N1

NOISE IN SC CIRCUITS

FIRST NEED TO LOOK AT SETTLING TIME-CONSTANTS

2 TYPES OF OPAMP ARCH.

CASE A) SINGLE STAGE WHERE OUTPUT CAPACITANCE SETS $\omega_{3dB} = \frac{1}{\tau}$

CASE B) 2 STAGE WHERE INTERNAL COMPENSATION CAPACITOR C_C SETS $\omega_{3dB} = \frac{1}{\tau}$

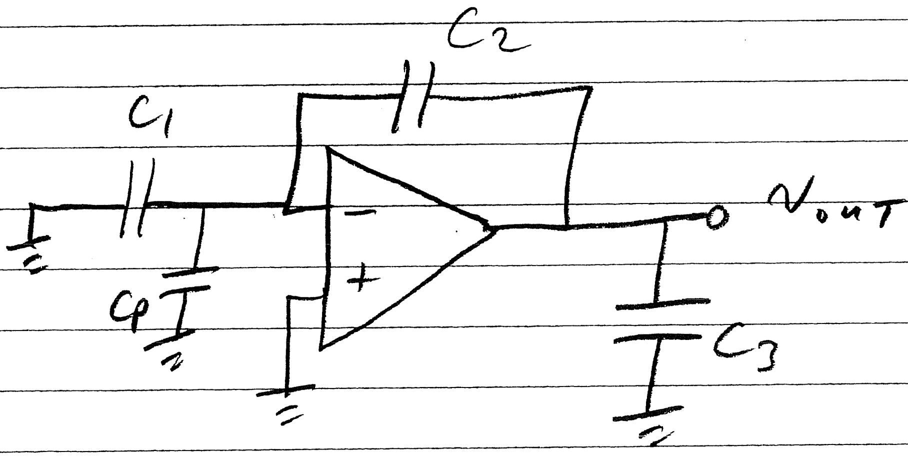
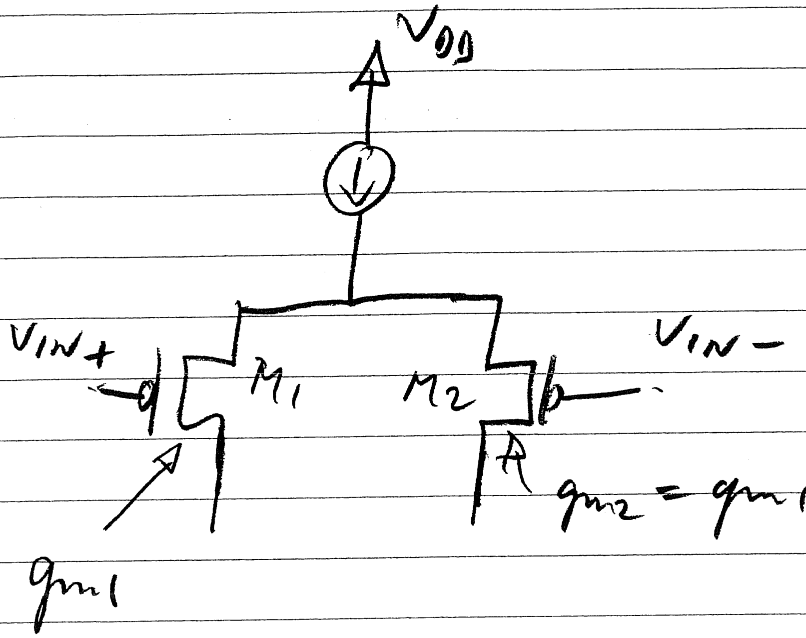


FIG 1

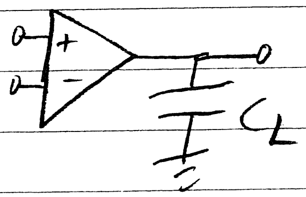
IN BOTH CASES, ASSUME INPUT STAGE OF OPAMP IS DIFFENTIAL STAGE WITH TRANCONDUCTANCE OF 1 TRANSISTOR BEING g_{m1}

N2



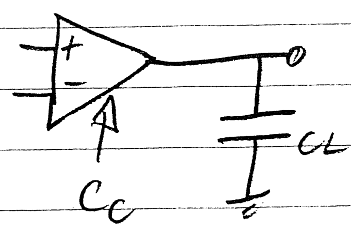
ALSO TRANSFER FUNCTION OF OPAMP MODELLED AS 1'ST ORDER SYSTEM WHERE

CASE A)



$$A(s) = \frac{g_{m1}}{sC_L}$$

CASE B)



$$A(s) = \frac{g_{m1}}{sC_c}$$

(INTERNAL COMPENSATION CAP)

N3

SO MODEL IS $A(s) = \dfrac{g_{m1}}{s C_0}$

WHERE $C_0 = C_L$ CASE A)
$C_0 = C_C$ CASE B)

SO FOR FIG ①, LOOP GAIN, $L(s)$, IS

$$L(s) = \beta(s) A(s)$$

WHERE $\beta(s) = \dfrac{C_2}{C_1 + C_2 + C_p} \equiv \beta$

NOT A FUNCTION OF "s"

AND $A(s) \equiv \dfrac{g_{m1}}{s C_0}$

WHERE

CASE A) $C_0 = C_3 + \dfrac{(C_1 + C_p) C_2}{(C_1 + C_2 + C_p)}$

CASE B) $C_0 = C_C$

N4

POLE OF CLOSED-LOOP SYSTEM FOUND BY

$$1 + L(s) = 0$$

$$1 + \frac{\beta g_{m1}}{s C_o} = 0 \Rightarrow s = -\frac{\beta g_{m1}}{C_o}$$

POLE ON NEGATIVE REAL AXIS AT $-\frac{\beta g_{m1}}{C_o}$

$$\Rightarrow \omega_{3dB} = \frac{\beta g_{m1}}{C_o} \equiv \frac{1}{\tau}$$

CLOSED-LOOP SYSTEM IS 1'ST ORDER WITH TIME-CONSTANT

$$\boxed{\tau = \frac{C_o}{\beta g_{m1}}} \qquad (1)$$

IF SC CIRCUIT, USUALLY HAVE $\frac{1}{2}$ CLOCK CYCLE TO SETTLE

DEFINE SAMPLE-RATE, $f_s = \frac{1}{T}$

CLOCK PERIOD, T

(2 PHASE SC CIRCUIT)

(N5)

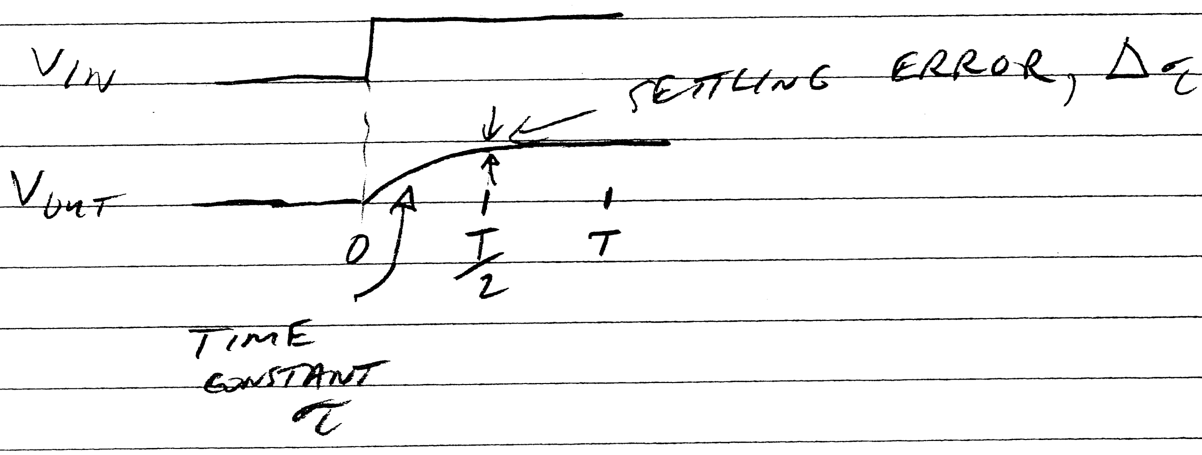
IF N_{EFF} IS NUMBER OF EFFECTIVE BITS OF RESOLUTION

WANT ERROR TO BE LESS THAN $2^{-(N_{EFF}+1)}$

$$e^{-\frac{T/2}{\tau}} < 2^{-(N_{EFF}+1)} \equiv \Delta_\tau$$

$$\frac{(T/2)}{\tau} > (N_{EFF}+1)\ln(2) \qquad (2)$$

DEFINE $N_\tau \equiv \left(\frac{T/2}{\tau}\right)$ # OF τ OF SETTLING



Δ_τ = SETTLING ERROR	$N_\tau = \left(\frac{T/2}{\tau}\right)$	N_{EFF} (BITS)
1%	4.6	5.6
0.1%	6.9	9.0
0.01%	9.2	12.3
0.001%	11.5	15.6

NSA

IF $T/2$ GIVEN & ACCURACY GIVEN

THEN $\tau < \dfrac{(T/2)}{(N_{EFF}+1)\ln(2)}$

SO IF C_0 DETERMINED (FROM NOISE ANALYSIS)

THEN $g_{m1} = \dfrac{C_0}{\beta\,\tau}$
FROM (1)

g_{m1} IS INVERSELY PROPORTIONAL TO β

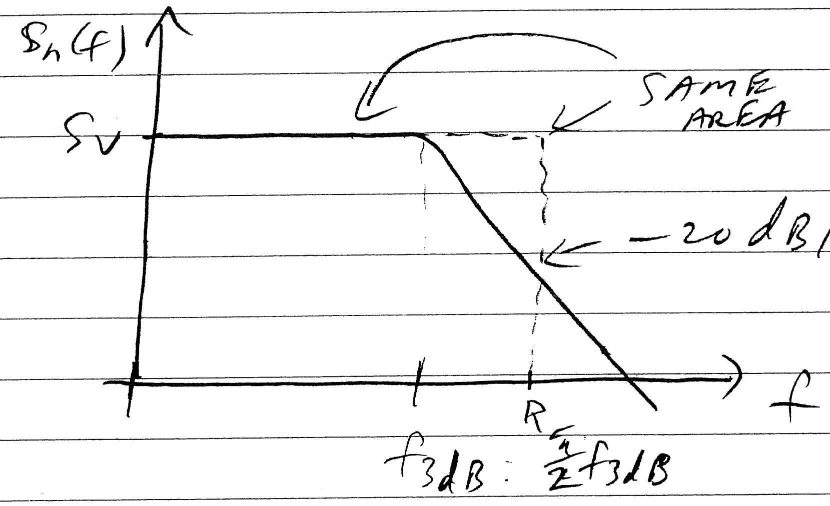
[GOOD TO KEEP β HIGH THOUGH $\beta \le 1$]

AND <u>POWER</u> IS PROPORTIONAL TO g_{m1}

NOISE

RECALL NOISE EQUIVALENT BANDWIDTH

GIVEN NOISE ROOT SPECTRAL DENSITY, $S_n(f)$



TOTAL NOISE, $\overline{v_o^2}$

$$\overline{v_o^2} = \int_0^\infty S_n^2(f)\, df$$

$$= \left(\frac{\pi}{2}\right) f_{3dB}\, S_V^2$$

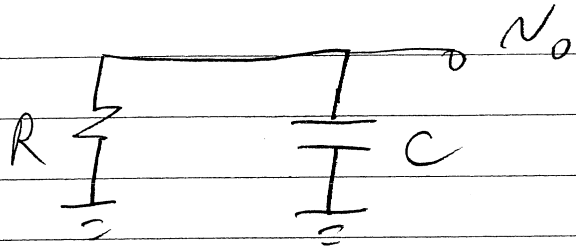
AND SINCE $f_{3dB} = \frac{\omega_{3dB}}{2\pi} = \frac{1}{2\pi\tau}$

$$\overline{v_o^2} = \left(\frac{1}{4\tau}\right)\left(S_V^2\right)$$

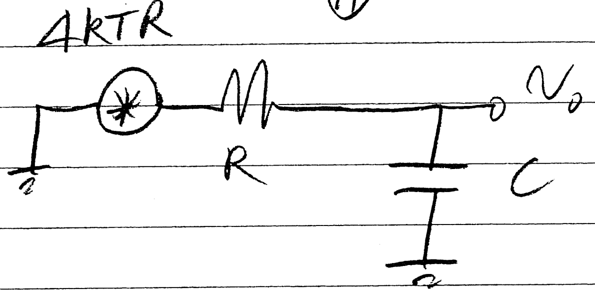
SO TOTAL NOISE IS dc SPECTRAL DENSITY TIMES $\frac{1}{4\tau}$

(N7A)

1F RC CIRCUIT



$\tau = RC$

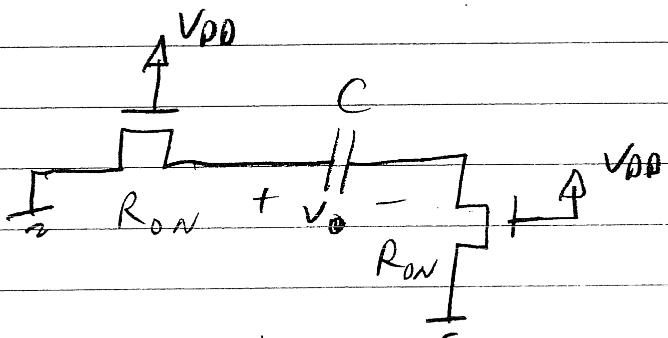


$S_v^2 = 4kTR$

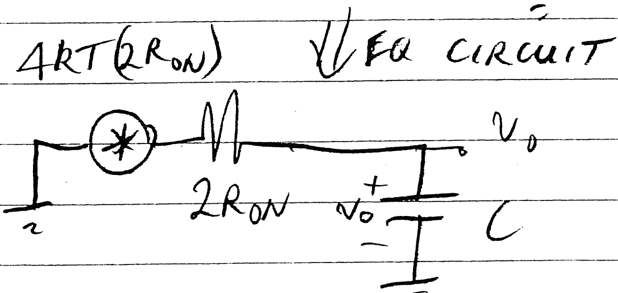
TOTAL NOISE

$$\overline{v_o^2} = \frac{1}{4(RC)}(4kTR) = \frac{kT}{C}$$

1F



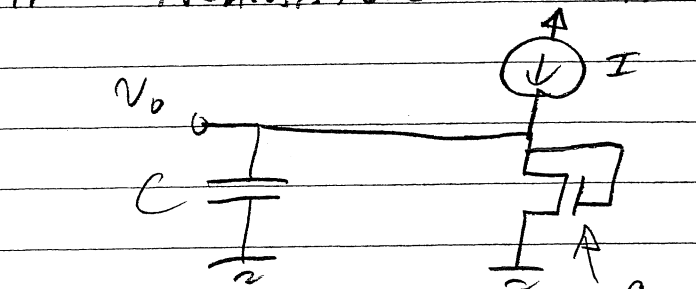
EQ CIRCUIT



$$\overline{v_o^2} = \frac{kT}{C}$$

N7B

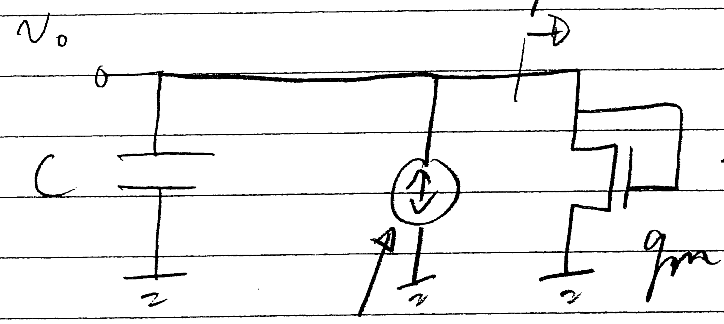
IF TRANSISTOR CIRCUIT



$$g_m = \frac{2I_0}{V_{eff}}$$

$$V_{eff} = V_{GS} - V_{tn}$$

$\Downarrow$



$$\tau = \frac{C}{g_m}$$

$$I_d^2(f) = \left(\frac{2}{3}\right) 4kT g_m$$

TOTAL NOISE

$$\overline{V_0^2} = \left(\frac{2}{3}\right)(4kT g_m)\left(\frac{1}{g_m^2}\right)\left(\frac{1}{4\tau}\right)$$

$$= \left(\frac{2}{3}\right)\frac{kT}{C}$$ SLIGHTLY LESS NOISE THAN A RC

BUT...

SHOULD INCLUDE NOISE OF CURRENT SOURCE

N7C

IF CURRENT SOURCE I
HAS SAME V_{eff} THEN ITS
NOISE CURRENT IS ALSO $\left(\frac{2}{3}\right) 4kT g_m$

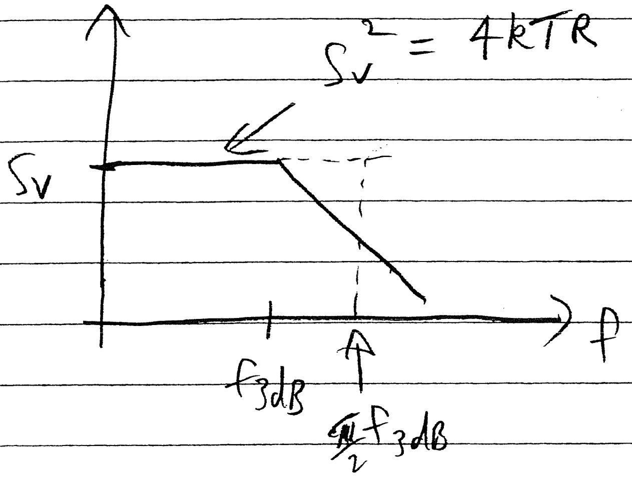
SO ...

$$\overline{v_o^2} = \left(\frac{4}{3}\right)\frac{kT}{C}$$

SLIGHTLY MORE NOISE THAN RC

N8

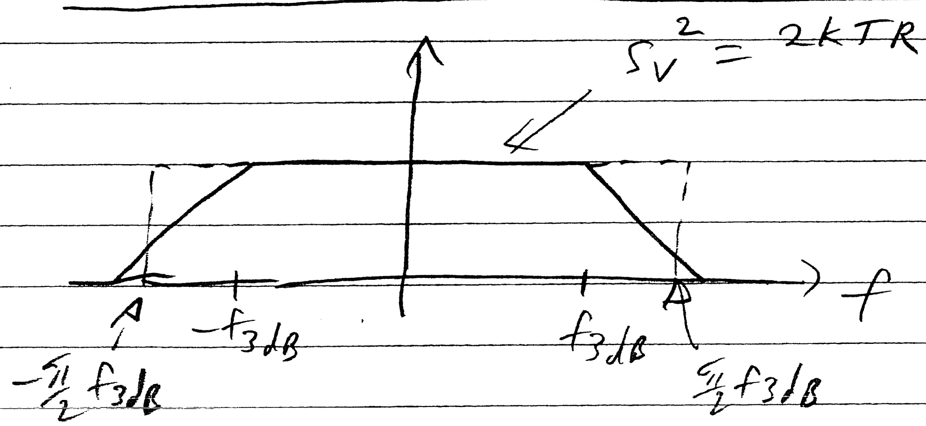
SINGLE-SIDED SPECTRUM DEFINITION (COMMONLY USED)



$$f_{3dB} = \frac{1}{2\pi RC}$$

$$\overline{V_o^2} = (4kTR)\left(\frac{1}{4RC}\right) = \frac{kT}{C}$$

DOUBLE-SIDE SPECTRUM DEFINITION (NOT COMMONLY USED)



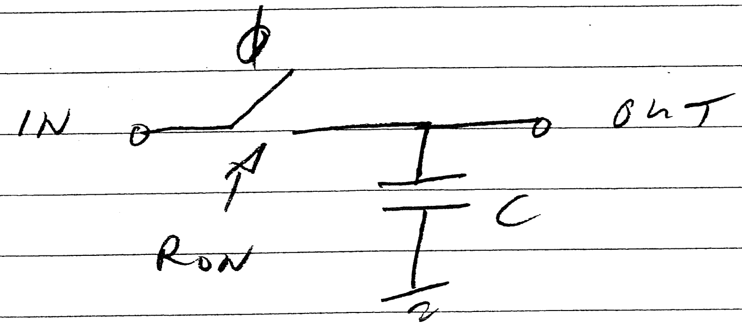
SPECTRAL DENSITY IS HALVED SINCE SPREAD OVER TWICE FREQ.

$$\overline{V_o^2} = (2kTR)\left(\frac{2}{4RC}\right) = \frac{kT}{C}$$

WILL USE DOUBLE-SIDE SPECTRUM ONLY FOR LOOKING AT ALIASING OF SAMPLED NOISE, OTHERWISE ALWAYS SINGLE-SIDE SPECTRUM USED.

N9A

SAMPLED – NOISE



$$\tau = R_{ON}\, C$$

RECALL $$N_\tau \equiv \frac{(T/2)}{\tau}$$

N_τ NORMALLY $5 \rightarrow 10$

WHICH IMPLIES

$$N_\tau = \frac{\left(\frac{1}{2 f_s}\right)}{\left(\frac{1}{2\pi f_{3dB}}\right)} = \frac{\pi\, f_{3dB}}{f_s}$$

$$\frac{f_s}{f_{3dB}} = \frac{\pi}{N_\tau} < 1$$

SIGNIFICANT ALIASING OF NOISE OCCURS.

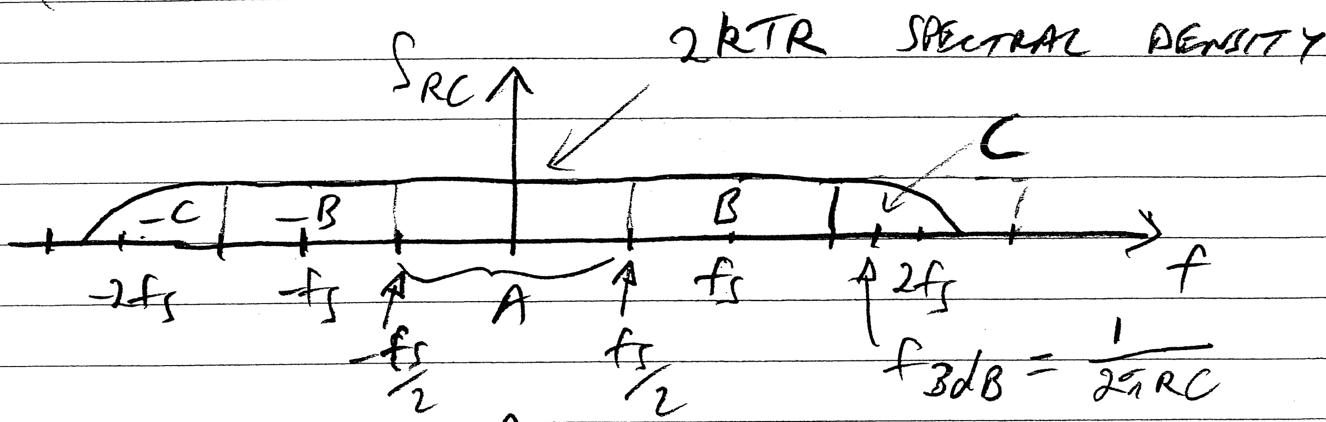
CORRELATION BETWEEN SAMPLES DEPENDS ON N_τ

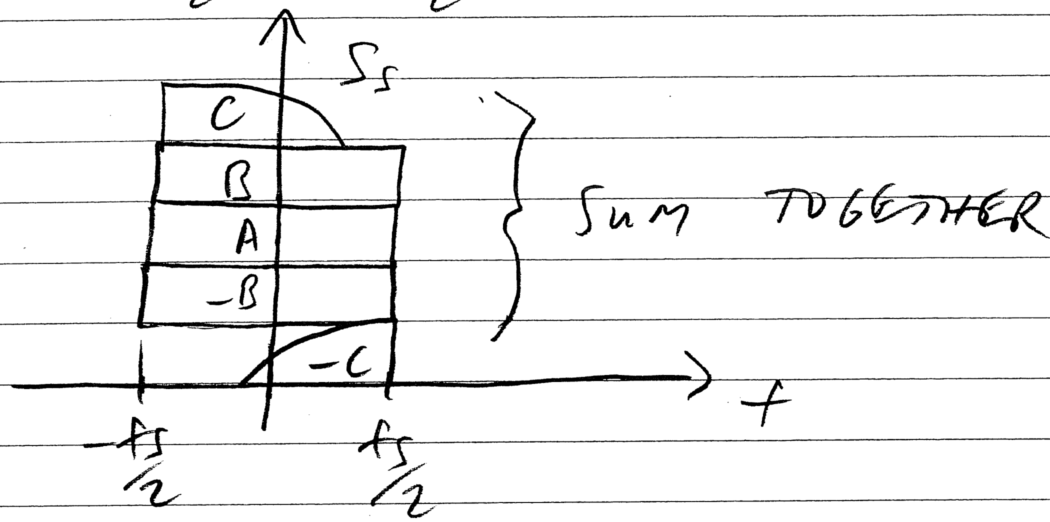
MORE ALIASING $\Rightarrow$ LESS CORRELATION

GENERALLY IF $N_\tau > 3$
THERE IS LITTLE CORRELATION BETWEEN
SAMPLES FOR $v_o(n)$

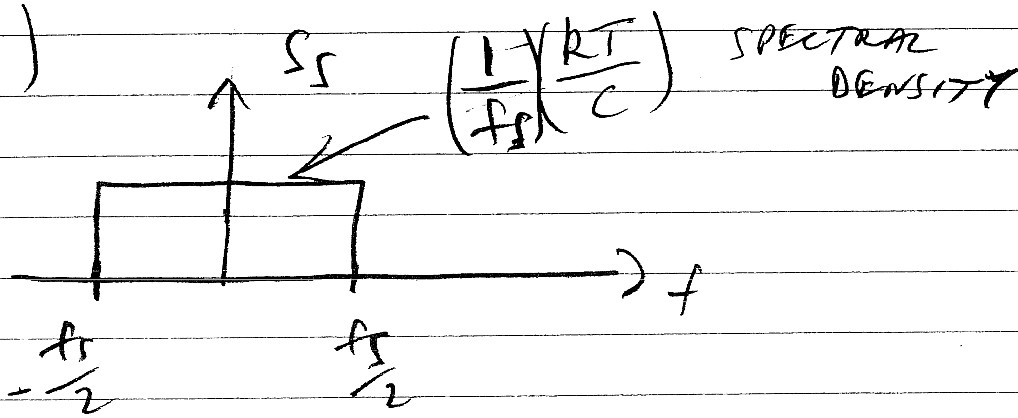
N10

(DOUBLE-SIDED)





(DOUBLE SIDED)



N11

IF ALL SINGLE-SIDE SPECTRUMS

THEN ORIGINAL SPECTRAL DENSITY

IS $S_{RC}^2 = 4kTR$

AND SAMPLED SPECTRAL DENSITY

IS $S_S^2 = \left(\frac{2}{f_S}\right)\left(\frac{kT}{C}\right)$

INCREASE IN SPECTRAL DENSITY OF

$$\frac{S_S^2}{S_{RC}^2} = \frac{\frac{2}{f_S}\left(\frac{kT}{C}\right)}{4kTR} = \frac{1}{2\tau f_S} = \frac{\pi f_{3dB}}{f_S}$$

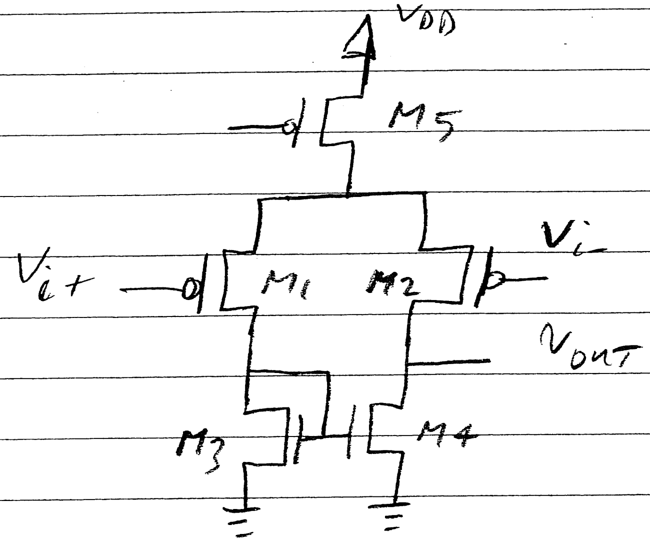
$$\boxed{\frac{S_S^2}{S_{RC}^2} = N_\tau}$$

DOES THIS MATTER? NOT REALLY

- CHOOSE CAPACITOR SIZE TO MEET NOISE PERFORMANCE
- CAN NOT IMPROVE PERFORMANCE BY REDUCING N_τ SINCE NOISE IS $\frac{kT}{C}$

N12

OPAMP NOISE (SINGLE-ENDED EXAMPLE)

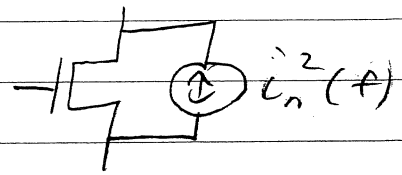


FIRST-STAGE OF 2-STAGE OPAMP

$M_1 = M_2$ (SIZES)
$M_3 = M_4$

EACH TRANSISTOR HAS CURRENT NOISE OF

$$\overline{i_n^2}(f) = \gamma\, 4kT g_m$$



USUALLY ASSUME $\gamma = \frac{2}{3}$

NOISE CURRENT FOR M5 IS COMMON MODE SO DOES NOT CONTRIBUTE MUCH OUTPUT NOISE

ASSUME OUTPUT RESISTANCE R_{OUT} AT V_{OUT}

OUTPUT NOISE

$$V_{on}^2 = \left(\frac{8}{3}kT g_{m1} + \frac{8}{3}kT g_{m2} + \frac{8}{3}kT g_{m3} + \frac{8}{3}kT g_{m4}\right) R_o^2$$

$$= \left(\frac{16}{3}kT g_{m1} + \frac{16}{3}kT g_{m3}\right) R_o^2$$

(N13)

EQUIVALENT INPUT NOISE v_{in}^2

$$v_n^2 = \frac{v_{on}^2}{g_{m1}^2 R_o^2}$$

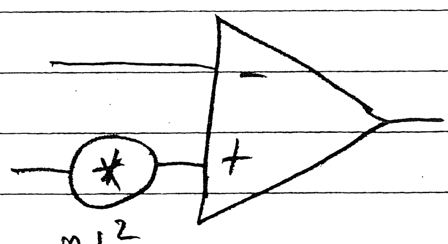
$$= \frac{16kT}{3g_{m1}}\left(1 + \frac{g_{m3}}{g_{m1}}\right)$$

DEFINE NOISE GAIN, n_f, TO BE INCREASE IN NOISE DUE TO OTHER TRANSISTORS IN OPAMP

IN THIS CASE $n_f = 1 + \frac{g_{m3}}{g_{m1}}$

$$\boxed{v_n^2 = \frac{16kT}{3g_{m1}} n_f}$$

IF $n_f = 1$ => NOISE ONLY DUE TO INPUT DIFF PAIR
- MINIMUM NOISE CASE

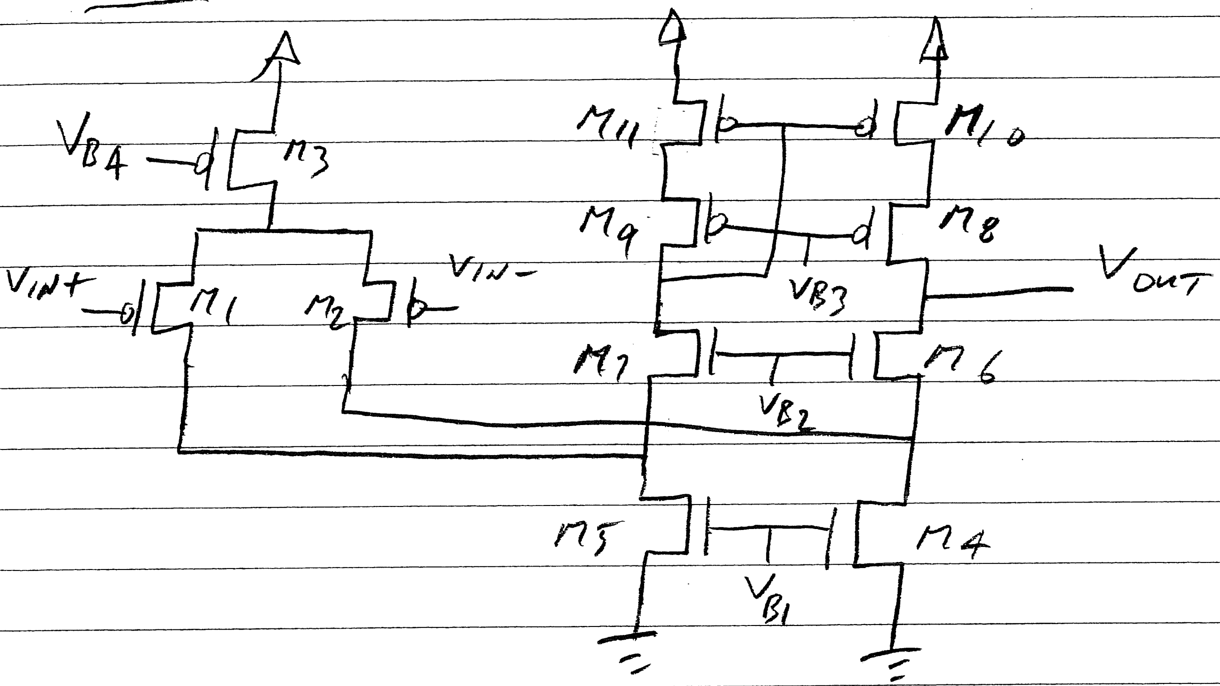


v_n^2

OPAMP MODEL

N14

n_f FOR FOLDED CASCODE



IGNORING CASCODE TRANSISTORS $M_6 - M_9$ NOISE

IF $V_{eff_1} = V_{eff_5} = V_{eff_{11}} \Rightarrow g_{m_1} = g_{m_{11}} = \frac{g_{m_5}}{2}$

$$n_f = 1 + \frac{g_{m_5}}{g_{m_1}} + \frac{g_{m_{11}}}{g_{m_1}} = 1 + 2 + 1 = 4$$

$n_f = 4 \Rightarrow$ 6 dB INCREASE IN NOISE WHEN OPAMP NOISE DOMINATES

MAY NEED MUCH LARGER CAPACITORS (UP TO 4 TIMES) WHICH IS MORE POWER (UP TO 4 TIMES)

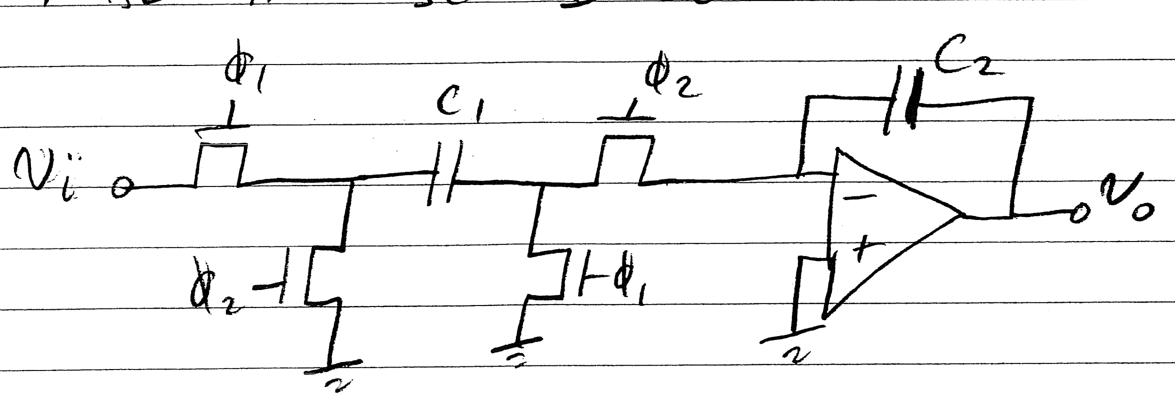
N15

IF POSSIBLE, KEEP N_f CLOSE TO ONE

FORTUNATELY, THIS EXTRA NOISE ONLY OCCURS IN CLOCK PHASE WHERE OPAMP NOISE IS IMPORTANT.

N16

NOISE IN SC INTEGRATOR



ϕ_1 HIGH $\Rightarrow$ SAMPLING PHASE

ϕ_2 HIGH $\Rightarrow$ INTEGRATING PHASE

WANT TO FIND INPUT REFERRED NOISE, $\overline{v_{ni}^2}$

BY FINDING OUTPUT NOISE, v_{no}^2 AND DIVIDING

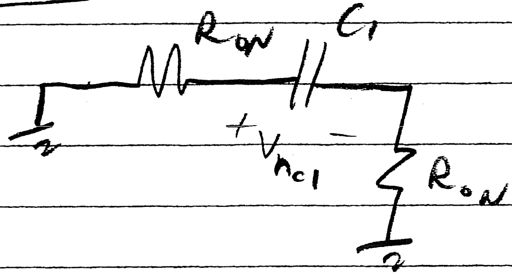
BY $\dfrac{v_o(f)}{v_i(f)}$ GAIN.

DEFINE $\quad H_I(f) = \dfrac{v_o(f)}{v_I(f)}$

FIND OUTPUT NOISE AT END OF ϕ_2 DUE TO:

ϕ_1 RESISTOR NOISE
ϕ_1 OPAMP NOISE
ϕ_2 RESISTOR NOISE
ϕ_2 OPAMP NOISE

Φ_1 RESISTOR NOISE



$$S^2_{nC1} = \left(\frac{2}{f_S}\right)\left(\frac{kT}{C_1}\right)$$

(SPECTRAL DENSITY)

$$S^2_{no} = \left(\frac{2}{f_S}\right)\left(\frac{kT}{C_1}\right) H_I^2(f)$$

$$S^2_{ni} = \frac{S^2_{no}}{H_I^2(f)} = \left(\frac{2}{f_S}\right)\left(\frac{kT}{C_1}\right)$$

$$\overline{v^2_{ni}} = \frac{kT}{C_1} \leftarrow \overline{v^2_{ni}} = \int_0^{f_S/2} S^2_{ni}(f)\, df$$

SWITCH Φ_1 NOISE

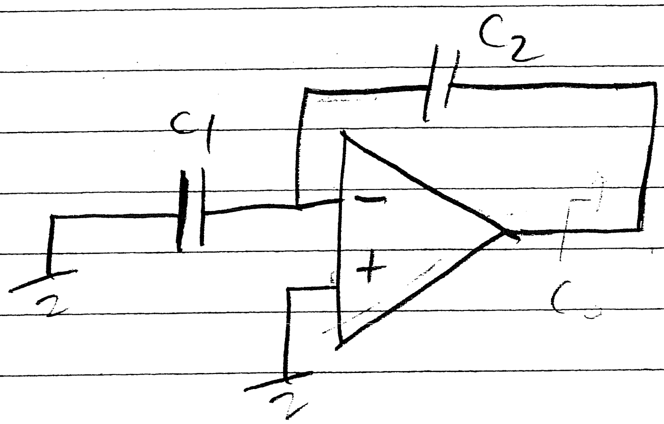
Φ_1 OPAMP NOISE

$\overline{v^2_{no}} = 0$ SINCE v_o HELD DURING Φ_1

$\overline{v^2_{ni}} = 0$

N18

ϕ_2 SETTLING TIME-CONSTANT



ASSUME OP AMP DOMINATES (OTHERWISE TOO MUCH POWER WASTED)

$$\tau = \frac{C_0}{\beta g_{m1}}$$

A) $C_0 = \dfrac{C_1 C_2}{C_1 + C_2} = C_2(1-\beta)$

B) $C_0 = C_C$

$$\beta = \frac{C_2}{C_1 + C_2}$$

RESISTOR

$$\overline{V_{ni}^2} = 4kT(2R_{ON})\left(\frac{\beta g_{m1}}{4 C_0}\right)$$

$$= \frac{kT}{C_0}\beta(2R_{ON} g_{m1}) \qquad \beta \le 1$$

IF $\dfrac{1}{g_{m1}} \gg 2R_{ON}$ THEN ABOVE NOISE $\ll \dfrac{kT}{C_0}$

Φ_2 PHASE
OPAMP

$$\frac{1}{4\tau}$$

$$\overline{v_{ni}^2} = \frac{16}{3}\frac{kT}{g_{m1}} n_f \left(\frac{\beta g_{m1}}{4 C_o} \right)$$

$$= \frac{kT}{C_o}\frac{4}{3} n_f \beta$$

NOTE IN CASE $\beta = \frac{C_2}{C_1+C_2}$ + $C_o = \frac{C_1 C_2}{C_1+C_2}$

$$\frac{\beta}{C_o} = \frac{1}{C_1}$$

$$\overline{v_{ni}^2} = \left(\frac{kT}{C_1}\right)\left(\frac{4}{3}\right) n_f$$

TOTAL INPUT NOISE

$$\overline{v_{ni}^2} \approx \left(\frac{KT}{C_1}\right)\left(1 + \frac{4 n_f}{3}\right)$$

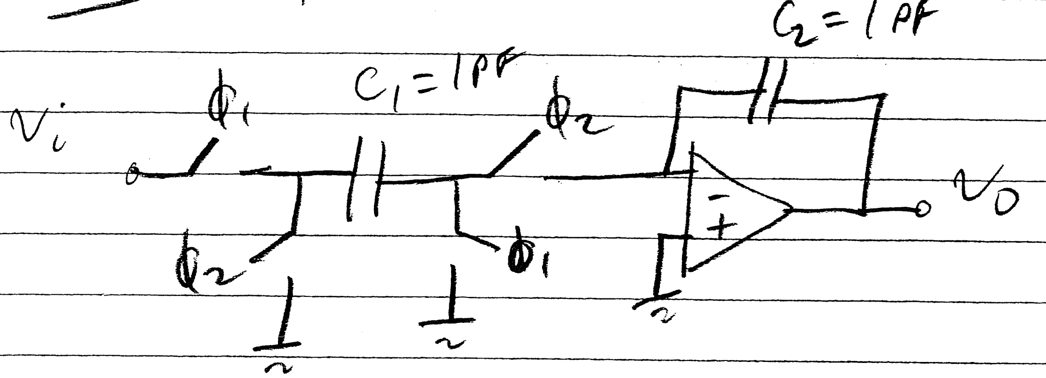
IF $n_f = 1$ (BEST CASE)

$$\overline{v_{ni}^2} = 2.33 \frac{kT}{C_1}$$

IF $n_f = 4$ (OTHER OPAMP TRANSISTORS CONTRIBUTE SIGNIFICANT NOISE)

$$\overline{v_{ni}^2} = 6.33 \frac{kT}{C_1}$$ (4.3dB WORSE THAN ABOVE)

Ex $\quad n_f = 1$



$\beta = 0.5$

$$\overline{v_{ni}^2} = 2.33 \frac{kT}{C_1}$$

$k = 1.38e^{-23}\ JK^{-1}$

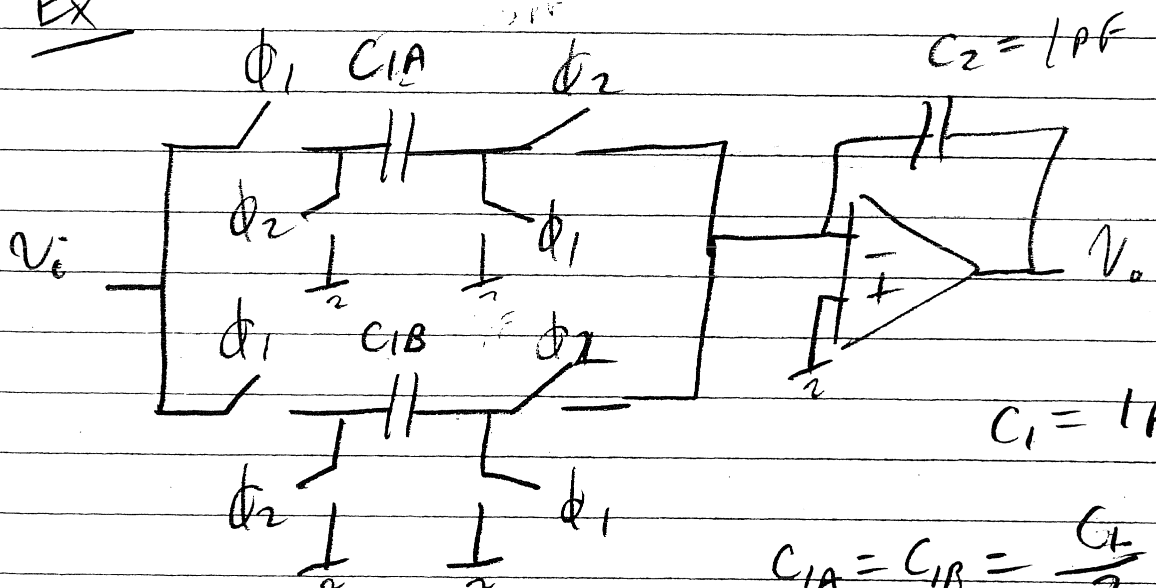
$T = 300\ ^\circ K$

$$H_I(z) = \frac{1}{z-1}$$

$$= 9.65e{-9}\ v^2$$

$v_{ni(RMS)} = 98\ \mu V$

Ex



$\beta = 0.5$

$C_1 = 1\,pF$

$$C_{1A} = C_{1B} = \frac{C_1}{2} = 0.5\,pF$$

$$H_I(z) = \frac{1}{z-1}$$ AS ABOVE

N22

$$S_{n_{1A}}^2 = \left(\frac{2}{f_S}\right)\left(\frac{2kT}{C_1} + \frac{4}{3}\left(\frac{2kT}{C_1}\right)\right)$$

$$S_{n_{1B}}^2 = S_{n_{1A}}^2$$

$$S_{n_o}^2 = \left(\frac{2}{f_S}\right)\left(\frac{4kT}{C_1} + \left(\frac{4}{3}\right)\left(\frac{4kT}{C_1}\right)\right)\left[\frac{1}{2} H_I(f)\right]^2$$

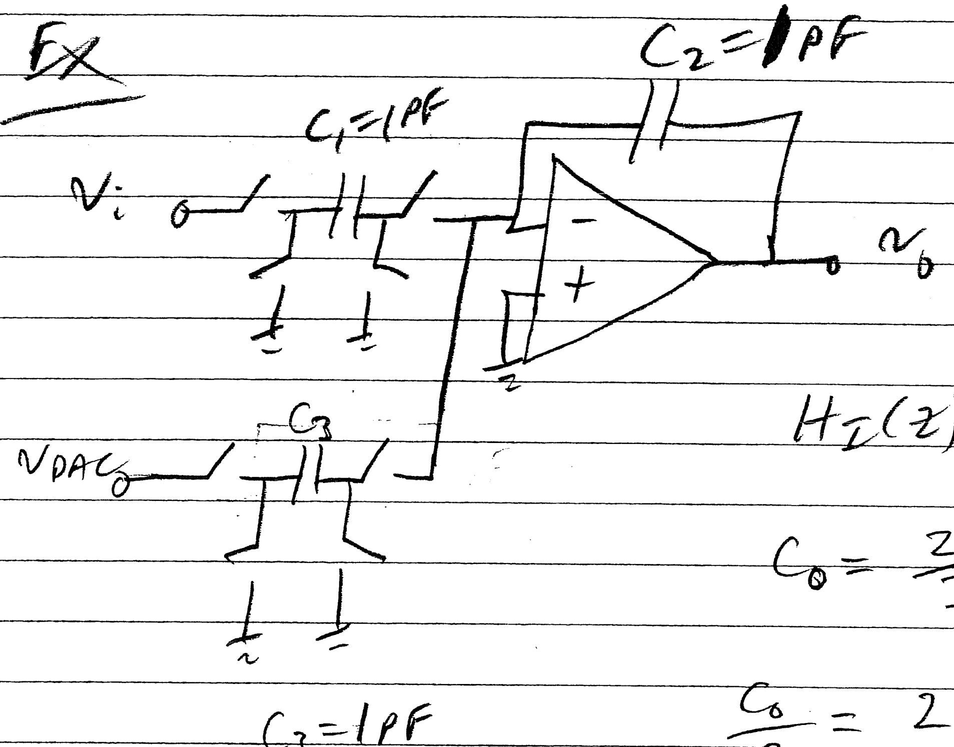
$$S_{n_i}^2 = \frac{S_{n_o}^2}{[H_I(f)]^2}$$

$$= \left(\frac{2}{f_S}\right)\left(\frac{kT}{C_1} + \left(\frac{4}{3}\right)\left(\frac{kT}{C_1}\right)\right)$$

$$\overline{v_{n_i}^2} = 2.33 \frac{kT}{C_1}$$ AS BEFORE

N23

EX



$\beta = \frac{1}{3}$

$H_I(z) = \frac{1}{z-1}$

$C_0 = \frac{2}{3}\,pF$

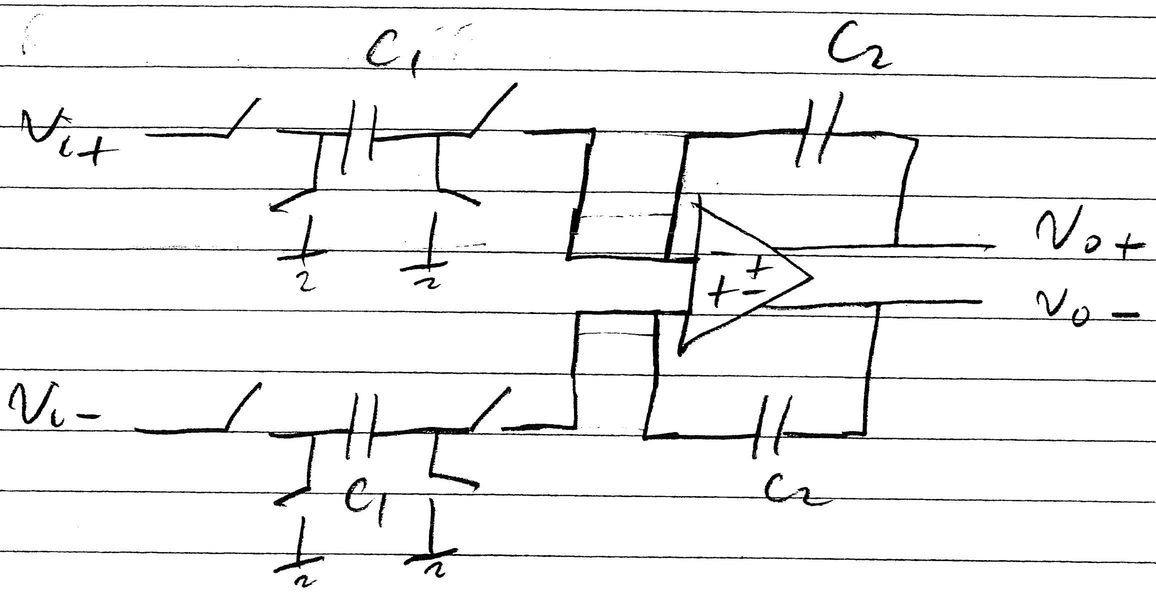
$\frac{C_0}{\beta} = 2 = 2C_1$

$$\overline{V_{ni}^2} = \frac{kT}{C_1} + \frac{4}{3}\frac{kT}{\left(C_0/\beta\right)} + \frac{kT}{C_3} + \frac{4}{3}\frac{kT}{\left(C_0/\beta\right)}$$

$$= 3.33\,\frac{kT}{C_1}$$

$V_{ni\,(RMS)} = 117\,\mu V$ (1.6 dB HIGHER NOISE)

SINGLE-ENDED VS DIFFERENTIAL

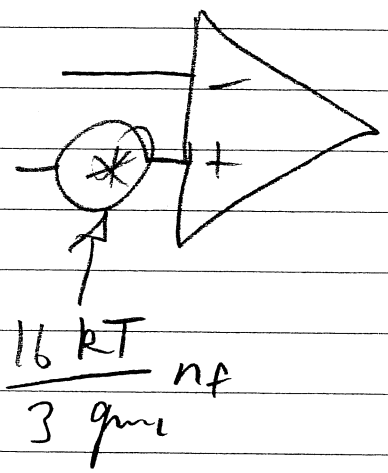


<u>RESISTOR NOISE</u>

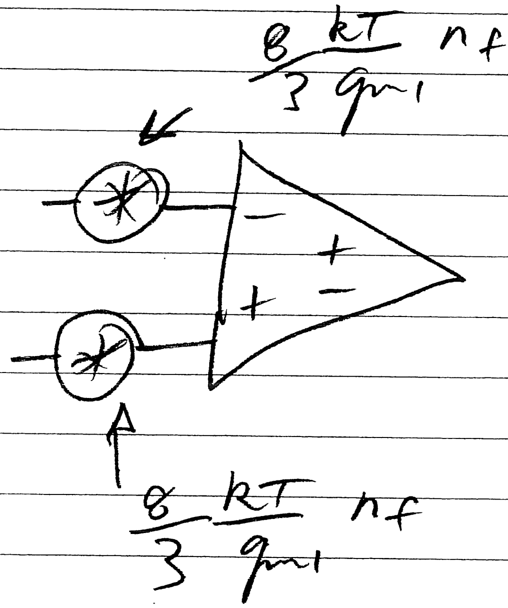
EACH SIDE SAME NOISE AS SINGLE-ENDED BUT 2 SIDES SO NOISE IS 3dB HIGHER. HOWEVER SIGNAL SWINGS ARE 6 dB HIGHER SO SNR IS 3dB HIGHER IF CAPS SAME SIZE AS SINGLE-SIDED. $\frac{1}{2}$ CAP SIZE & SNR SAME.

OPAMP NOISE

SINGLE-SIDED



DIFF



NOISE IS $\frac{1}{2}$ THAT OF SINGLE-SIDED.

LESS NOISE FOR DIFFERENTIAL