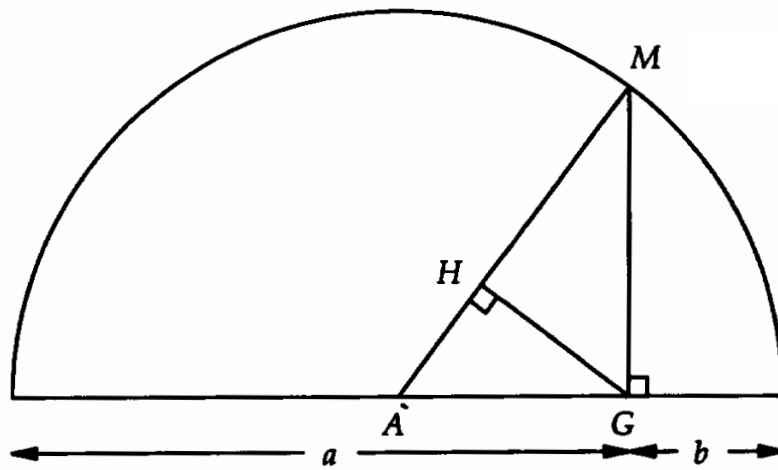


## The Arithmetic–Geometric–Harmonic Mean Inequality

$$a, b > 0 \Rightarrow \frac{a+b}{2} \geq \sqrt{ab} \geq \frac{2ab}{a+b}$$



$$\overline{AM} = \frac{a+b}{2}, \quad \overline{GM} = \sqrt{ab}, \quad \overline{HM} = \frac{2ab}{a+b},$$
$$\overline{AM} \geq \overline{GM} \geq \overline{HM}.$$

—Pappus of Alexandria (circa A.D. 320)