt{a^2 + 4b}}{b}} \end{aligned}$$

2) The parameters M_2, M_3, C_2, C_3 of $M(t) = M_2 \tan^{-1} C_2 t - M_3 \tan^{-1} C_3 t$

$$M_2 = K \cdot \frac{b}{\sqrt{a^2 - 4b}} \cdot \frac{1}{\sqrt{b \cdot f}}; \quad f = \frac{a}{2} - \frac{1}{2} \sqrt{a^2 - 4b}$$

$$M_3 = K \cdot \frac{b}{\sqrt{a^2 - 4b}} \cdot \frac{1}{\sqrt{b \cdot g}}; \quad g = \frac{a}{2} + \frac{1}{2} \sqrt{a^2 - 4b}$$

$$C_2 = \sqrt{\frac{b}{f}}$$

$$C_3 = \sqrt{\frac{b}{g}}$$

REFERENCES

- [1] J. R. Morrison and D. E. Speliotis, "Study of peak-shift in thin recording surfaces," *IEEE Trans. Magn.*, vol. MAG-3, no. 3, pp. 208-211, Sept. 1967.
- [2] C. S. Chi and D. E. Speliotis, "Dynamic self-consistent iterative simulation of high bit density digital magnetic recording," *IEEE Trans. Magn.*, vol. MAG-10, no. 3, pp. 765-768, Sept. 1974.
- [3] D. E. Speliotis, "Digital recording theory," *Annals N.Y. Acad. Sci.*, vol. 189, pp. 21-51, Jan. 1972.
- [4] N. Curland and D. E. Speliotis, "An iterative hysteretic model for digital magnetic recording," *IEEE Trans. Magn.*, vol. MAG-7, no. 3, pp. 538-543, Sept. 1971.
- [5] H. M. Sierra, "An analytical estimate of bit shift and bit crowding in digital magnetic recording," IBM Tech. Rep. TR02.245, pp. 1-7, Jan. 24, 1963.
- [6] A. J. Kosters and D. E. Speliotis, "Predicting magnetic recording performance by using single pulse superposition, presented at 1971 Intermag Conf., Denver, Colorado, April 13-16, 1971, Paper 14.9.
- [7] N. D. MacIntosh, "A superposition based analysis of pulse slimming techniques for digital recording," in *Video and Data Recording Conf.*, Southampton Univ., pp. 121-146, July 1979.
- [8] D. F. Eldridge, "Magnetic recording and reproduction of pulses," *IRE Trans. Audio*, pp. 47-52, Aug. 8, 1960.
- [9] R. I. Potter, "Analysis of saturation magnetic recording based on arctangent magnetization transitions," *J. Appl. Physics*, vol. 41, pp. 1647-1651, 1970.
- [10] J. R. Morrison, "An analysis of recording demagnetization," *IEEE Trans. Magn.*, vol. MAG-5, no. 4, pp. 948-954, 1969.
- [11] N. Curland and D. E. Speliotis, "An iterative hysteretic model for digital magnetic recording," *IEEE Trans. Magn.*, vol. MAG-7, no. 3, pp. 538-543, 1971.
- [12] A. V. Davies, "The influence of some head and coating properties on pulse resolution in NRZ digital recording," in *Int. Conf. Digital Recording*, London, pp. 68-71, 1964.
- [13] N. Curland and D. E. Speliotis, "A theoretical study of an isolated transition using an iterative hysteretic model," *IEEE Trans. Magn.*, vol. MAG-6, no. 3, pp. 640-645, Sept. 1970.
- [14] I. S. Gradshteyn and I. M. Ryzhik, *Table of Integrals Series and Products*. New York: Academic, 1979, p. 67.
- [15] R. O. McCary, "Saturation magnetic recording process," *IEEE Trans. Magn.*, vol. MAG-7, no. 1, pp. 4-16, Mar. 1971.