Circuit Review

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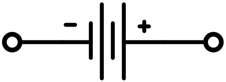
Voltage

- difference in electric potential between 2 points
- often a common reference (such as ground) used as 1 of the points
- units: volts [V]
  - named after Volta who invented the battery in 1799
  - static electricity known almost 2000 years earlier
- Battery symbol



Current

- net rate of flow of electric charge across a surface
- units: amps or ampere [A]
  - named after Ampere who created Ampere's Law in 1823
  - Ampere's Law are formulae relating magnetic fields and electric currents
- charge of electron = 1.6022×10^{-19} coulombs
- 1A = 1 coulomb/s
- 1A = $\frac{1}{1.6022 \times 10^{-19}} = 6.24 \times 10^{18}$ electrons per second crossing a surface
- **We define current as going from positive to negative voltage**
  - due to Ben Franklin defining positive and negative static charges and calling electricity fluid flow before electrons were discovered (1750).

Conductor/Insulator

Conductor

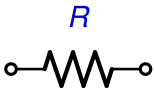
- Allows electrons to flow from one place to another
- Discovered in 1729 using static electricity (before the battery was discovered)
- Conductors are usually metal
- **Wire**
  - An excellent metal conductor is called a ”wire”
  - Common wire material are copper and aluminum
- Wire cross sectional size and material determines how much current can flow through a wire without damage

Insulator

- Blocks the flow of electrons
- Good insulators are glass (silicon dioxide), rubber, plastic, ceramic

Resistor

- A two terminal device that "resists" the flow of current
- Discovered in 1827 by Georg Ohm
- A resistance of 1 ohm will allow 1A of current flow through it when 1V is applied across the resistor
- Resistors are typically made from carbon film or metal film
  - Film is a thin resistive material deposited on an insulating material
- Resistors dissipate energy when current flows through them
- Resistor symbol (units: Ohm [Ω])



Unexpected Resistors

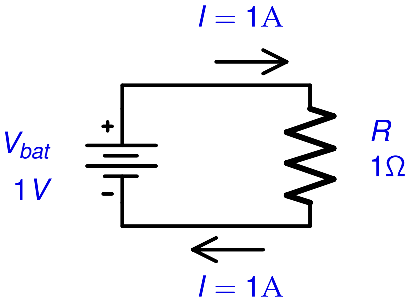
- Physically large resistors for braking when ”regen” used in diesel electric locomotives

from Animagraffs youtube channel

- Large value resistor used when docking to the international space station so no static spark occurs

Simple Circuit

- A simple battery resistor circuit

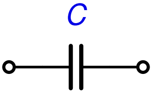


Capacitor

- A two terminal device that fills up with charge
- Discovered in 1745 by van Musschenbroek and von Kleist (independently)
- van Musschenbroek wrote to a friend
  - ”I would like to tell you about a new but terrible experiment, which I advise you never to try yourself, nor would I, who have experienced it, and survived by the grace of God, do it again for all the kingdom of France.”
- He filled a jar with water and an electrode in the water, then applied static electricity to the electrode. Nothing happened.
- He then held the jar with his hand (completing the circuit), applied static electricity and the capacitor charged up.
  - He got a **very large shock** when he touched the electrode

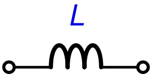
Capacitor

- A capacitor is typically made up of 2 metal plates with a dielectric between the metal plates
- A dielectric is an insulator that can be polarized (i.e. positive and negative centers of the atom (or molecule) become shifted apart)
- An ideal capacitor does NOT dissipate energy
- Capacitor energy is stored in the electric field as well as polarization of the dielectric
- Different dielectrics have more/less polarization
- Capacitor symbol (units: Farad [F]) (named after Michael Faraday)



Inductor

- A two terminal device that stores energy in its magnetic field
- Discovered in 1831 by Joseph Henry
- Inductors are typically made from wire wound around a ferrite core or air core
  - Use of a ferrite core increases the inductance
- An ideal inductor does NOT dissipate energy
- Inductor symbol (units: Henry [H])



DC/AC

- DC - Direct Current
  - A non-changing voltage or current over time
    — a static signal
- AC - Alternating Current
  - A changing voltage or current over time
    — a time varying signal
  - Often a sinusoidal signal but in general, any changing signal over time (voice, video, data, etc)

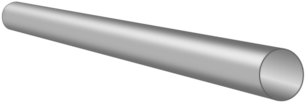
Electricity/Water Analogy

- Use water analogy to visualize how electricity works
- Water system consists of pipes, pumps, valves, etc all completely filled with water (no air in the system).

Electricity	Water System
Voltage	Pressure
Current	Flow
Electrons	Water molecules
Wire	Large Pipe
Switch	Valve
Resistor	Pipe with restrictions
Capacitor	Tank with flexible membrane
Inductor	Tank with heavy paddle wheel
Current Source	Pump
Voltage Source	Pump with feedback

Electricity/Water Analogy

- Wire $\Leftrightarrow$ Large pipe

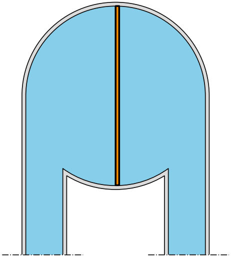


- Resistor $\Leftrightarrow$ Pipe with restrictions

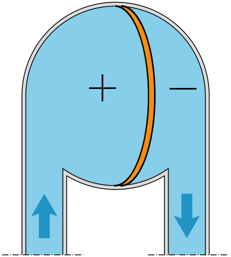
  - Energy is dissipated in restricted pipe

Electricity/Water Analogy

- Capacitor ⇔ Tank with flexible membrane



Uncharged

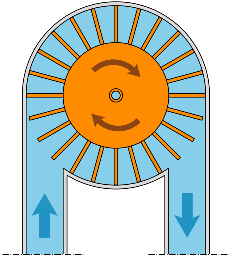


Charged

- Energy stored in flex of membrane due to pressure difference
- AC can flow through the tank while charging/discharging
- DC charges or uncharges the tank
- Water pressure can not increase or decrease instantaneously

Electricity/Water Analogy

- Inductor ⇔ Tank with heavy paddle wheel



- Paddle wheel weight determines the acceleration of the paddle wheel rotation due to water pressure
- Energy is stored as angular momentum of paddle wheel
- Water flow can not start or stop instantaneously

Electricity/Water Analogy

- Current Source ⇔ Water pump
  - Water pump sets water flow to a constant value
- Voltage Source ⇔ Water pump with feedback
  - Feedback used around the pump to keep fixed water pressure across the water pump
  - If the pressure is too low (high), feedback will cause the pump to increase (decrease) flow.

Ohm's Law and Power Dissipation

- $I/V/R/P$ — current/voltage/resistance/power
- Ohm's Law

$$I = \frac{V}{R} \tag{1}$$

- Power dissipation (Joule's Law)

$$P = I^2 R \tag{2}$$

Ohm's Law and Power Dissipation

- Combining the above 2 eqn, we also have

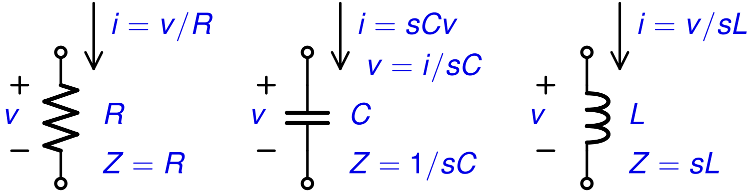
$$P = IV \tag{3}$$

$$P = V^2/R \tag{4}$$

- Power (units: Watts [W])

Impedance

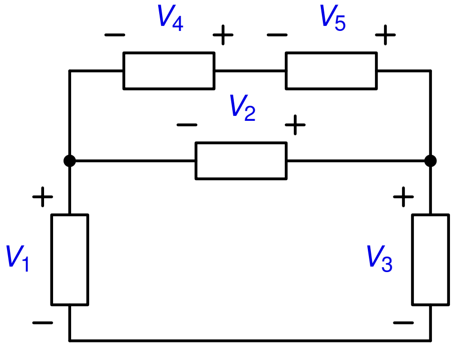
- Impedance of resistor/capacitor/inductor and Ohm's law
- Makes use of the Laplace transform variable "s"



- Capacitor: voltage is integral of current
- Inductor: current is integral of voltage
- Power dissipation is zero for both the capacitor and inductor
  - Due to voltage and current being 90 degrees out of phase with each other

Kirchhoff's Voltage Law (KVL)

- The sum of all voltages around a loop must equal zero



- There are 3 loops here
  - V_1, V_2, V_3
  - V_2, V_4, V_5
  - V_1, V_4, V_5, V_3

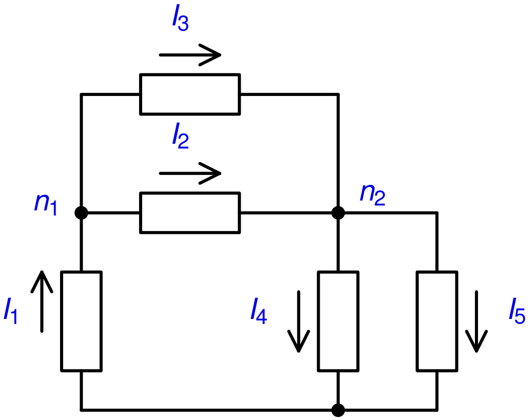
Kirchhoff's Voltage Law (KVL)

$$
\begin{aligned}
V_1 + V_2 - V_3 &= 0 \\
V_4 + V_5 - V_2 &= 0 \\
V_1 + V_4 + V_5 - V_3 &= 0
\end{aligned}
$$

- The third equation is NOT independent of the other 2 equations
  - It is the sum of the first 2 equations
  - So we only need to write the first 2 equations in this case

Kirchhoff's Current Law (KCL)

- The sum of currents flowing into a node equals the sum of currents flowing out of that node.
- Or letting outward currents be negative inward currents...
  - The sum of all currents flowing into a node equals zero

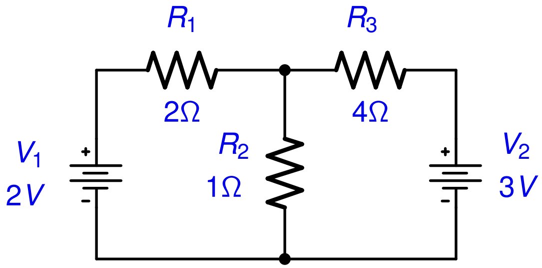


Kirchhoff's Current Law (KCL)

- Node n_1
  - positive
    $I_1 = I_2 + I_3$
  - Outward is negative inward current
    $I_1 - I_2 - I_3 = 0$

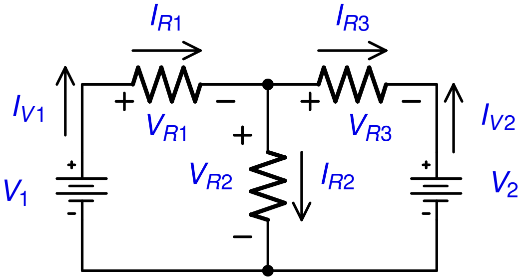
- Node n_2
  - positive outward currents
    $I_2 + I_3 = I_4 + I_5$
  - Outward is negative inward current
    $I_2 + I_3 - I_4 - I_5 = 0$

Example with KVL, KCL and Ohm's Law



- Need to assign currents and voltages of the resistors
- For load elements (such as resistors), the current should flow from positive to negative through the resistor
- For source elements (such as batteries), the current should flow from positive to negative OUT of the battery

Example with KVL, KCL and Ohm's Law



$$I_{R1} = V_{R1}/R_1$$
$$I_{R2} = V_{R2}/R_2$$
$$I_{R3} = V_{R3}/R_3$$

$$I_{V1} = I_{R1}$$
$$I_{V2} = -I_{R3}$$

Example with KVL, KCL and Ohm's Law

$$V_1 - V_{R1} - V_{R2} = 0$$
$$V_{R2} - V_{R3} - V_2 = 0$$
$$I_{R1} - I_{R2} - I_{R3} = 0$$

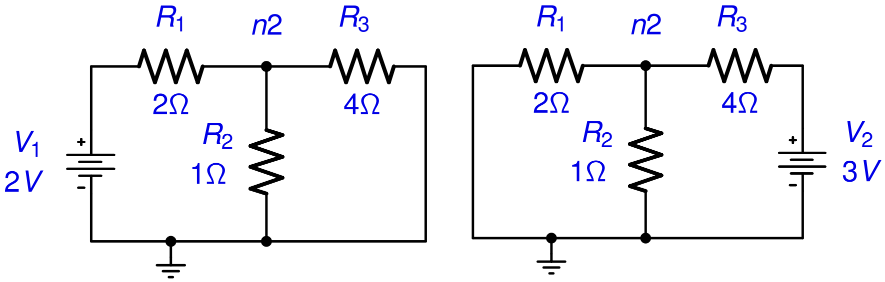
- We have 8 equations and 8 unknowns, so they can all be combined to find the solution
- Result

$$I_{R1} = 0.5\mathrm{A} \qquad V_{R1} = 1\mathrm{V}$$
$$I_{R2} = 1\mathrm{A} \qquad V_{R2} = 1\mathrm{V}$$
$$I_{R3} = -0.5\mathrm{A} \qquad V_{R3} = -2\mathrm{V}$$
$$I_{V1} = 0.5\mathrm{A} \qquad I_{V2} = 0.5\mathrm{A}$$

Superposition

- Above is fine for a computer but messy for a human
- Can use **superposition** for a linear circuit
- Superposition Theorem
  - For a linear circuit with multiple independent sources, the voltage (or current) in any branch equals the algebraic sum of the voltage (or current) due to each individual source acting alone.
  - When zeroing a voltage source, replace with a short circuit.
  - When zeroing a current source, replace with an open circuit.

Superposition Example



$$V'_{n2} = V_1(R_2||R_3)/(R_2||R_3 + R_1)$$
$$V'_{n2} = 2(0.8)/(2.8)$$
$$V'_{n2} = 0.571\mathrm{V}$$

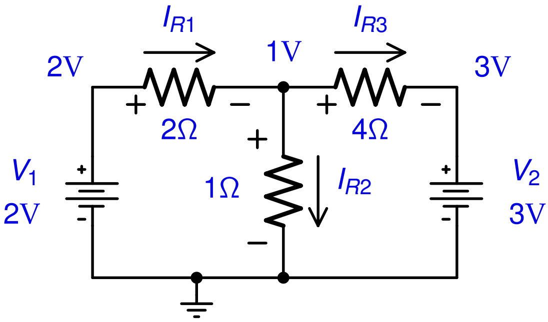
$$V''_{n2} = V_2(R_1||R_2)/(R_1||R_2 + R_3)$$
$$V''_{n2} = 3(0.667)/(4.667)$$
$$V''_{n2} = 0.429\mathrm{V}$$

- Combining we find node n2 voltage to be

$$V_{n2} = V'_{n2} + V''_{n2} = 1\mathrm{V}$$

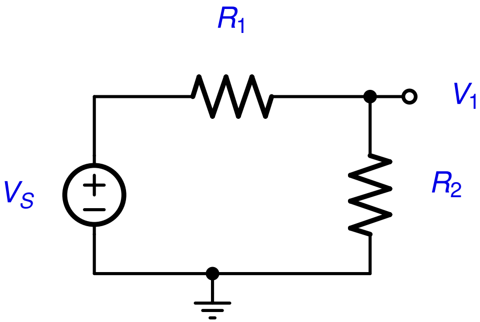
Superposition Example

- Above we defined the lowest node to be ground (i.e. 0V)

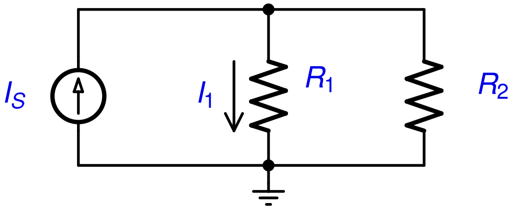


$$I_{R1} = (2 - 1)/2 = 0.5\text{A}$$
$$I_{R2} = (1 - 0)/1 = 1\text{A}$$
$$I_{R3} = (1 - 3)/4 = -0.5\text{A}$$

Voltage Divider

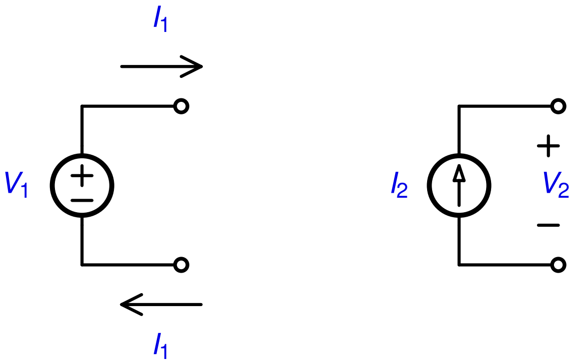


$$V_1 = \frac{R_2}{R_1 + R_2} V_S \tag{5}$$



$$I_1 = \frac{R_2}{R_1 + R_2} I_S \qquad (6)$$

Independent Sources



Dependent Sources

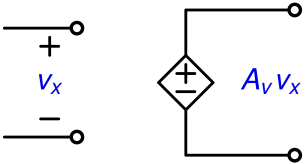
There are 4 types of dependent sources

1. VCVS (voltage controlled voltage source)
2. VCCS (voltage controlled current source)
3. CCVS (current controlled voltage source)
4. CCCS (current controlled current source)

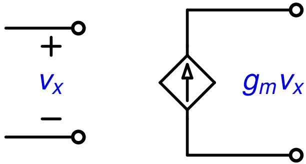
- VCVS and VCCS are commonly used
- CCVS and CCCS are not used very often

Dependent Sources

VCVS



VCCS

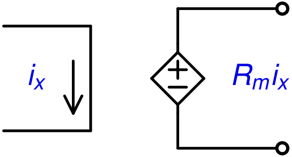


A_V is voltage gain
unitless or $[V/V]$

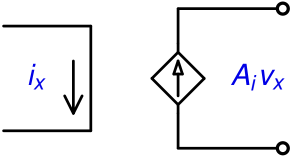
g_m is transconductance gain
units: $[A/V]$ or $[\Omega^{-1}]$

Dependent Sources

CCVS



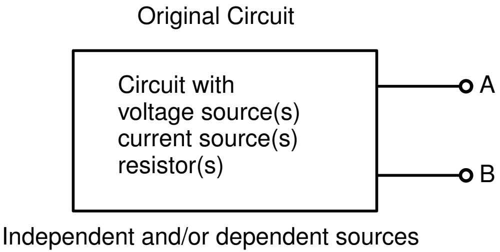
CCCS



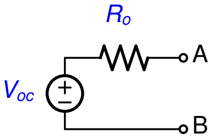
R_m is transresistance gain
units: $[V/A]$ or $[\Omega]$

A_i is current gain
unitless or $[A/A]$

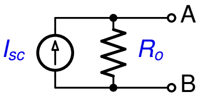
Thevenin/Norton Equivalent



- Above circuit can be replace with either ...



Thevenin Equivalent



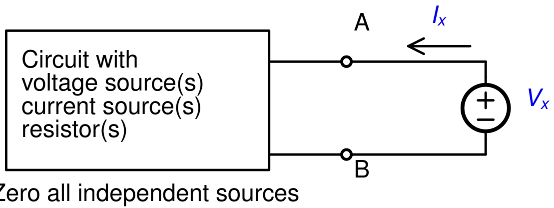
Norton Equivalent

Thevenin/Norton Equivalent

- Thevenin's theorem used to replace a multiple element circuit with a single voltage source and resistor
- Norton's theorem used to replace a multiple element circuit with a single current source and resistor
- Which one to use?
  - Better intuitive understanding if the one best suited for the load being driven is used.
  - Thevenin: If load is much greater than R_o
  - Norton: if load is much less than R_o
  - Either if load is similar in size to R_o
  - If wrong one is used, it will work but voltage/current values may be crazy large and not physically sensible.
  - Could end up with 100V or 100A inside a microchip

Thevenin/Norton Equivalent

- Finding R_o



- Apply arbitary voltage V_x at port A-B and determine current I_x

$$R_o = \frac{V_x}{I_x} \tag{7}$$

- **Zero independent sources** while leaving dependent sources in the circuit

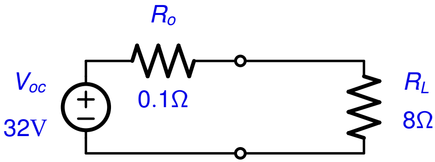
Thevenin/Norton Equivalent

- Finding V_{oc}
  - Disconnect any elements attached to port A-B and find open circuit voltage at port A-B
- Finding I_{sc}
  - Short circuit port A-B and find the short circuit current flowing from A to B
- Relationship between V_{oc} and I_{sc}
  - Since Thevenin and Norton circuits are equal to each other

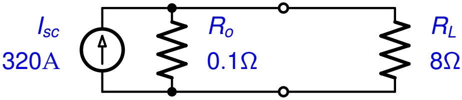
$$I_{sc} = \frac{V_{oc}}{R_o} \tag{8}$$

Example 1

- Show the Thevenin and Norton equivalent circuits for an audio amplifier with a 32V source and a 0.1 ohm output resistance driving an 8 ohm loudspeaker
- Thevenin equivalent circuit



- Norton equivalent circuit

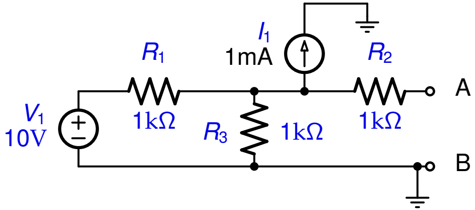


Example 1

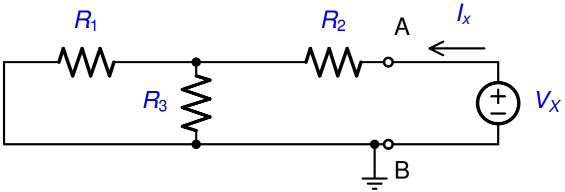
- Norton equivalent circuit is correct but shows 320A of current
- May give the false impression that some wires may have to carry 320A of current
- In fact, wires to speaker only need carry $32/8 = 4\mathrm{A}$ of current and there is no large current of $320\mathrm{A}$ anywhere in the amplifier
- Power dissipation of Thevenin circuit is $32^2/8.1 = 128\mathrm{W}$
- Power dissipation of Norton circuit is $320^2 \times (0.1||8) = 10,113\mathrm{W}$
- Power dissipation is not preserved with Thevenin/Norton equivalent circuits

Example 2

- Find the Thevenin and Norton equivalent circuits for ...



- To find R_o, we set $V_1 = 0$ and $I_1 = 0$ and apply V_x to port A-B

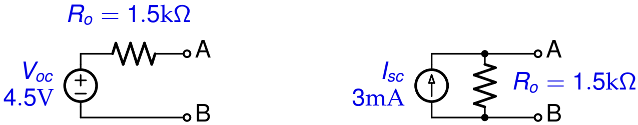


- $R_o = R_2 + R_1 || R_3 = 1.5\text{k}\Omega$

Example 2

- To find V_{oc}, we use superposition
  $V_{oc,V1} = (R_3/(R_1 + R_3)) * V_1 = 5\text{V}$
  $V_{oc,I1} = (R_1||R_3) * (-I_1) = -0.5\text{V}$

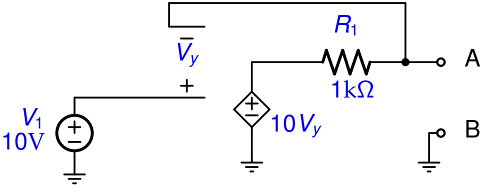
  $V_{oc} = V_{oc,V1} + V_{oc,I1} = 4.5\text{V}$
- To find I_{sc}, we have
  $I_{sc} = V_{oc}/R_o = 3\text{mA}$
- The 2 circuits are:



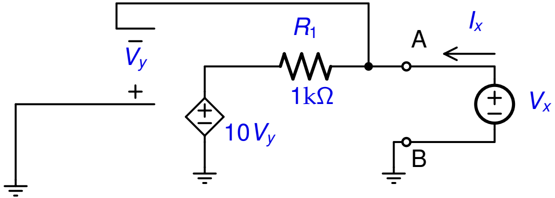
Thevenin Equivalent   Norton Equivalent

Example 3

- Find the Thevenin and Norton equivalent circuits for ...

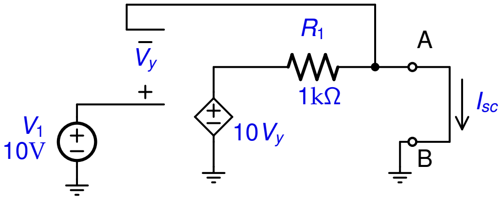


- To find R_o, we set $V_1 = 0$ resulting in



Example 3

- We have $V_y = -V_x$ leading to
  $I_x = \frac{V_x - (10\,V_y)}{R_1} = \frac{V_x + 10\,V_x}{R_1} = \frac{11\,V_x}{R_1}$
  $R_o = V_x / I_x = R_1/11 = 90.91\Omega$
- We see that R_o is smaller than R_1.
  This is due to feedback in the above circuit.
- To find I_{sc}, we use the following circuit



Example 3

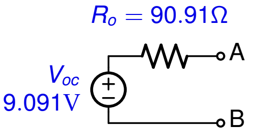
$$V_y = V_1 = 10\mathrm{V}$$

$$I_{sc} = \frac{10V_y - 0}{R_1} = 0.1\mathrm{A}$$

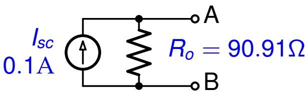
- and we can find V_{oc} from

$$V_{oc} = I_s c * R_o = 9.091\mathrm{V}$$

- The 2 circuits are:



Thevenin Equivalent



Norton Equivalent

Topics Covered

- Basic electricity concepts
- Voltage, current, insulator, wire
- Resistor, Capacitor, Inductor
- Electricity/water analogy
- Dependent/independent sources
- Thevenin/Norton Equivalence