University of Toronto
Faculty of Applied Science and Engineering
Midterm Examination

ECE 241S - Digital Systems
Examiner: Belinda Wang, Jianwen Zhu

6:00 - 7:00pm, March 03, 2003
Duration: 60 minutes

ANSWER QUESTIONS ON THESE SHEETS, USING THE BACKS IF NECESSARY.

1. No calculator is allowed.

2. Weight for each question is indicated in []. Attempt all questions, since a blank sheet will certainly get a zero.

Maximum grade = 60

Last Name: _____

First Name: _____

Student Number: _____

Lecture Section:

Section 01 (Zhu)   [   ]

Section 02 (Wang)   [   ]

Question	Mark
1	
2	
3	
4	
5	
6	
7	
8	
Total	

Problem 1 [5] Find all the decimal equivalents of the following binary numbers represented in two's complement scheme.

1. 1010101 _____

2. 1111010 _____

3. 0000001 _____
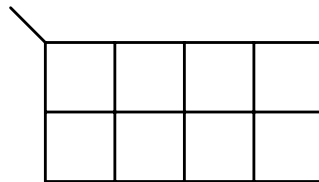
4. 100000 _____

5. 000000 _____

Problem 2 [6] For the logic expression $f = x \oplus y \oplus z$,

1. Express f in canonical SOP form using boolean algebraic manipulation.

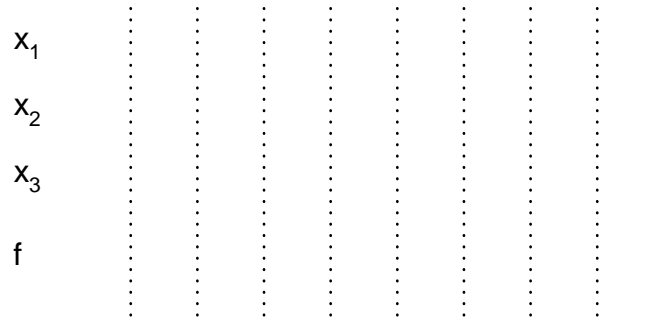2. Draw the corresponding K-map for the logic function f.

3. Write f in canonical POS form.

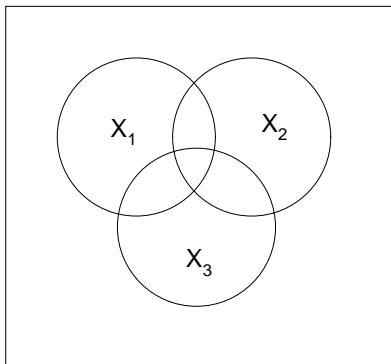**Problem 3 [13]** Given the canonical SOP form of a function

$$f = \overline{x_1}\,\overline{x_2}\,x_3 + \overline{x_1}\,x_2\,\overline{x_3} + \overline{x_1}\,x_2\,x_3 + x_1\,x_2\,\overline{x_3} + x_1\,x_2\,x_3$$

1. Represent the function f in a timing diagram

   x_1

   x_2

   x_3

   f

2. Use algebraic manipulation to find the minimum-cost SOP for f.

3. Show in the following Venn diagram where all five minterms (in the canonical SOP form of f) are located.

   X$_1$ X$_2$

   X$_3$

4. Write the minimum-cost POS form for the function f.

Problem 4 **[3]** Draw an equivalent logic circuit as the one shown below using only NAND gates. (as fewer gates as possible, assuming that both the variables and their complements are available as primary inputs.)

$$
\begin{array}{l}
x_1 \\
x_2 \\
\overline{x}_3
\end{array}
\longrightarrow f
$$
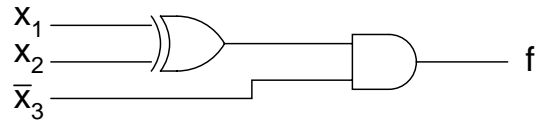
Problem 5 **[8]** Draw a CMOS implementation of the logic function $f = (x_1 \oplus x_2)\, \overline{x_3}$ using as fewer transistors as possible.

Problem 6 [7] A logic function $f(x_1,x_2,x_3,x_4)$ is equal to 1 if the 4-bit number represented by $x_1x_2x_3x_4$ is a prime number.

1. Generate a K-map for the function f.

$x_3 x_4$ \ $x_1 x_2$	00	01	11	10
00				
01				
11				
10				

2. Write the minimum-cost expression for f.

3. Evaluate the cost of implementing the function f assuming that both x_i and $\overline{x_i}$ are available as inputs.

Problem 7 [13] One of your classmates designed the following circuits. Your task is to make improvements in order to reduce overall cost of implementing both logic functions f_1 and f_2.
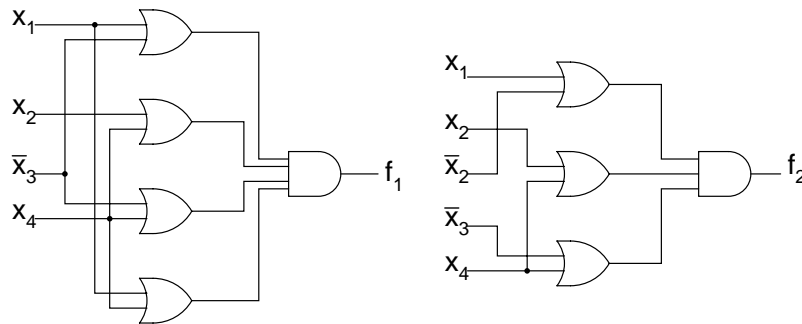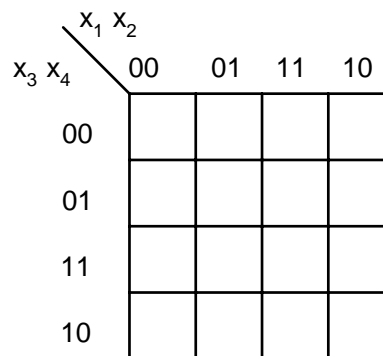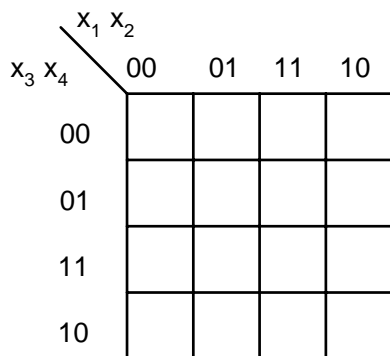

1. Determine the total cost of implementing both f_1 and f_2.

2. Write the logic expression for f_1 and f_2.

   $f_1 =$

   $f_2 =$

3. Draw the K-maps representing f_1 and f_2.

4. Determine the minimum cost expressions for both f_1 and f_2.

   $f_1 =$

   $f_2 =$

5. Draw the improved logic circuits for f_1 and f_2 and find the overall costs

Problem 8 **[5]** For the circuit implemented on the PLA below, write the logic expression for the output f.

x_3 x_2 x_1 x_0

f