Nyquist-Rate D/A Converters

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D/A Converter Basics.

- $B_{in}$ is a digital signal (or word),
  \[ B_{in} = b_12^{-1} + b_22^{-2} + \ldots + b_N2^{-N} \]  
  (1)
- $b_i$ equals a “1” or a “0” (i.e. a binary digit).
- $V_{ref}$ — an analog reference; $V_{out}$ — output.
  \[ V_{out} = V_{ref}(b_12^{-1} + b_22^{-2} + \ldots + b_N2^{-N}) \]  
  (2)
- Define $V_{LSB}$ to be LSB signal change, $V_{LSB} \equiv V_{ref}/2^N$
**D/A Converter Basics**

- For errors, define “units” of LSB $1 \text{ LSB} = 1/2^N$
- A **multiplying** D/A allows $V_{\text{ref}}$ to be a **varying input** — $V_{\text{out}}$ proportional to multiplication of $V_{\text{ref}}$ and $B_{in}$.
- For **ideal** D/A, output signal is a **well defined value** — no quantization error!

![Graph showing relationship between $V_{\text{ref}}$, $V_{\text{out}}$, and $B_{in}$]

**D/A Resistor-String (Hamadé, JSSC, Dec. 1978)**

- Guaranteed monotonic
- Integrated with better than 10-bits absolute accuracy.
- Delay through the switch network major speed limitation
- Resistors might be realized using polysilicon
- If n-channel only used, can be laid out small
- Requires $2^N$ resistors

![Diagram of D/A Resistor-String with 3-bit resolution and $2^N$ resistors]
**D/A Resistor-String — Digital Decoding**

- Higher speed implementation (less resistance thru transistors)
- Large cap load on buffer input
- Can pipeline digital decoding for faster speed
- Requires \(2^N\) resistors

**Folded-resistor-string D/A**

- (Abrial, JSSC, Dec. 1988)
- Less capacitance load over the single bus approach
- Requires \(2^N\) resistors
Binary-Weighted Resistor D/A’s.

- Only N resistors
- Resistor and current ratios are on the order of $2^N$
- No guarantee of monotonicity.
- Prone to glitches (more later).

$$V_{out} = -R_F V_{ref} \left( -\frac{b_1}{2R} - \frac{b_2}{4R} - \frac{b_3}{8R} - \ldots \right) \quad (3)$$

Reduced Spread Binary Resistor D/A

- Reduced resistor spread
- Keep repeating this procedure $\rightarrow$ R-2R ladder

$$V_A = \frac{1}{4} (-V_{ref})$$
R-2R Based D/A Converters

- Small size, good matching (only R and 2R)

\[
R'_4 = 2R \\
R_4 = 2R \parallel 2R = R \\
R'_3 = R + R_4 = 2R \\
R_3 = 2R \parallel R'_3 = R
\]

R-2R Based Resistor Ladders

- Example D/A converter

- Currents through the switches are scaled
- Should scale switch sizes for good accuracy
- No node voltage changes except for output —> fast
**R-2R Based Resistor Ladders**

- Slower circuit having *equal* current through switches

![Resistor Ladder Diagram]

- Node voltages change — slower circuit
- No need to scale switch sizes (smaller size)

**Glitches**

- Different delays for switching the different currents
- MSB change often worst case

![Glitch Diagram]

- Glitches can be minimized by limiting the bandwidth but that slows down circuit
- Use thermometer code to reduce glitches
**Charge-Redistribution SC D/A’s**

- Programmable SC gain amplifier.
- Sign bit realized by interchanging input phases
- Carefully clock-waveforms required to minimize voltage dependency of clock-feed-through.
- Digital codes should be changed when input side of capacitors are connected to ground. Requires extra digital complexity.

![Diagram](image1)

**Thermometer D/A Converters**

<table>
<thead>
<tr>
<th>Decimal</th>
<th>Binary $b_1$ $b_2$ $b_3$</th>
<th>Thermometer Code $d_1$ $d_2$ $d_3$ $d_4$ $d_5$ $d_6$ $d_7$</th>
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<td>7</td>
<td>1 1 1</td>
<td>1 1 1 1 1 1 1</td>
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</table>

![Diagram](image2)
**Thermometer Code D/A Converter**

- Top Capacitors are Connected to Ground
- Bottom Capacitors are Connected to V_ref

- $2^N$ unit sized caps
- Guaranteed monotonic
- Much lower glitching
- Low DNL

**Current-Mode D/A's**

- Thermometer-code
- High-speed, output feeds directly to resistor
- Important that delay to all the switches are equal.
- Overlapped clocks much better than having non-overlapped clocks.
Current-Steered D/A [Colles, 88]

- Operates as cascode current sources.
- For max speed, keep voltage swing at source of Q1 small (just turned off)
- Switching feed-through from the digital input enhances switching speed.

![Current-Steered D/A Circuit Diagram](image)

Segmented D/A

- Schoeff, 79; Saul, 85; Grebene, 84

- Combine thermometer and binary
- Accuracy needed for LSB reduced
- Glitches reduced
- Very popular
Dynamically-Matched Current Sources

- Schouwenaar, 88

- Each current source is calibrated with a single reference
- 64 used so D/A can continue operating
- Achieved 92 dB SNDR, and 20 mW with 3V.
- Used for audio application

- Dynamic technique with current switching for realizing very well-matched current sources
- Up to 16 bit accuracy

- Minimize clock-feedthrough and charge-injection by having capacitance $C_{gs}$ and bias voltage $V_{GS}$ large
- Implies voltage error causes less current deviation.