# University of Toronto <br> Faculty of Applied Science and Engineering Final Examination 

ECE 241S - Digital Systems 2003
Examiner: Belinda Wang, Jianwen Zhu
Duration: 2.5 Hours

ANSWER QUESTIONS ON THESE SHEETS, USING THE BACKS IF NECESSARY.

1. No calculator is allowed.
2. Weight for each question is indicated in []. Attempt all questions, since a blank sheet will certainly get a zero.

## Last Name:

First Name:

## Student Number:

$\qquad$

## Lecture Section:

$$
\begin{array}{ll}
\text { Section } 01 \text { (Zhu) } & \text { [ ] } \\
\text { Section } 02 \text { (Wang) } & \text { [ ] }
\end{array}
$$

Maximum grade $=100$

| Question | Mark |
| :---: | :---: |
| 1 |  |
| 2 |  |
| 3 |  |
| 4 |  |
| 5 |  |
| 6 |  |
| 7 |  |
| 8 |  |
| Total |  |

$\qquad$ ID:

Problem 1 [20] A 2-bit digital output of each of two sensors along an assembly line conveyor belt is proportional to the number of parts that pass by on the conveyor belt in a $30-\mathrm{s}$ period. Design a logic circuit that reports an error if the outputs of the two sensors differ by more than one part per 30-s period.

1. Develop a truth table.
2. Express the error function $(f)$ in SOP form by making use of a K-map.

$f=$
$\qquad$

$$
I D:
$$

$\qquad$
3. Use only 4-to-2 and/or 2-to-1 multiplexers, but as fewer as possible to implement the error function $f$.

Problem 2 [5] Flip-flop and counters.


Draw a timing diagram (four complete clock cycles) for $Q_{0}, Q_{1}$ and $Q_{2}$. Assume that all initial values are 0 . Note that all flip-flops are negative edge-triggered.

$Q_{0}$
$Q_{1}$
$Q_{2}$


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Problem 3 [25] Given the state table as follows.

| Present <br> State | Next State/Z |  |
| :--- | :--- | :--- |
|  | $\mathrm{x}=0$ | $\mathrm{x}=1$ |
| A | $\mathrm{B} / 0$ | $\mathrm{G} / 0$ |
| B | $\mathrm{B} / 1$ | $\mathrm{H} / 1$ |
| C | F/1 | $\mathrm{D} / 0$ |
| D | B/0 | $\mathrm{H} / 0$ |
| E | F/1 | $\mathrm{D} / 0$ |
| F | F/0 | $\mathrm{C} / 1$ |
| G | E/0 | $\mathrm{A} / 0$ |
| H | E/0 | $\mathrm{A} / 0$ |

1. Obtain a minimal reduced state table (letting new state A be the block in which the old state A appears and so on).
2. Draw the equivalent state diagram using the reduced states.
3. For the following input sequence ( X ) with the machine initially in state A , determine the resulting output sequence.

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4. Draw the state-assignment table.
5. Determine the minimum-cost expression for all next state variables and the output $(Z)$.
$\qquad$
ID:

Problem 4 [10] A synchronous sequential circuit has two input lines, $x_{1}$ and $x_{2}$, an output line, $Z$. The data line is $x_{1}$ and $x_{2}$ is reset line. Whenever $x_{2}=1$, the circuit is reset. When $x_{2}$ becomes 0 , the first 4 bits on the data line constitute a message word. The output is to become 1 if the message received is 1010 . At the end of the fourth bit of any word received when $x_{2}=0$, the circuit is to enter a waiting state, where it remains until it is reset and where the output is 0 for any input bits after the fourth bit.

1. Construct an ASM chart.
2. Construct an equivalent state diagram.
