University of Toronto
Faculty of Applied Science and Engineering
Midterm Examination

ECE 241S - Digital Systems
Examiner: Belinda Wang, Jianwen Zhu

February 27th, 2002
Duration: 90 minutes

ANSWER QUESTIONS ON THESE SHEETS, USING THE BACKS IF NECESSARY.

1. No calculator is allowed.
2. Weight for each question is indicated in []. Attempt all questions, since a blank sheet will certainly get a zero.

Last Name: _____

First Name: _____

Student Number: _____

Lecture Section:

Section 01 (Zhu) [ ]

Section 02 (Wang) [ ]

Maximum grade = 100

Question	Mark
1	
2	
3	
4	
5	
6	
Total	

Problem 1 [20 marks] Circle T (for True) or F (for False) in statement below.

IMPORTANT NOTE: For each question, right answer gets "2", wrong answer gets "-2", no answer (no circle) gets "0".

T F The following are all valid representations of Boolean function.

- Truth table
- Karnaugh map
- Venn diagram
- Timing diagram
- Sum-of-product Boolean expression

T F Any Boolean function can be implemented exclusively by NAND gates.

T F Any Boolean function can be expressed as the sum of its essential prime implicants.

T F $(00011010)_2 = (32)_8 = (1B)_{16} = (26)_{10}$

T F For a complex logic function, a multi-level implementation is often favored over two-level implementation due to its lower cost.

T F Any logic function can be implemented by a multi-level network of multiplexors.

T F In CMOS technology, a 2-input AND gate is faster than a 2-input NAND gate, provided that all transistors have equal sizes.

T F For any Boolean function f, there exists one and only one optimal PLA implementation.

T F The pull-down network of a CMOS gate can be implemented by PMOS transistors equally well as the NMOS transistors.

T F A multi-level logic circuit can be improved by applying logic transformations such as functional decomposition and factoring.

Problem 2 [10 marks] Reduce the following Boolean expressions to the indicated number of literals.

(1) $\overline{\overline{A}\,\overline{B}+C}+C+AB+CD$ (to three literals). [5 marks]

(2) $\overline{A}B(\overline{D}+\overline{C}D)+B(A+\overline{A}CD)$ (to one literal). [5 marks]

Problem 3 [15 marks] Obtain the truth table of the function f and express f in canonical sum-of-product (sum-of-minterm) and canonical product-of-sum (product-of-maxterm) forms.

$$f = (x_1x_2 + x_3)(x_2 + x_1x_3)$$

(1) Truth table. [5 marks]

(2) Sum-of-minterm form. [5 marks]

(3) Product-of-maxterm form. [5 marks]

Problem 4 [20 marks] Given the Boolean function

$$f = x_1\bar{x_2}x_3 + \bar{x_1}\bar{x_2}x_3 + \bar{x_4}x_1x_2 + x_4\bar{x_1}x_2 + x_4x_1x_2$$

(1) Obtain the K-map of the function. [5 marks]

(2) Simplify the function to a minimal number of literals. [5 marks]

(4) Show the logic diagram from the simplified expression, assuming for all i, both x_i and $\bar{x_i}$ are available as the primary inputs. [5 marks]

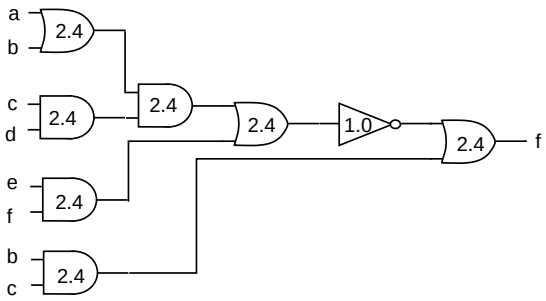
(4) Evaluate the cost of the logic diagram in (3). [5 marks]

Problem 5 [15 marks]

(1) Show the implementation of a CMOS gate $Y = \overline{ABC + (C+D)E}$ as a network of NMOS and PMOS transistors. Identify the pull-up network and pull-down network. [5 marks]

(2) Assuming all transistors have the same size, How does the delay of an gate in (1) compared to the delay of an inverter? State your reason. [5 marks]

(3) For the following logic diagram, where each gate is labeled with its delay, calculate the total delay from primary inputs to primary outputs f. [5 marks]



Problem 6 [20 marks]

Implement a logic circuit that can detect whether a binary coded decimal (BCD) digit (from 0 to 9), which can be represented by the four Boolean variables x_1, x_2, x_3, x_4, is a prime number. Note that a prime number is a number that can only be factored as a product of 1 and itself. Also note that 0 and 1 are not considered prime numbers.

(1) Obtain the K-map of the circuit. [5 marks]

(2) Obtain the simplified Boolean expression. [10 marks]

(3) Implement the simplified Boolean expression using NOR gates, assuming for all i, both x_i and $\overline{x_i}$ are available as primary inputs. [5 marks]