LG1

LOOP GAIN ANALYSIS

FOR ANALYSIS OF FEEDBACK SYSTEMS

THERE ARE GENERALLY 3 THINGS TO FIND IN FEEDBACK CIRCUITS

1) CLOSED-LOOP GAIN OR TRANSFER FUNCTION

2) PORT IMPEDANCE (INPUT OR OUTPUT IMPEDANCE)

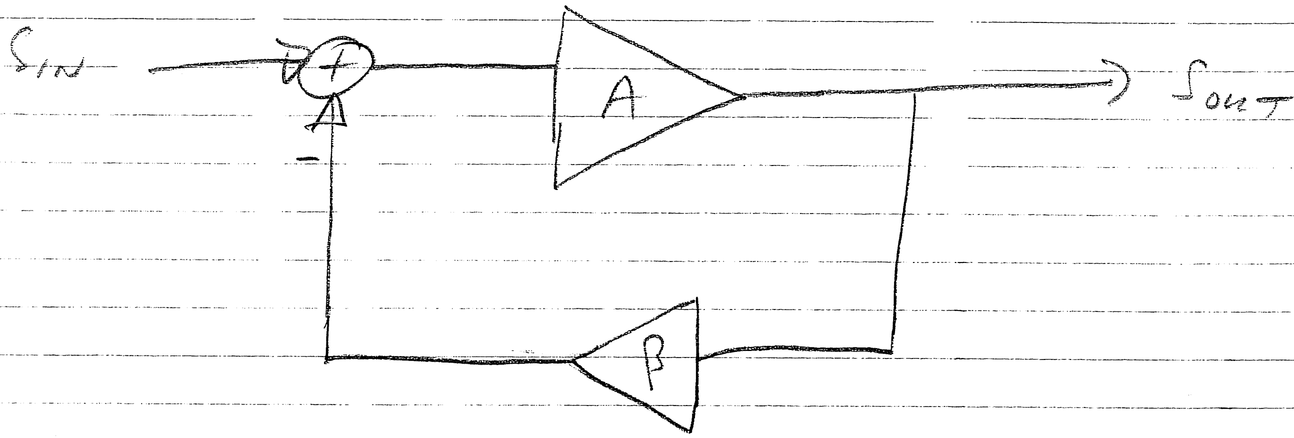
3) STABILITY ANALYSIS

IS THE SYSTEM STABLE

AND <u>HOW</u> STABLE IS IT?

CLOSED-LOOP GAIN

IDEAL CASE



A, β ARE UNIDIRECTIONAL

S_{IN} ⇒ INPUT VOLTAGE OR CURRENT

S_{OUT} ⇒ OUTPUT VOLTAGE OR CURRENT

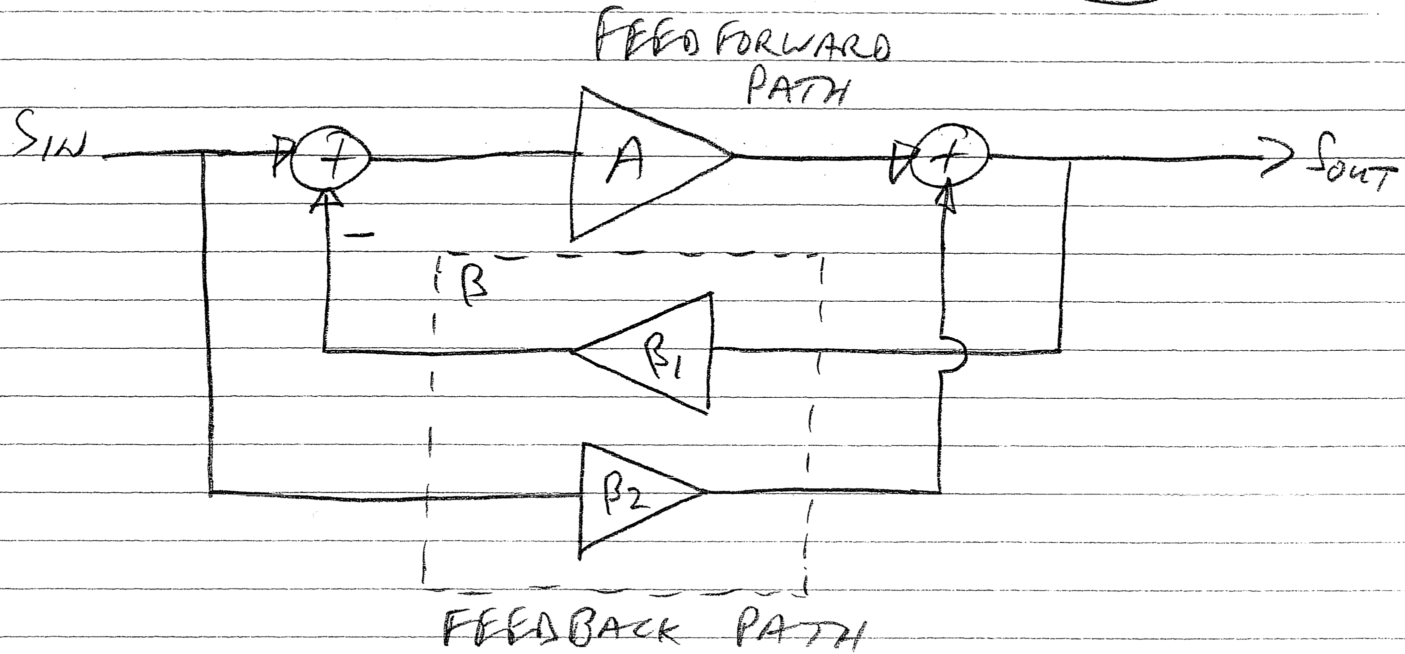
HOWEVER, IN PRACTICE

GENERALLY A IS UNIDIRECTIONAL

WHILE β IS BIDIRECTIONAL

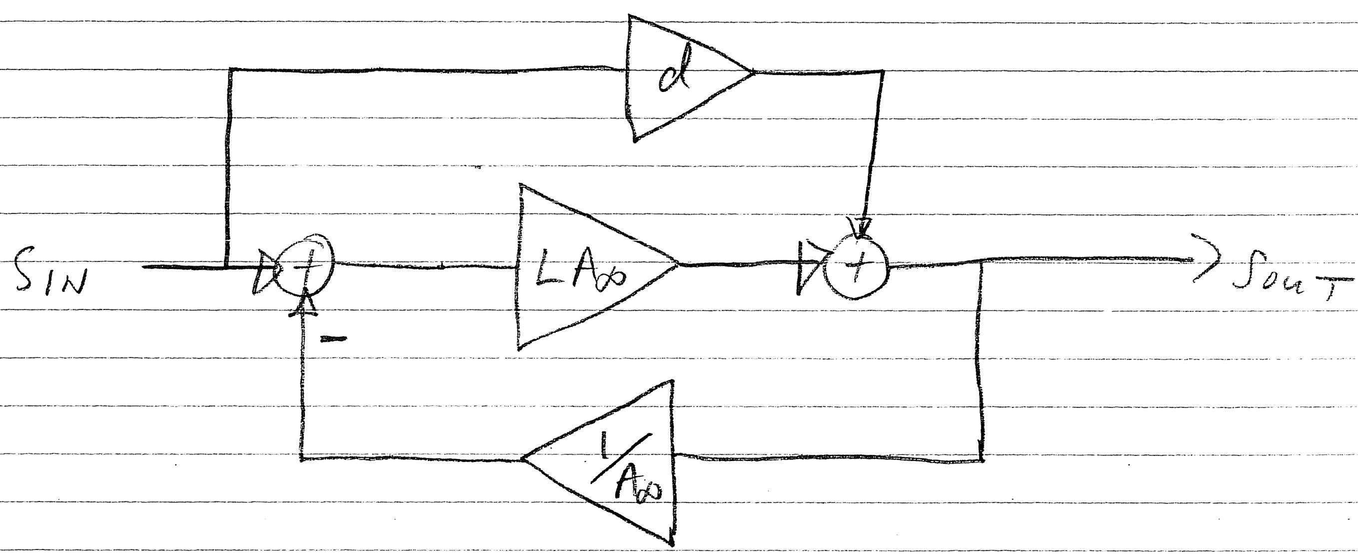
LEADING TO ...

LG3



THE ABOVE CAN BE RE-ARRANGED AND DEFINE $A_\infty \equiv \frac{1}{\beta_1}$ $d \equiv \beta_2$ $L \equiv A\beta_1$

CALLED ASYMPTOTIC GAIN MODEL



CAN SHOW THE CLOSED-LOOP GAIN, A_{CL}
IS EQUAL TO

$$A_{CL} \equiv \frac{S_{OUT}}{S_{IN}} = A_{\infty}\left(\frac{L}{1+L}\right) + d\left(\frac{1}{1+L}\right)$$

WHERE

L IS LOOP-GAIN WHEN $S_{IN} = 0$

$$A_{\infty} \equiv \left.\frac{S_{OUT}}{S_{IN}}\right|_{L \to \infty}$$

$$d \equiv \left.\frac{S_{OUT}}{S_{IN}}\right|_{L \to 0}$$

SO TO FIND A_{CL} $\Rightarrow$ FIND L, A_{∞}, d

LOOP GAIN: L

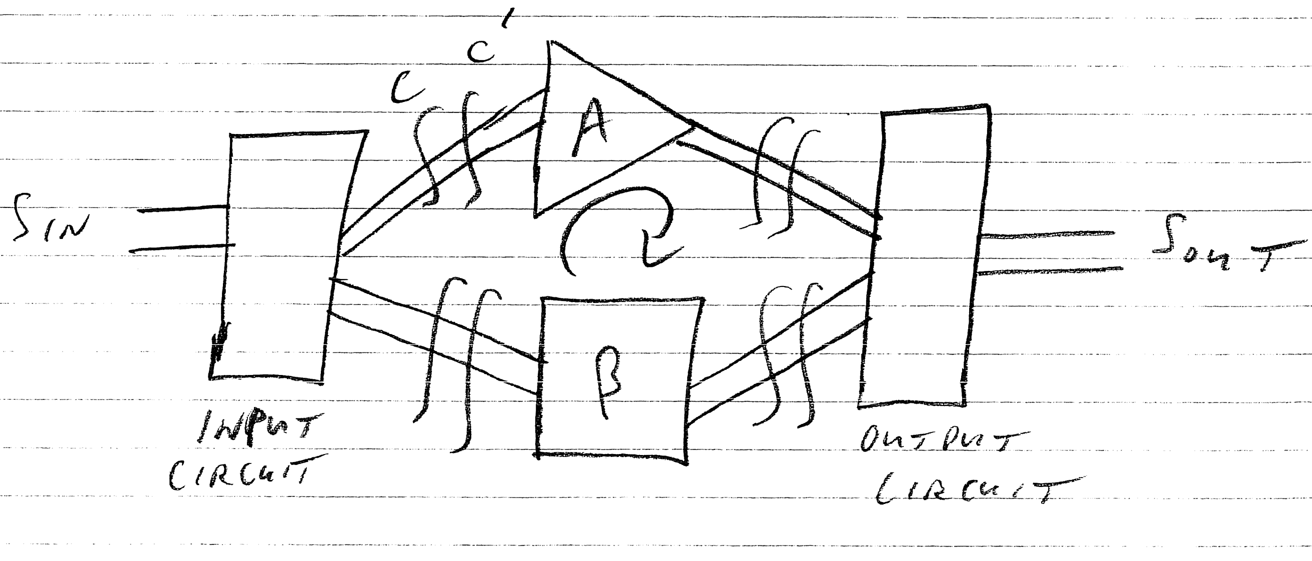
⇒ SET $S_{IN} = 0$

⇒ BREAK FEEDBACK LOOP
FIND IMPEDANCE AT BREAK POINT
& TERMINATE LOOP WITH THIS
IMPEDANCE

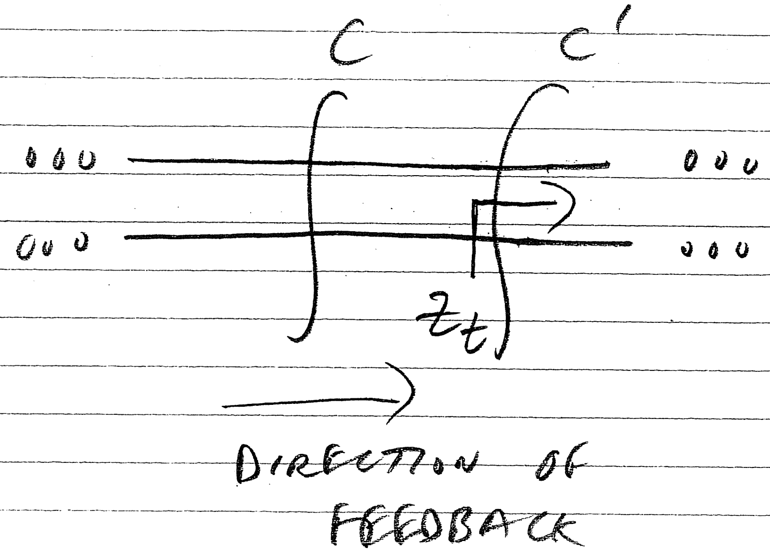
⇒ INSERT TEST SIGNAL EITHER v_t OR i_t

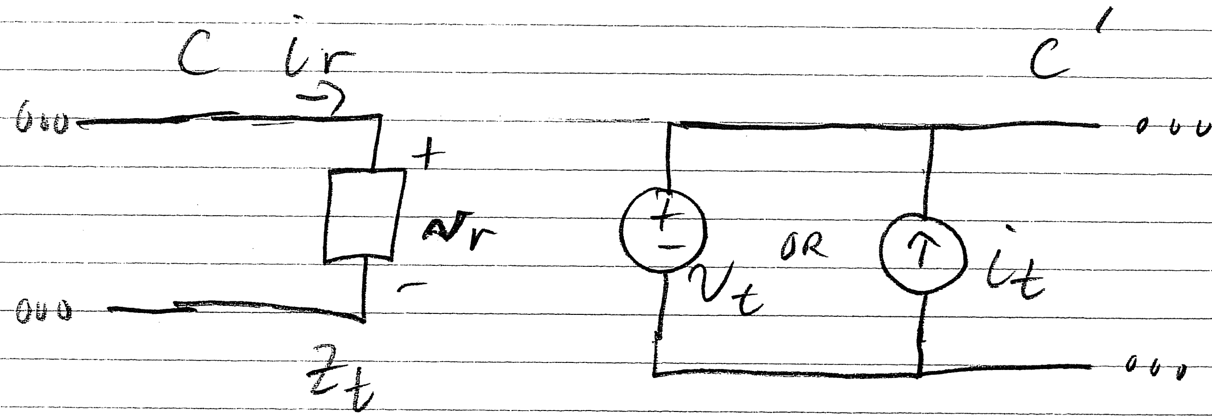
AND FIND v_r OR i_r

$$L \equiv -\frac{v_r}{v_t} \quad \text{OR} \quad L \equiv -\frac{i_r}{i_t}$$



BREAK AT ONE OF $\int\int$





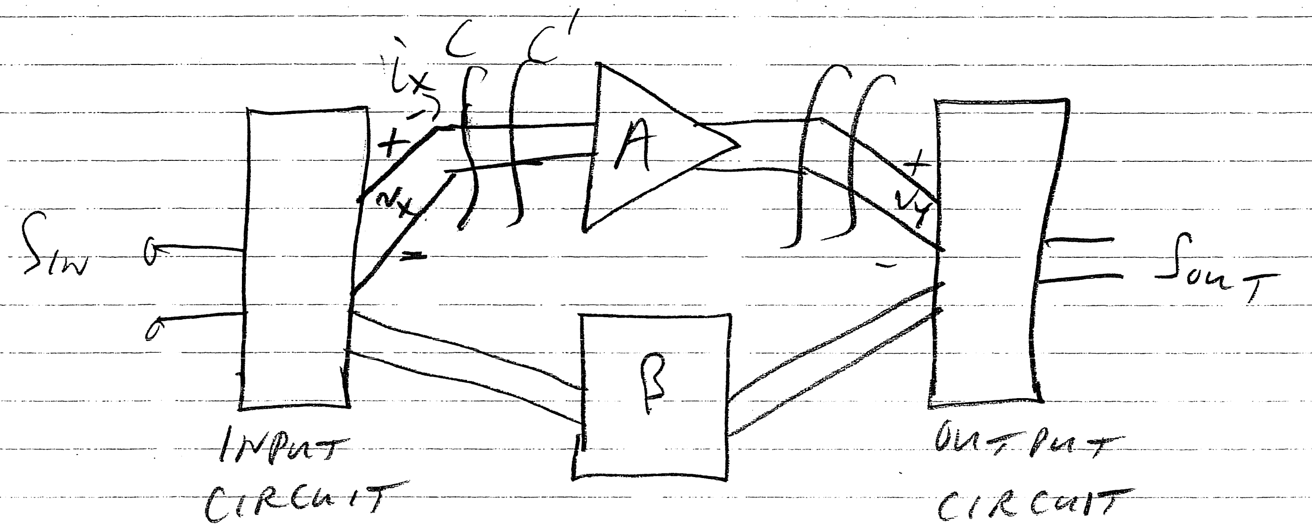
$$L \equiv -\frac{v_r}{v_t} \quad \text{OR} \quad L \equiv -\frac{i_r}{i_t}$$

CAN BREAK LOOP ANYWHERE

USUALLY FIND A SPOT WHERE Z_t IS EASILY CALCULATED

$L > 0$ FOR NEGATIVE FEEDBACK

ASYMPTOTIC GAIN, A_∞



BREAK AT ONE OF ∫∫

INSERT IDEAL VOLTAGE AMP, GAIN A_v

LET $A_v \to \infty \Rightarrow L \to \infty$

DUE TO FEEDBACK V_Y IS FINITE

SO IF $A_v \to \infty \Rightarrow v_x \to 0$
$\quad$ & $i_x \to 0$

WE HAVE

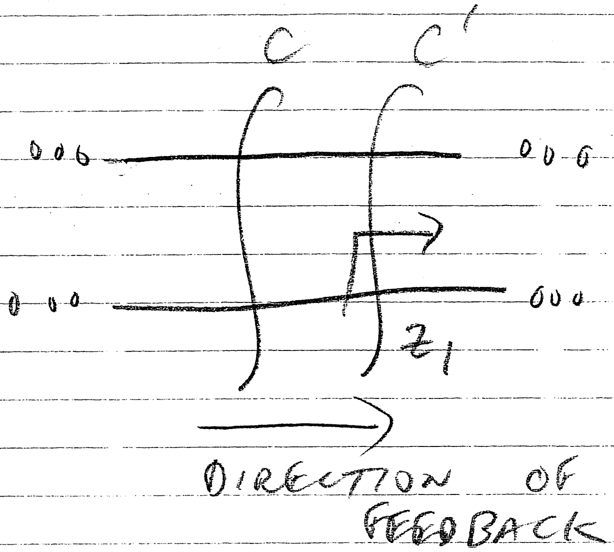
$$A_\infty \equiv \left.\frac{S_{OUT}}{S_{IN}}\right|_{L \to \infty \Rightarrow v_x \to 0,\ i_x \to 0}$$

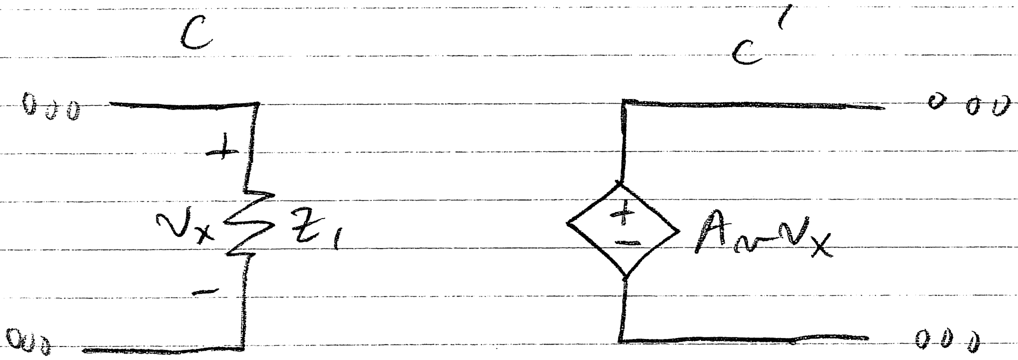
USE RESULT THAT $V_x \to 0$ & $i_x \to 0$

TO FIND A_∞

LG7A

A_∞



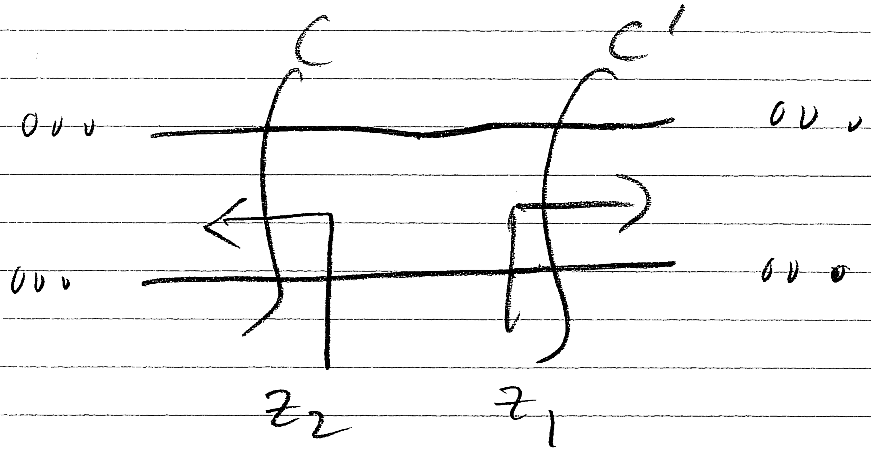


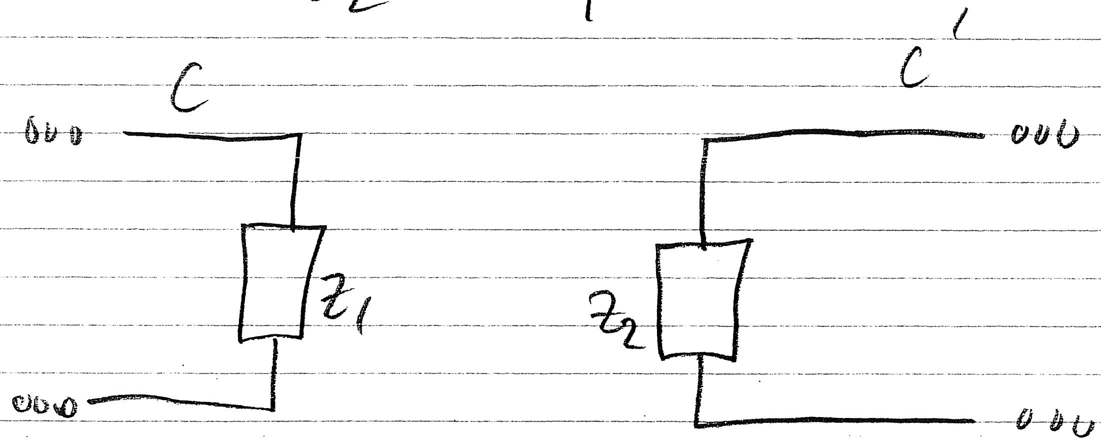
LET $A_v \to \infty$

FEED THROUGH TERM, d

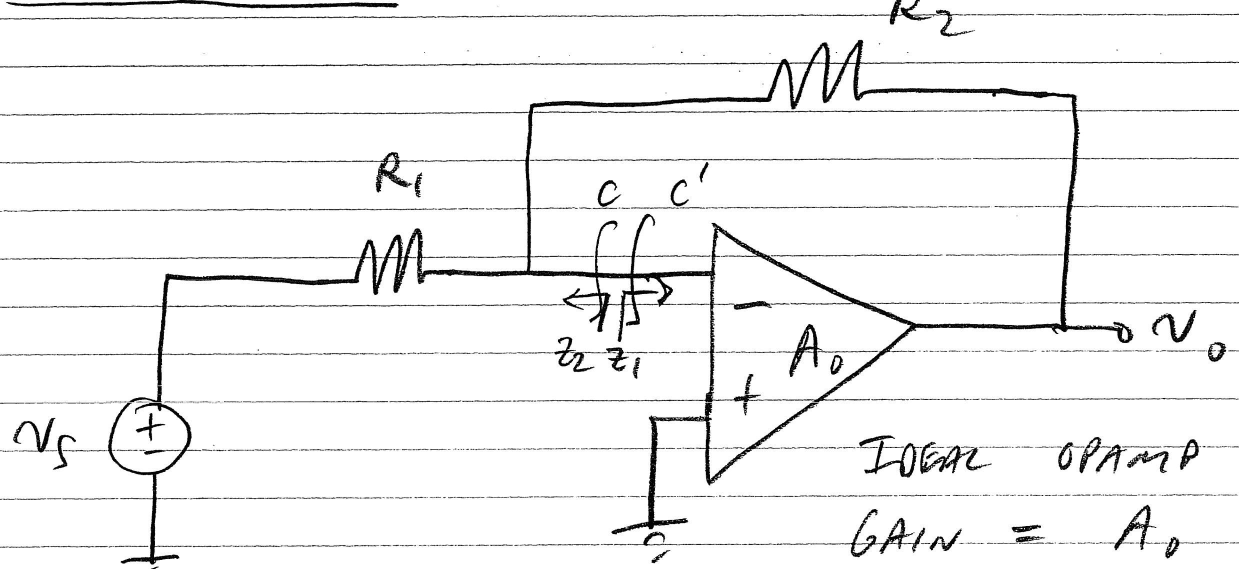
BREAK IN FEEDTHROUGH PATH (SAME AS A_∞ BREAK) & ADD APPROPRIATE TERMINATIONS (WHICH CAUSES $L \to 0$)

$$d \equiv \left. \frac{S_{OUT}}{S_{IN}} \right|_{L \to 0}$$





EXAMPLE 1



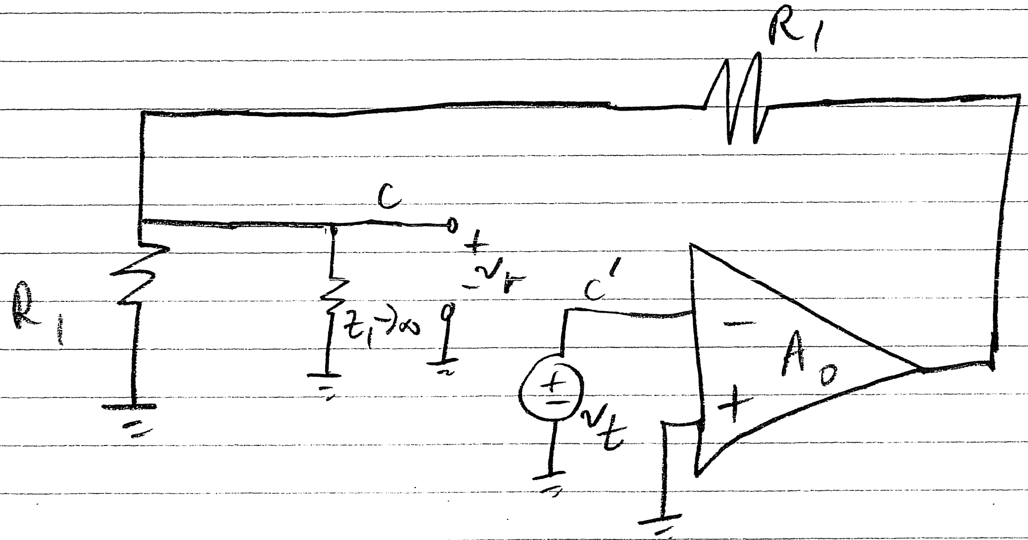
IDEAL OPAMP

GAIN = A_0

$z_1 \to \infty$

$z_2 = R_1 \| R_2$

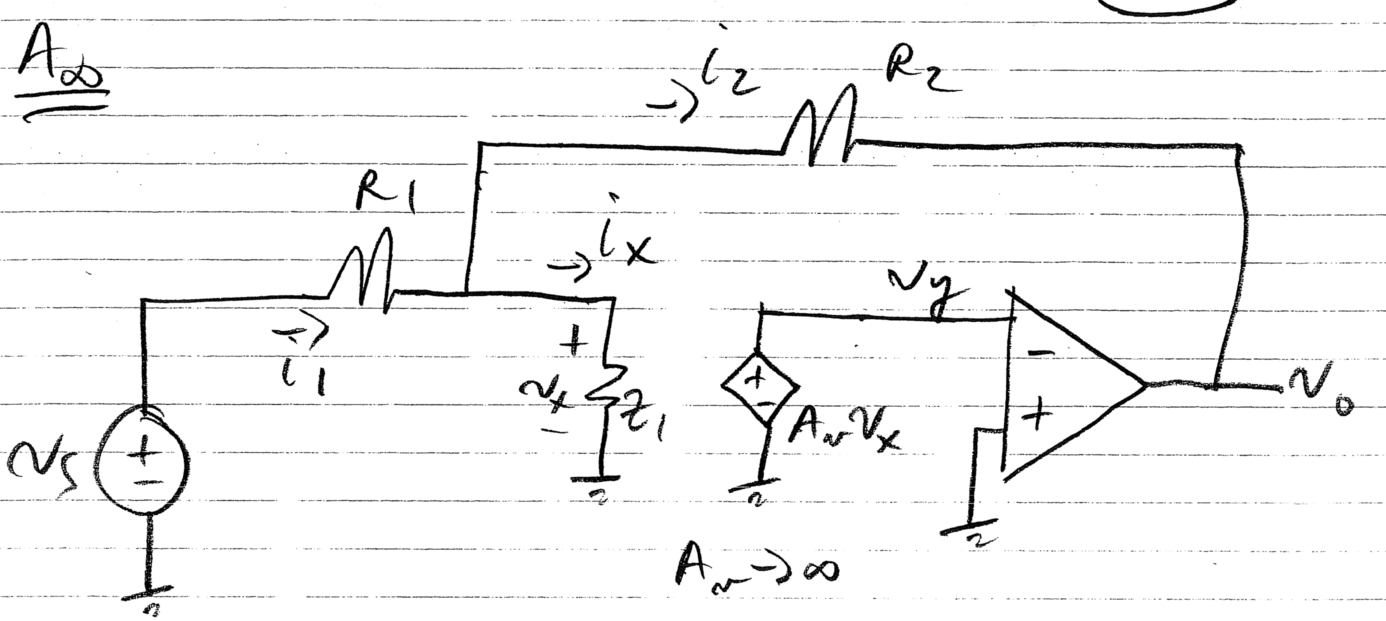
L



$$L \equiv -\frac{v_r}{v_t} = A_0\left(\frac{R_1}{R_1+R_2}\right)$$

LG10

A_∞



FEEDBACK $\Rightarrow$ v_o FINITE $\Rightarrow$ v_y FINITE

$\Rightarrow$ $v_x = 0$ $\Rightarrow$ $i_x = 0$

$$i_1 = \frac{v_s}{R_1} \Rightarrow i_2 = i_1$$

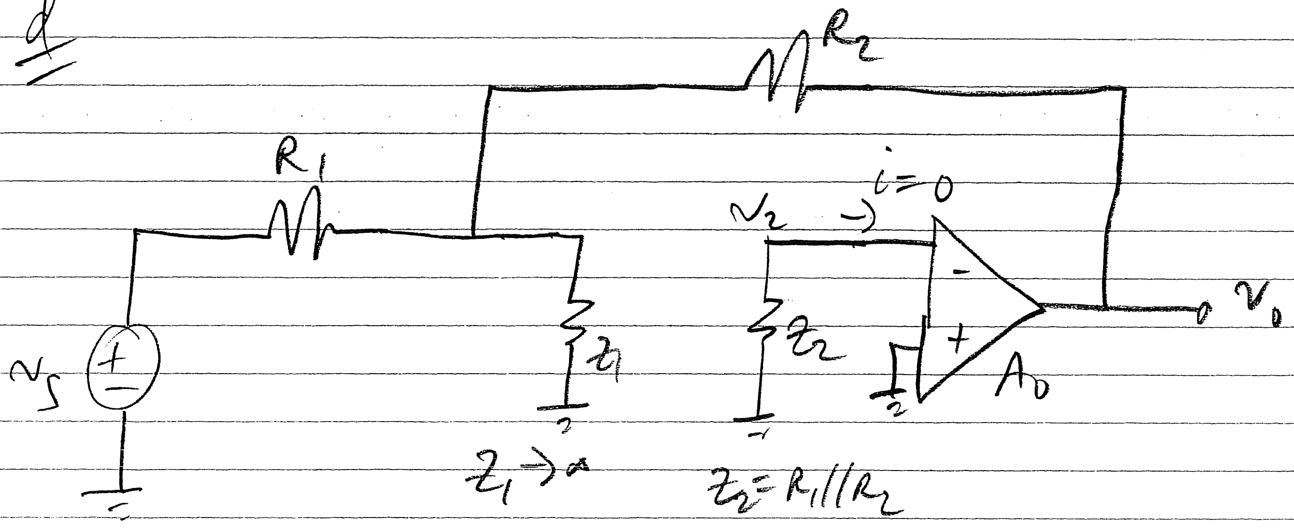
$$v_o = 0 - i_2 R_2 = 0 - v_s\left(\frac{R_2}{R_1}\right)$$

$$\frac{v_o}{v_s} = -\frac{R_2}{R_1} \qquad A_\infty \equiv \left.\frac{v_o}{v_s}\right|_{A_v \to \infty}$$

$$A_\infty = -\frac{R_2}{R_1}$$

LG11

$\underline{\underline{d}}$



SINCE $i=0 \Rightarrow v_2 = 0 \Rightarrow v_0 = 0$

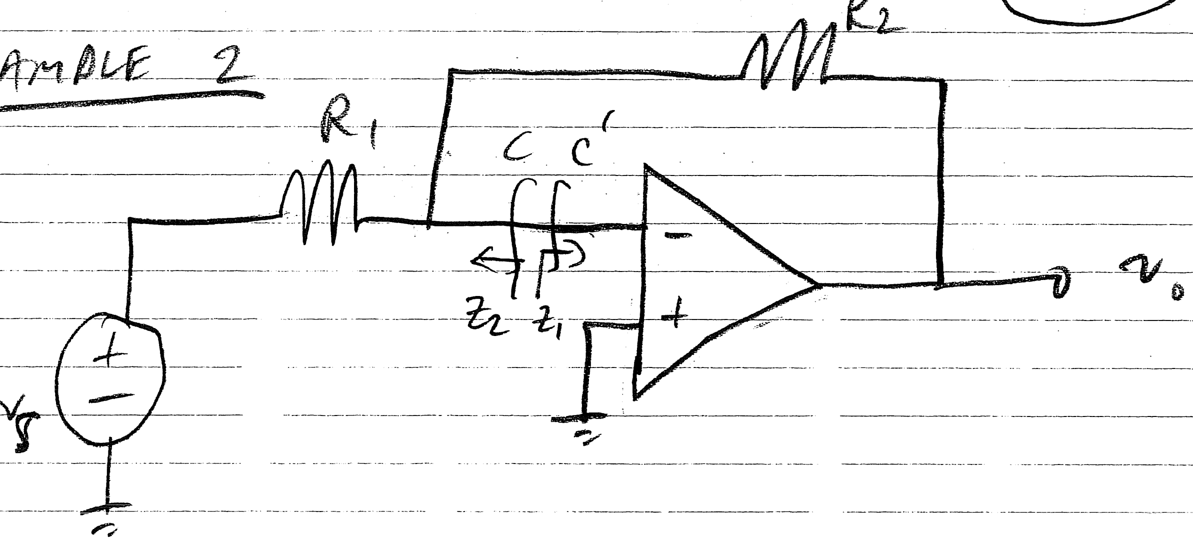
$$d \equiv \left.\frac{v_0}{v_S}\right|_{L \to 0} = 0$$

$$A_{CL} = A_\infty \left(\frac{L}{1+L}\right) + d\left(\frac{1}{1+L}\right)$$

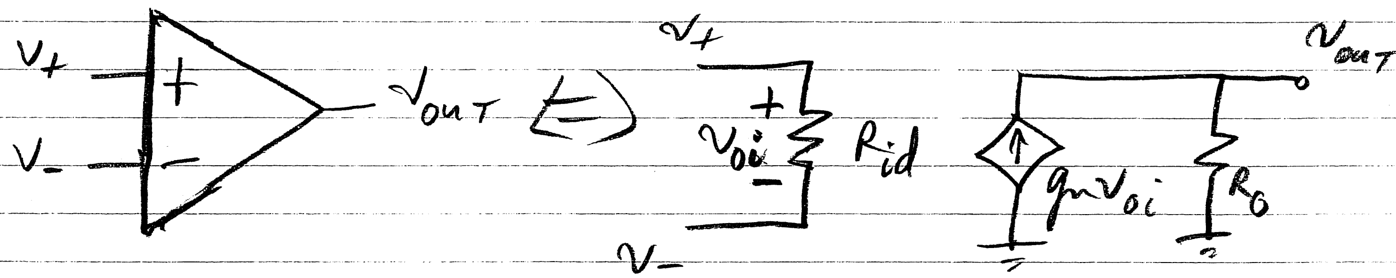
$$\underline{\underline{A_{CL} = -\left(\frac{R_2}{R_1}\right)\left(\frac{A_0\left(\frac{R_1}{R_1+R_2}\right)}{1 + A_0\left(\frac{R_1}{R_1+R_2}\right)}\right)}}$$

LG12

EXAMPLE 2



WITH OPAMP MODEL

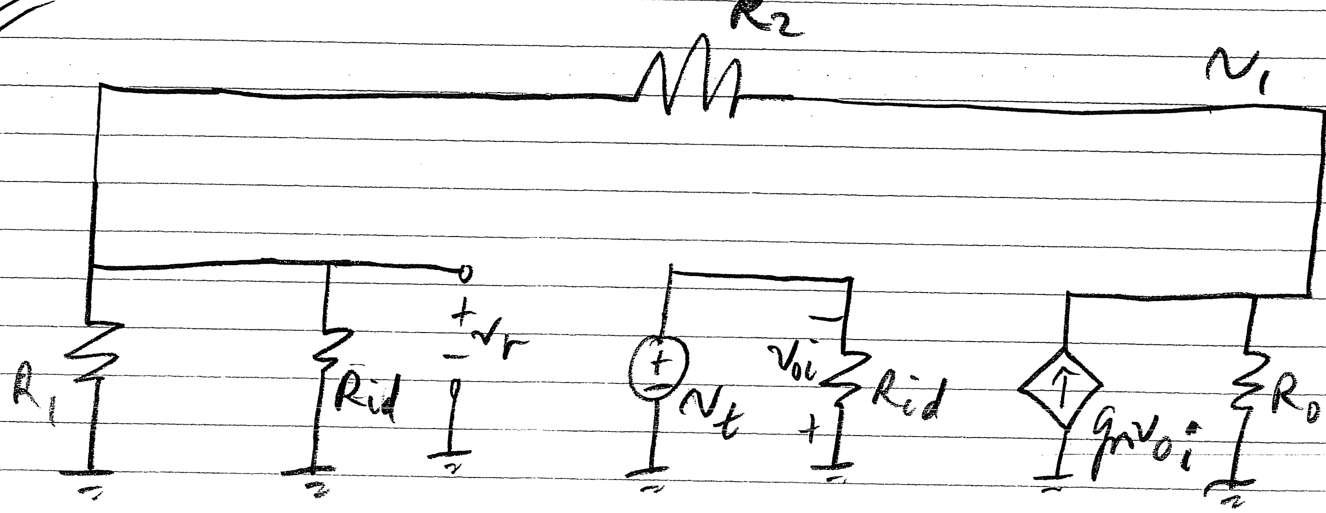


So $\quad Z_1 = R_{id}$

$$Z_2 = R_1 \parallel (R_2 + R_o)$$

L613

L



$$v_{oi} = -v_t$$

$$v_1 = g_m v_{oi} \left(R_o \,||\, \left[R_2 + (R_1 || R_{id})\right]\right)$$

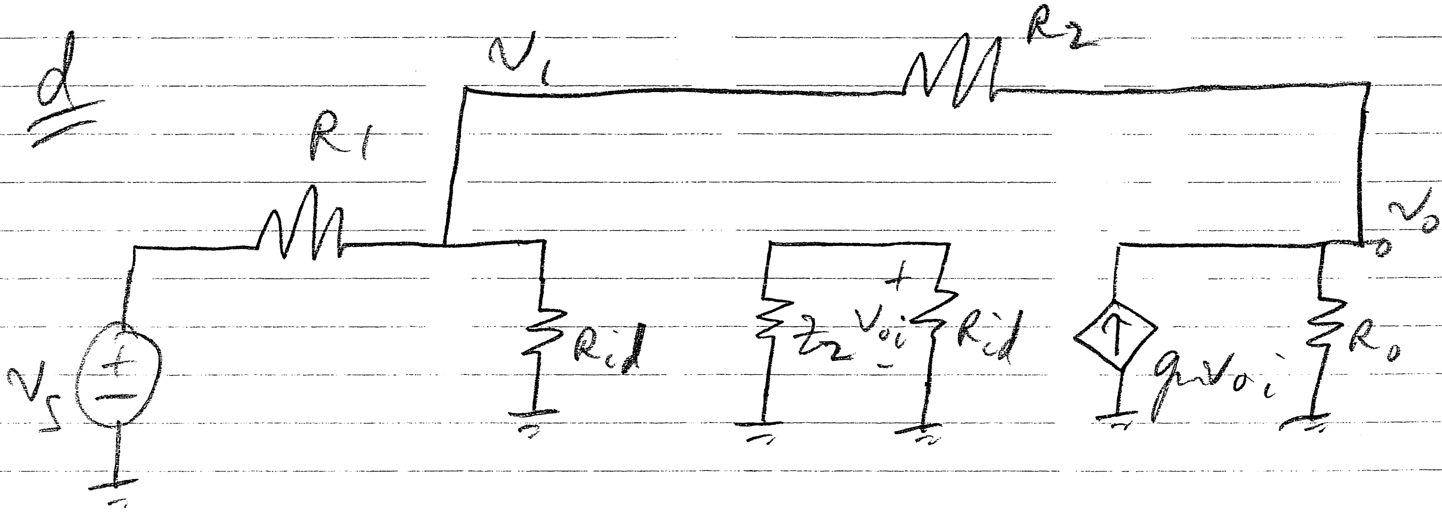
$$v_r = \frac{R_1 || R_{id}}{(R_1 || R_{id}) + R_2} \, v_1$$

$$L = -\frac{v_r}{v_t} = \frac{g_m R_o (R_1 || R_{id})}{R_o + R_2 + (R_1 || R_{id})}$$

NOTE $L > 0$ AS EXPECTED

$\underline{A_\infty}$ AS BEFORE $A_\infty = -\left(\frac{R_2}{R_1}\right)$

$\underline{d}$



$$v_{oi} = 0 \Rightarrow g_m v_{oi} = 0$$

$$v_1 = \frac{\left(R_{id} \| (R_2 + R_0)\right)}{\left[R_{id} \| (R_2 + R_0)\right] + R_1} v_s$$

$$v_0 = \left(\frac{R_0}{R_0 + R_2}\right) v_1$$

$$d \equiv \left.\frac{v_o}{v_s}\right|_{L \to 0} = \frac{\left[R_{id} \| (R_2 + R_0)\right] (R_0)}{\left(\left[R_{id} \| (R_2 + R_0)\right] + R_1\right)(R_0 + R_2)}$$

$$A_{CL} = A_\infty \left(\frac{L}{1+L}\right) + d\left(\frac{1}{1+L}\right)$$

LG15

IF $R_2 = R_1 = 1K$

$R_{id} = 1K$ $g_m = 20\ mA/V$

$R_o = 100K$

$L = 9.85$

$A_\infty = -1$ $d = 0.493$

$$A_{CL} = A_\infty \left(\frac{L}{1+L}\right) + d \left(\frac{1}{1+L}\right)$$

$$= -0.862$$

NOTE

IF d IGNORED $A_{CL} \simeq A_\infty \left(\frac{L}{1+L}\right) = -0.908$

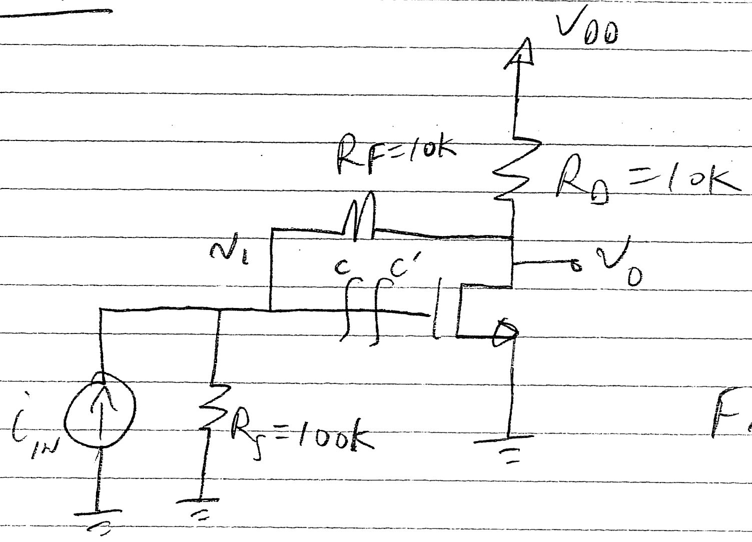
$\simeq 5\%$ ERROR

IN MANY CASES CAN IGNORE d

BUT IN SOME CASES d VERY IMPORTANT

LG16

Ex 3



$g_m = 1\,mA/V$

$r_o = 50k$

FIND $A_{CL} \equiv \frac{v_o}{i_{IN}}$

$$L = g_m \left[R_D \| r_o \| (R_F + R_S)\right] \left(\frac{R_S}{R_S + R_F}\right)$$

$$= 7.04 \ V/V$$

$$A_\infty = -R_F = -10k$$

$$d \equiv \left.\frac{v_o}{i_{IN}}\right|_{L \to 0} \Rightarrow v_1 = i_{IN}\left(R_S \| \left[R_F + (R_D \| r_o)\right]\right)$$

$$v_1 = i_{IN}\ (15.3k)$$

$$v_o = \left(\frac{R_D \| r_o}{(R_D \| r_o) + R_F}\right) v_1 = 0.4545\, v_1$$

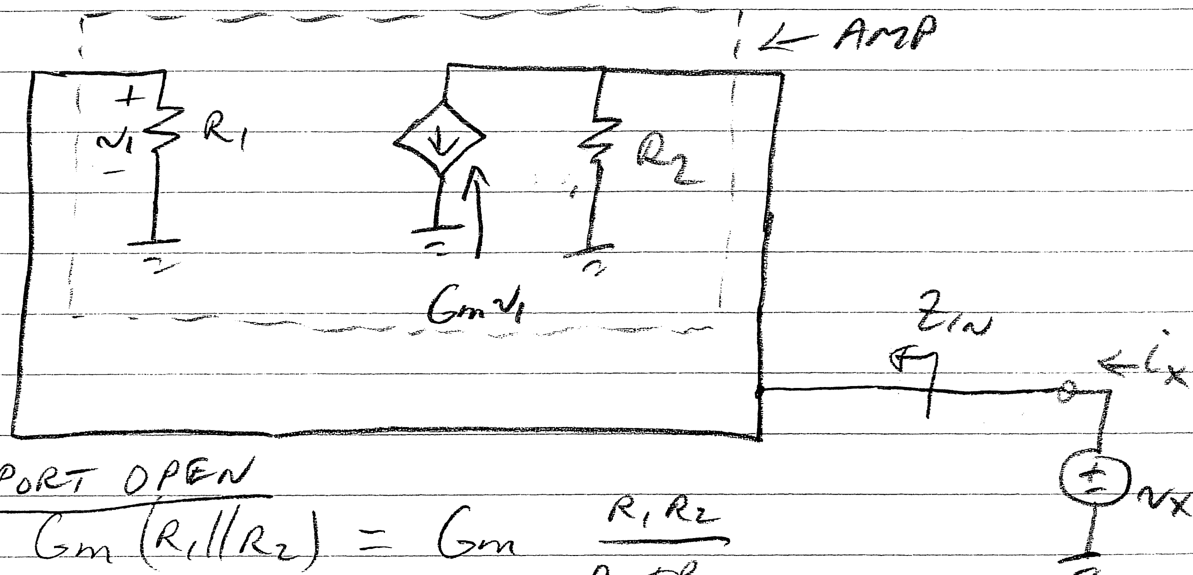
$$d = \frac{v_o}{i_{IN}} = 7.05k$$

$$A_{CL} = A_\infty\left(\frac{L}{1+L}\right) + d\left(\frac{1}{1+L}\right) = (-10)\left(\frac{7.04}{8.04}\right) + 7.05\left(\frac{1}{8.04}\right)$$

$$A_{CL} = -7.88 \ V/A$$

FINDING CLOSED-LOOP INPUT/OUTPUT IMPEDANCE

CONSIDER PORT IMPEDANCE (SHUNT)



WITH PORT OPEN

$$L_0 = G_m (R_1 \| R_2) = G_m \frac{R_1 R_2}{R_1 + R_2}$$

$$G_m = \left(\frac{R_1 + R_2}{R_1 R_2}\right) L_0$$

$$Z_{IN} \equiv \frac{v_x}{i_x}$$

$$i_x = \frac{v_x}{R_1} + \frac{v_x}{R_2} + G_m v_x$$

$$= v_x \left(\frac{1}{R_1} + \frac{1}{R_2} + \left(\frac{R_1 + R_2}{R_1 R_2}\right) L_0 \right)$$

$$= v_x \left(\frac{R_1 + R_2 + (R_1 + R_2) L_0}{R_1 R_2} \right)$$

LGMA

$$i_x = \left(\frac{R_1+R_2}{R_1R_2}\right)(1+L_0)\, v_x$$

$$Z_{IN} \equiv \frac{v_x}{i_x} = \frac{R_1R_2}{(R_1+R_2)(1+L_0)} = \frac{R_1 \| R_2}{(1+L_0)}$$

DEFINE $Z_{IN}^0 = R_1 \| R_2$

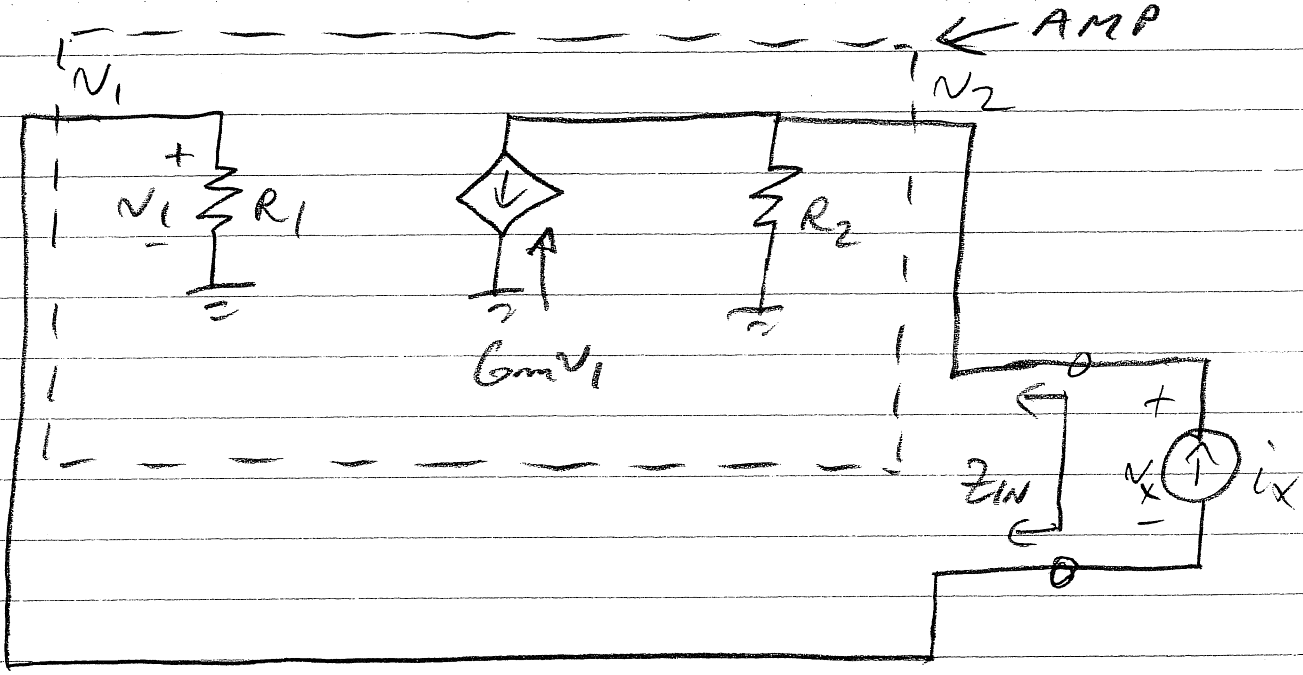
IMPEDANCE LOOKING INTO PORT WHEN FEEDBACK LOOP BROKEN INSIDE AMP ($L=0$)

$$Z_{IN} = \frac{Z_{IN}^0}{(1+L_0)}$$

L_0 IS LOOP GAIN WITH PORT OPEN

LG17B

CONSIDER PORT IMPEDANCE (SERIES)



WITH PORT SHORTED $\Rightarrow L_S = G_m (R_1 \| R_2)$

$$G_m = \left(\frac{R_1 + R_2}{R_1 R_2}\right) L_S$$

$$Z_{IN} \equiv \frac{v_x}{i_x}$$

$$v_1 = -i_x R_1$$

$$v_2 = i_x R_2 - G_m v_1 R_2$$

$$= i_x R_2 + i_x G_m R_1 R_2$$

$$= i_x R_2 + i_x (R_1 + R_2) L_S$$

LG17C

$$v_x = v_2 - v_1$$

$$v_x = i_x \left(R_2 + (R_1 + R_2) L_S + R_1 \right)$$

$$Z_{IN} = \frac{v_x}{i_x} = R_1 + R_2 + (R_1 + R_2) L_S$$

$$Z_{IN} = (R_1 + R_2)(1 + L_S)$$

DEFINE $Z_{IN}^0 = (R_1 + R_2)$

IMPEDANCE LOOKING INTO PORT WHEN FEEDBACK LOOP IS BROKEN INSIDE AMP ($L = 0$)

$$Z_{IN} = Z_{IN}^0 (1 + L_S)$$

L_S IS LOOP GAIN WITH PORT SHORTED

FINDING CLOSED-LOOP INPUT/OUTPUT IMPEDANCE IN GENERAL

FOR A PORT Z_P

$$Z_P = Z_P^0 \left[\frac{1 + L_S}{1 + L_O} \right]$$

$L_S \Rightarrow$ LOOP GAIN WITH PORT SHORTED

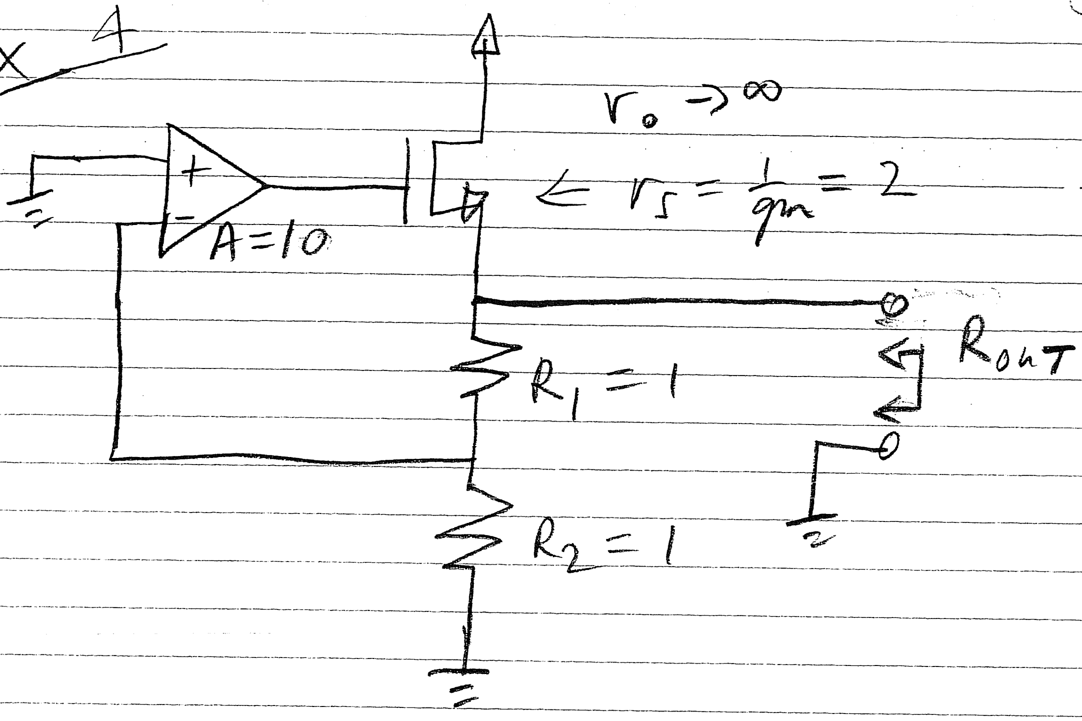
$L_O \Rightarrow$ LOOP GAIN WITH PORT OPENED

$Z_P^0 \Rightarrow$ PORT IMPEDANCE WITH LOOP BROKEN INSIDE AMP

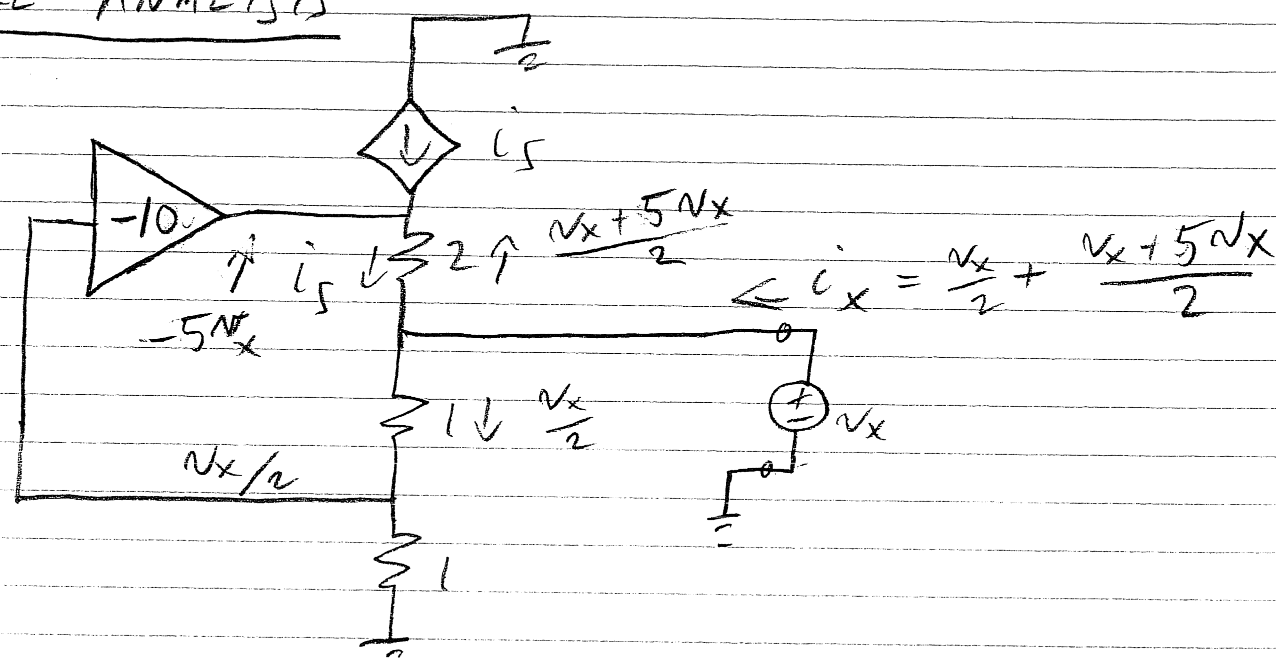
CAN USUALLY MOVE PORT CLOSER TO FEEDBACK SUCH THAT ONE OF L_S OR L_O IS ZERO

AND OTHER IS L (THE LOOP GAIN)

Ex 4



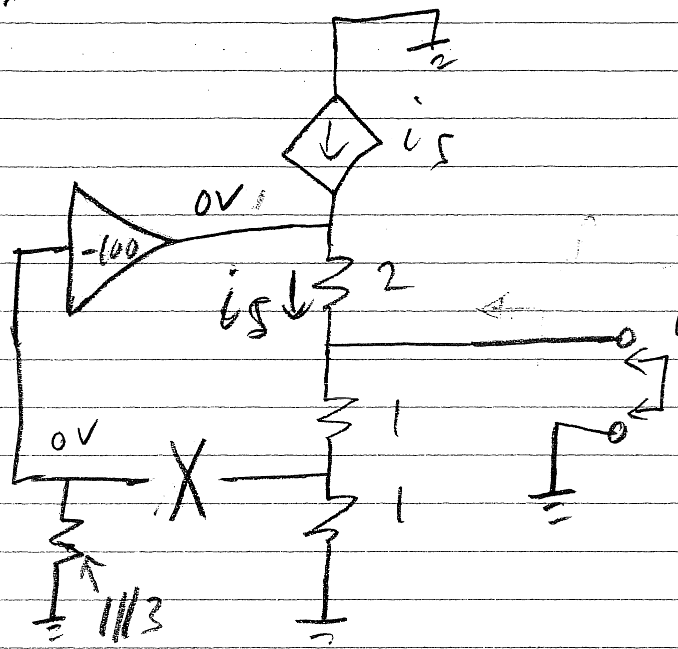
FULL ANALYSIS



$$R_{OUT} \equiv \frac{v_x}{i_x} = \left(\frac{1}{2} + \frac{1}{2} + \frac{5}{2}\right)^{-1} = \frac{2}{7}$$

(LG19)

LG ANALYSIS



$L = \left(\frac{1}{2}\right)\left(\frac{1}{2}\right)(10) = 5/2$

$R_1(L=0) = 2 \| 2 = 1\,\Omega$

$L_S = 0$

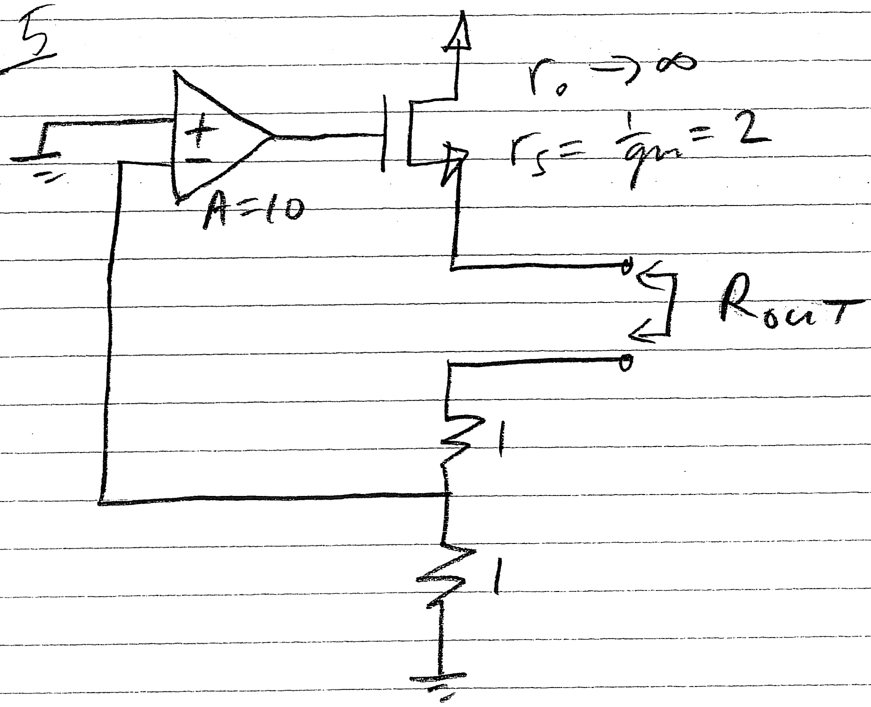
$L_O = L$

$R_{OUT} = R_1(L=0)\left[\frac{1+L_S}{1+L_O}\right] = (1)\left(\frac{1+0}{1+5/2}\right)$

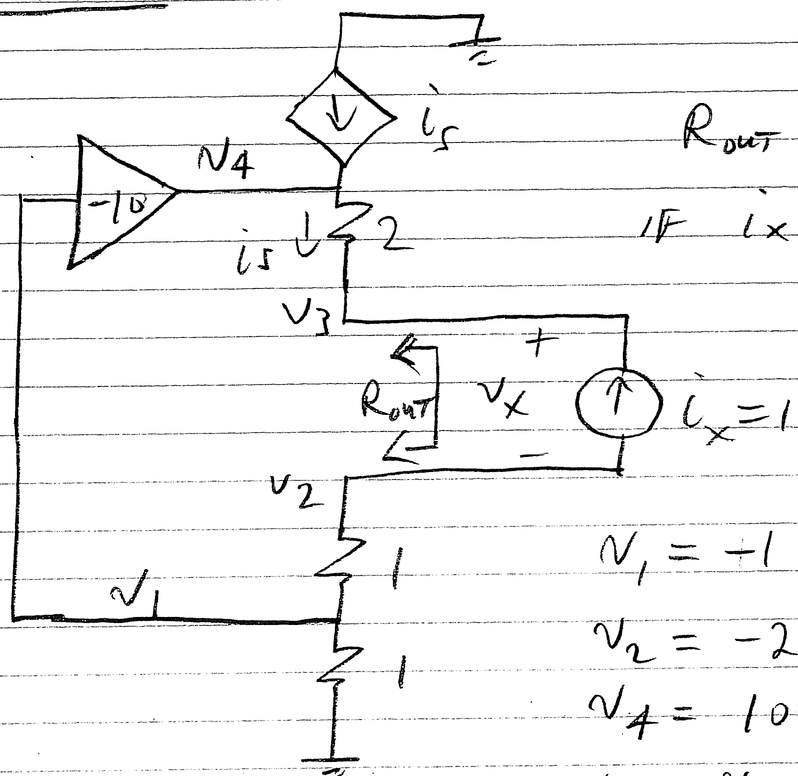
$R_{OUT} = \frac{2}{7}$ ✓

LG20

Ex 5



FULL-ANALYSIS



$R_{OUT} = \frac{v_x}{i_x}$

IF $i_x = 1 \Rightarrow R_{OUT} = v_x$

$v_1 = -1\ V$

$v_2 = -2\ V$

$v_4 = 10\ V$

$v_3 = v_4 + 2 i_s = 10 + 2$

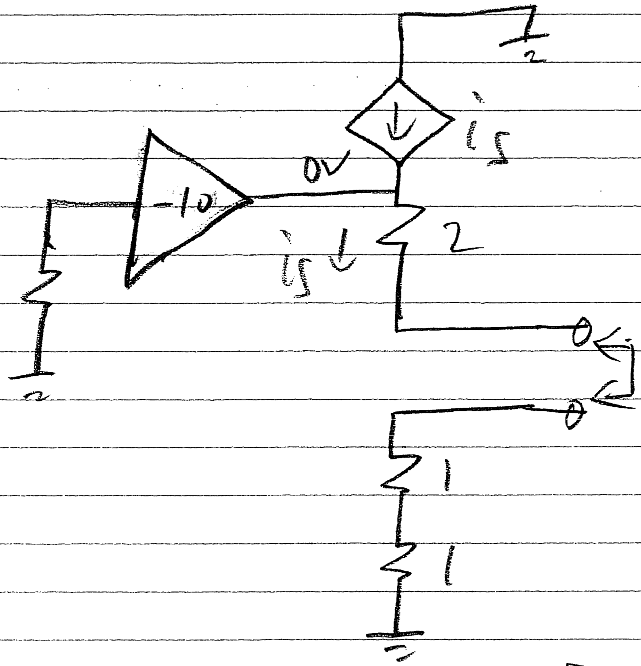
$= 12\ V$

$v_x = v_3 - v_2 = 14\ V$

$R_{OUT} = \underline{14\ \Omega}$

LG21

LG ANALYSIS



$L = 5/2$

$L_s = L = 5/2$

$L_0 = 0$

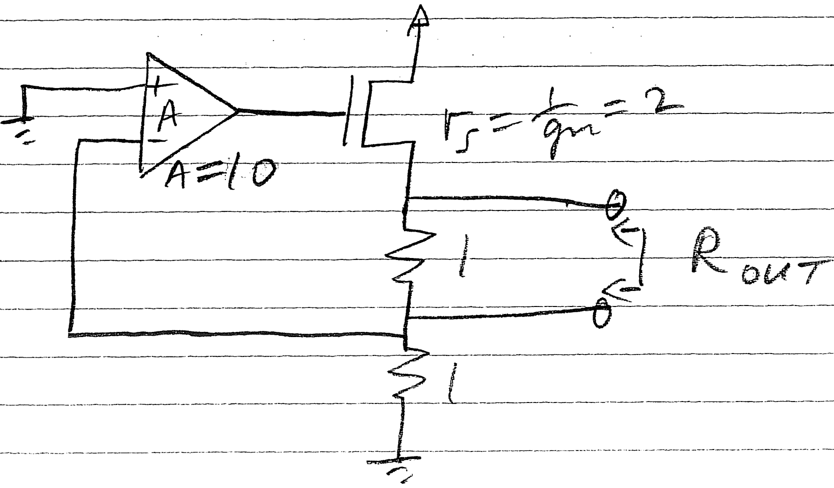
$R_1(L=0) = 2+2 = 4$

$$R_{OUT} = R_1(L=0)\left[\frac{1+L_s}{1+L_0}\right] = (4)\left(\frac{1+5/2}{1+0}\right)$$

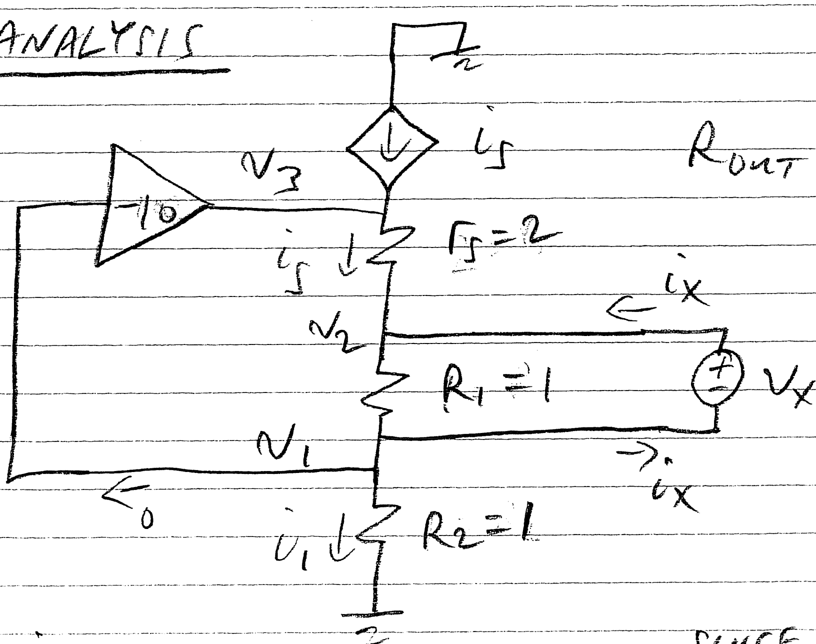
$$= 14\,\Omega$$

EX 6

A CASE WHERE NEITHER L_s OR L_o EQUALS 0



FULL ANALYSIS



$$R_{OUT} \equiv \frac{v_x}{i_x}$$

$i_s = i_1 \Rightarrow v_3 - v_2 = 2v_1$ SINCE $r_s = 2R_2$

$$v_1 + v_x + 2v_1 = -10v_1 \Rightarrow v_x = -13v_1 \quad (1)$$

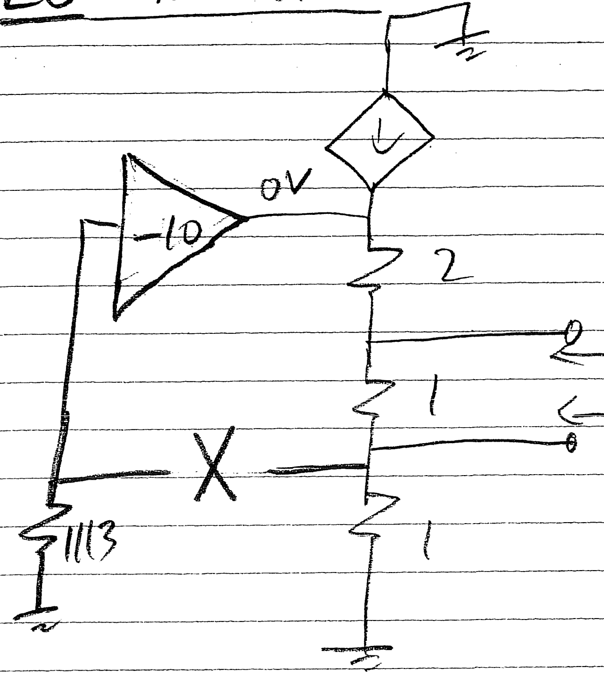
$$\frac{v_x}{R_1} = i_x + \frac{v_1}{R_2} \Rightarrow v_x = i_x + v_1 \Rightarrow v_1 = v_x - i_x \quad (2)$$

PUT (2) INTO (1) $\Rightarrow 14v_x = 13i_x \Rightarrow R_{OUT} = \frac{v_x}{i_x}$

$$R_{OUT} = \frac{13}{14}$$

LG23

LG ANALYSIS



$R_1(L=0) = 1 \| 3 = \frac{3}{4}$

$L_0 = L = \frac{5}{2}$

$L_S = \left(\frac{1}{3}\right)(10) = \frac{10}{3}$

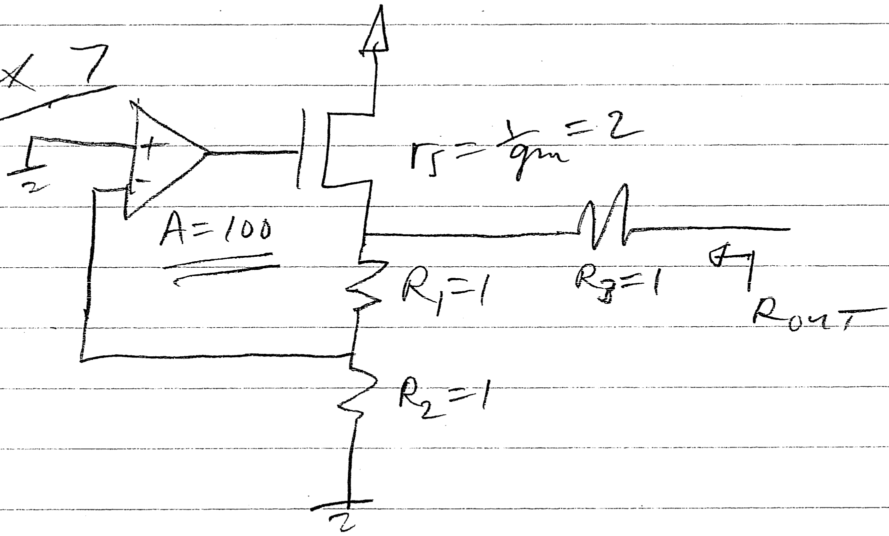
$$R_{OUT} = R_1(L=0)\left[\frac{1+L_S}{1+L_0}\right] = \left(\frac{3}{4}\right)\left(\frac{1+\frac{10}{3}}{1+\frac{5}{2}}\right)$$

$$R_{OUT} = \left(\frac{3}{4}\right)\left(\frac{\left(\frac{13}{3}\right)}{\left(\frac{7}{2}\right)}\right) = \frac{13}{14}\ \Omega$$ ✓

LG24

WHAT IF PORT IMPEDANCE TO BE FOUND WHERE EXTRA RESISTANCE NOT IN FEEDBACK LOOP EXISTS?

EX 7



CAN USE LG ANALYSIS DIRECTLY

$$R_1(L=0) = R_3 + [r_s \| (R_1+R_2)] = 2\,\Omega$$

$$L_0 = L = 25 \qquad L_s = \left(\frac{R_3\|(R_1+R_2)}{R_3\|(R_1+R_2) + r_s}\right)\left(\frac{R_2}{R_2+R_1}\right)A$$

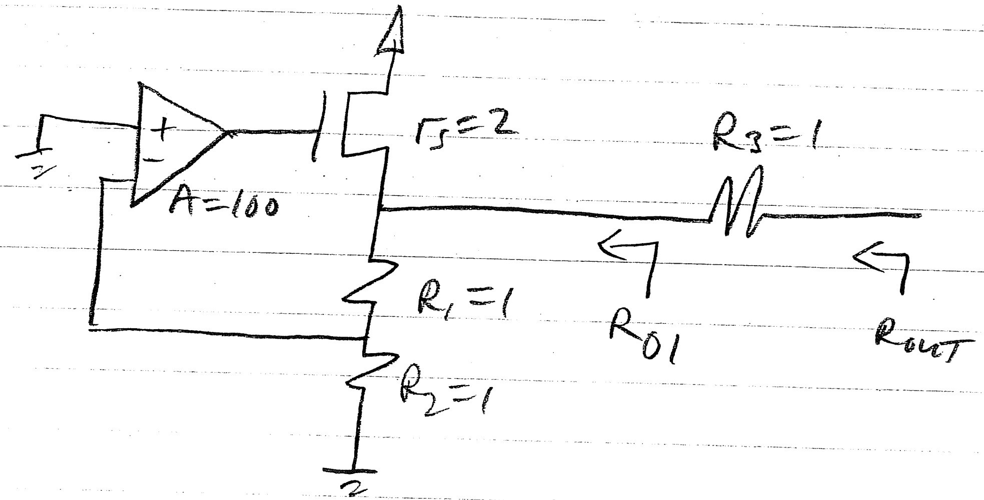
$$= 12.5$$

$$R_{OUT} = R_1(L=0)\left[\frac{1+L_s}{1+L_0}\right] = 2\left[\frac{1+12.5}{1+25}\right]$$

$$= 1.0385$$

HOWEVER EASIER TO FIND IMPEDANCE CLOSER TO FEEDBACK LOOP & THEN INCLUDE R_3

LG25



$$R_{01} = 1\left[\frac{1+L_s}{1+L_o}\right] = 1\left[\frac{1+0}{1+25}\right] = \frac{1}{26}$$

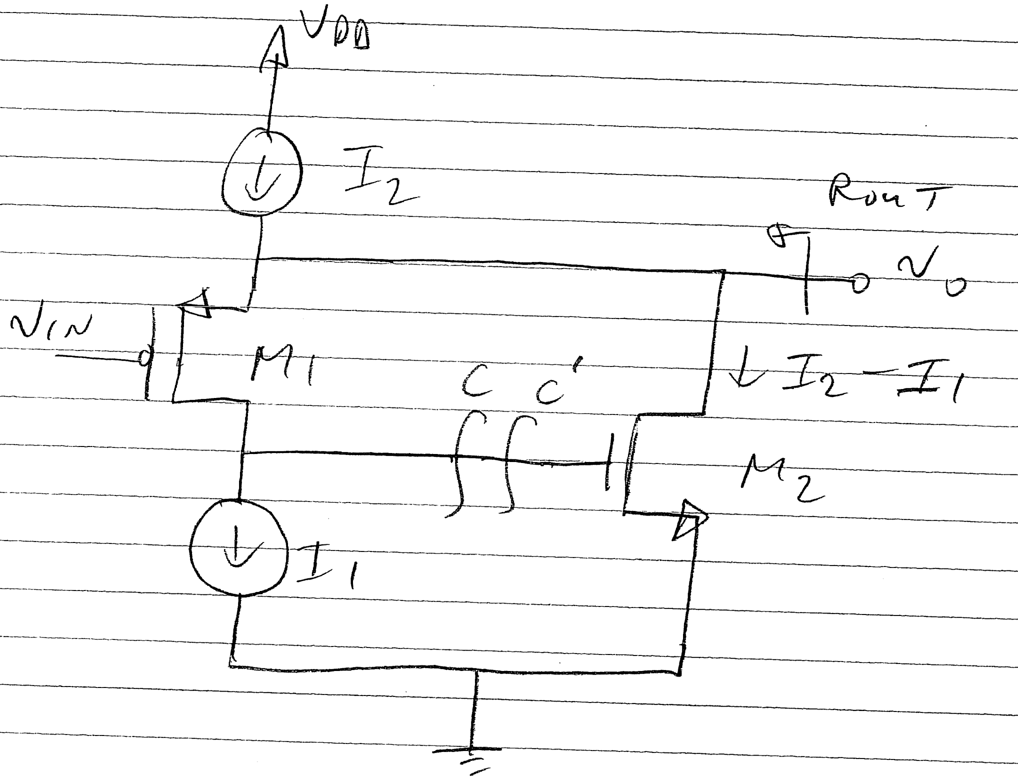
$$R_{OUT} = R_3 + R_{01} = 1.0385 \checkmark$$

IN GENERAL, TRY TO FIND PORT IMPEDANCES CLOSER TO FEEDBACK LOOP SUCH THAT ONE OF L_s OR L_o GOES TO <u>ZERO</u> AND OTHER IS $\underline{\underline{L}}$

LG26

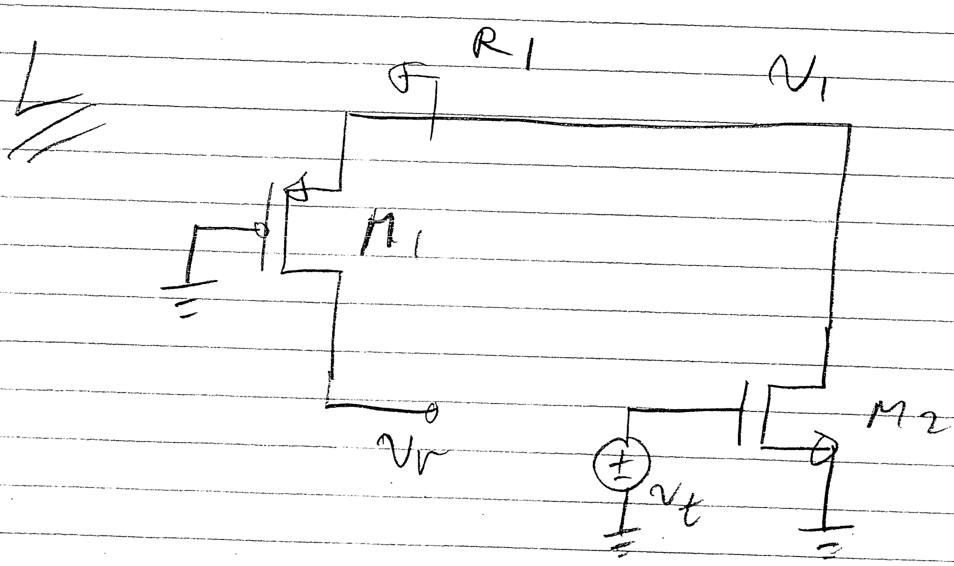
EXAMPLE 8

SUPER SOURCE FOLLOWER



ASSUME I_1, I_2 IDEAL

INCLUDE r_{o1}, r_{o2}



LG 27

$R_1 \to \infty$

$v_1 = -g_{m2} r_{o2}$

$v_r = i_{sc1} r_o \qquad i_{sc1} = \frac{g_m r_o + 1}{r_o} v_1$

(SEE MS NOTES)

$v_r = (1 + g_{m1} r_{o1}) v_1$

$$L \equiv -\frac{v_r}{v_t} = g_{m2} r_{o2} (1 + g_{m1} r_{o1})$$

$$L \approx g_{m1} g_{m2} r_{o1} r_{o2}$$

$$R_{OUT} = R_{1,0} \left[\frac{1 + L_S}{1 + L_O} \right] \qquad L_S = 0 \qquad L_O = L$$

$R_{1,0} = r_{o2}$

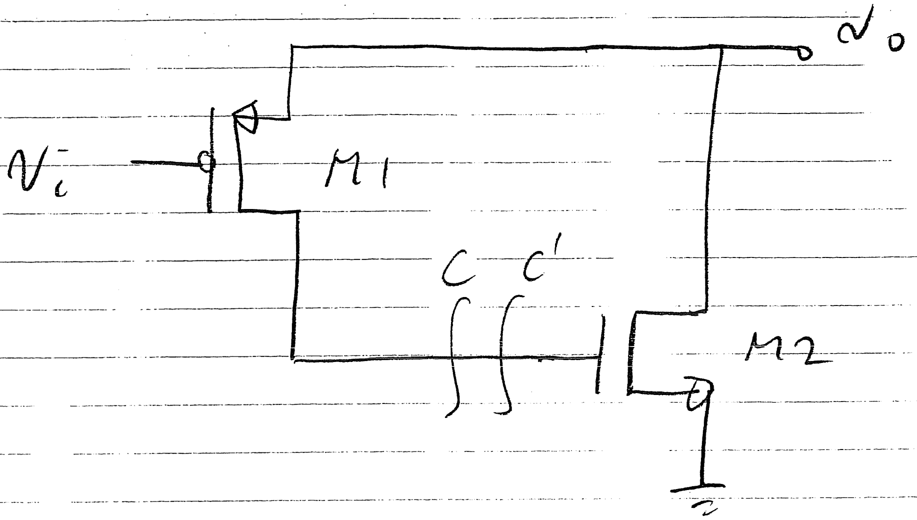
$$R_{OUT} = \frac{r_{o2}}{1 + g_{m1} g_{m2} r_{o1} r_{o2}} \approx \frac{r_{o2}}{g_{m1} g_{m2} r_{o1} r_{o2}}$$

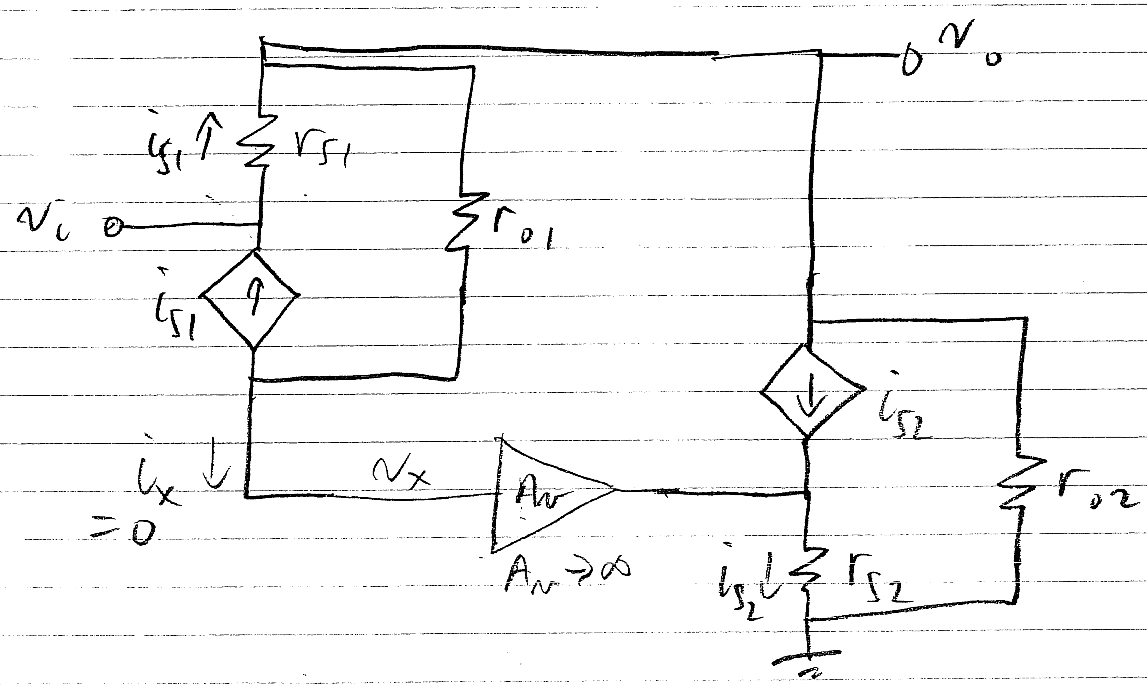
$$\boxed{R_{OUT} \approx \frac{1}{g_{m2}(g_{m1} r_{o1})}}$$

REDUCED BY $g_{m1} r_{o1}$ COMPARED TO SOURCE FOLLOWER

Ex 8 (CONTINUED)

A_∞





$A_v \to \infty \Rightarrow v_x \to 0$

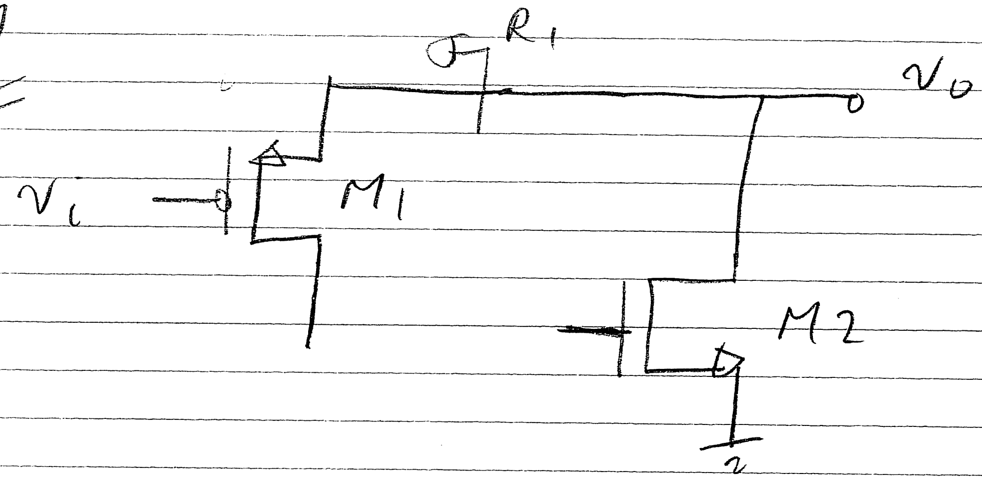
$$\frac{v_o - v_x}{r_{o1}} = i_{s1} = \frac{v_i - v_o}{r_{s1}} \quad + v_x = 0$$

(LG29)

$$\frac{v_o}{r_{o1}} + \frac{v_o}{r_{s1}} = \frac{v_i}{r_{s1}}$$

$$A_\infty \equiv \left.\frac{v_o}{v_i}\right|_{L\to\infty} = \frac{(r_{o1} \| r_{s1})}{r_{s1}} \approx 1$$

d



$$v_o = v_{oc1}\left(\frac{r_{o2}}{r_{o2} + R_1}\right) \qquad R_1 \to \infty$$

$$v_o = 0$$

$$d = 0$$

LG30

$$A_{CL} = A_\infty \left(\frac{L}{1+L} \right) + d \left(\frac{1}{1+L} \right)$$

$$A_{CL} \cong \left(\frac{r_{o1} \| r_{S1}}{r_{S1}} \right) \left(\frac{g_{m2} r_{o2} (1 + g_{m1} r_{o1})}{g_{m2} r_{o2} (1 + g_{m1} r_{o1}) + 1} \right)$$

$$A_{CL} \approx \underline{1}$$