Mosfet Small Signal Modelling

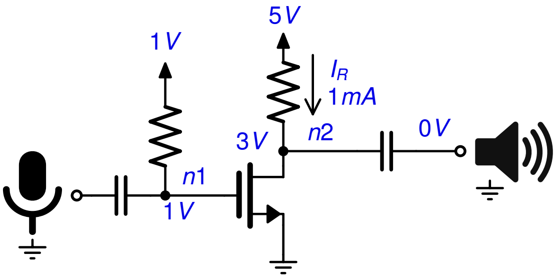
David Johns

University of Toronto

david.johns@utoronto.ca

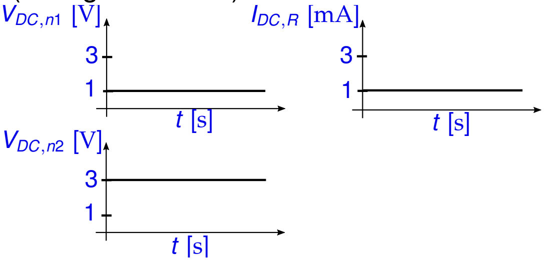
Small Signal Analysis

- Large Signal Analysis
  - Uses non-linear large signal equations to find DC operating point
- Small Signal Analysis
  - Linearize the non-linear behavior and look at **variations** in the voltage/current values from their bias values
- One transistor amp

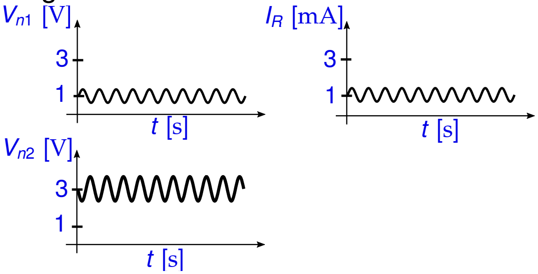


Small Signal Analysis

- DC bias voltage (no signal on Mic)

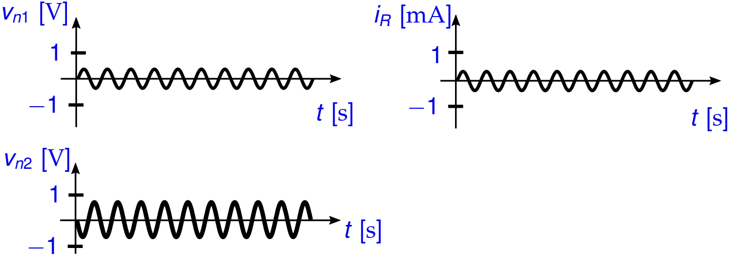


- With a sinusoidal signal on Mic



Small Signal Analysis

- Small signal voltage

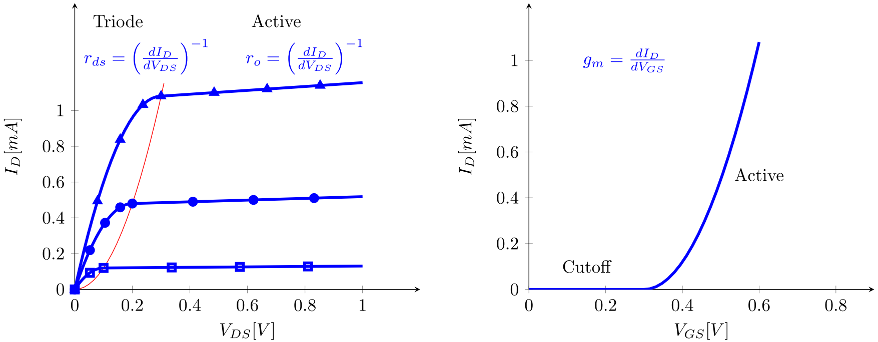


- $v_{n1} = V_{n1} - V_{DC,n1}$
- In general: $v = V - V_{DC}$
- The small-signal voltage is the difference between the actual signal, V, and the dc bias voltage, V_{DC}

Small Signal Analysis

- Independent voltage/current sources do not change their values due to input signal
  - Except for the independent input signal ...
  - All independent sources are set to zero
- To find small signal models
  - Find derivatives dI_D/dV_{GS} and dI_D/dV_{DS} at the DC operating point for each transistor
  - In other words, linearize the large signal models in each of the regions of operation.

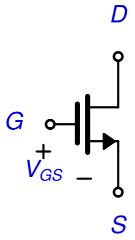
Models vs I_D Plots



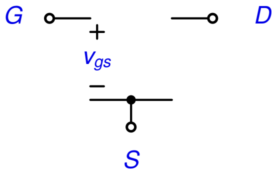
- Cutoff: $I_D = 0$ so all derivatives = 0
- Triode: r_{ds} in triode region
- Active: r_o and g_m in active region

Model: Cutoff Region

- Since $I_D = 0$ ...



NMOS - cutoff

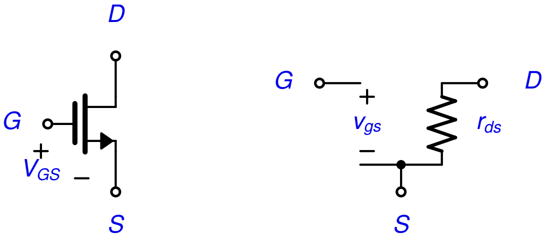


small-signal model - cutoff

- Open circuit at all nodes

Model: Triode Region

$$I_D = \mu_n C_{ox}(W/L)(V_{ov} - 0.5V_{DS})V_{DS}$$



NMOS - triode   small-signal model - triode

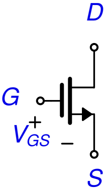
- Looks like a resistor between Drain/Source for small V_{DS}

$$r_{ds} = (\mu_n C_{ox}(W/L)V_{ov})^{-1} \tag{1}$$

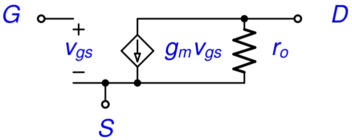
- Becomes more accurate for small V_{DS}

Model: Active Region

$$I_D = 0.5\mu_n C_{ox}(W/L)V_{ov}^2(1 + \lambda_n V'_{DS})$$



NMOS - active



small-signal model - active

- Looks like a resistor and dependent current source between Drain/Source

$$r_o = (\lambda_n I_D)^{-1} \tag{2}$$

$$g_m = \mu_n C_{ox}(W/L)V_{ov}$$

$$g_m = 2I_D/V_{ov} \tag{3}$$

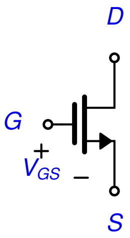
$$g_m = \sqrt{2\mu_n C_{ox}(W/L)I_D}$$

Model: Active Region

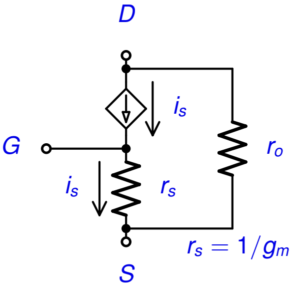
- For g_m, ignore the small $(1 + \lambda_n V'_{DS})$ term
- 3 different (but equivalent) ways of finding g_m.
- Gives insight when designing for a value of g_m.
- For example, if a designer keeps V_{ov} constant (which is a common practice), then g_m is proportional to I_D
- Active region is where the transistor is most commonly used in an analog circuit
  - Results in gain in the circuit

Model: Active Region

- Alternative model



NMOS - active



small-signal T-model - active

- Equivalent to other active model
- Gate current zero since current source i_s forced to equal the current through resistor r_s
- This model useful when resistors are in the source lead to ground. Makes some analysis easier.

PMOS Transistors

- Small signal model
  - The **SAME** models are used for PMOS as for NMOS
  - There are **NO SIGN CHANGES**
    (except that $|\lambda_p|$ should be used to keep r_o positive)

λ is a Function of Channel Length

- λ is given for a transistor with a **given channel length**
- However, λ is inversely proportional to channel length
- Define new parameter, λ', where $\lambda' \equiv \lambda L$
  - Units of λ' are [m/V]

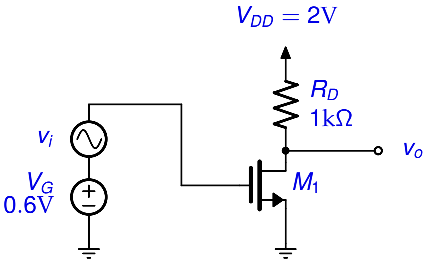
$$\boxed{\lambda = \frac{\lambda'}{L}} \tag{4}$$

$$\boxed{r_o = \frac{L}{\lambda' I_D}} \tag{5}$$

- $r_o \to \infty$ as L increases
  - A large r_o makes the transistor more ideal in terms of output impedance
  - However, a longer channel length generally results in lower speed and higher power dissipation

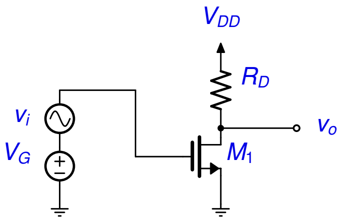
Example 1

- Find the small signal gain v_o/v_i for the circuit shown below.



$V_{tn} = 0.3\text{V}$
$\mu_n C_{ox} = 240\mu\text{A}/\text{V}^2$
$W = 4\mu\text{m}$
$L = 200\text{nm}$
$\lambda_n' = 50\text{nm}/\text{V}$

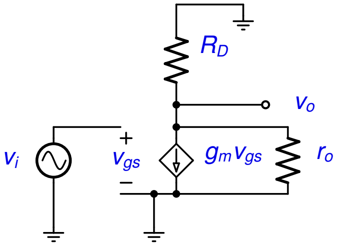
Example 1



- First we find the dc operating point...
- Set the small-signal, $v_i = 0$
  - since nothing is changing while we find the dc operating point
- $V_{ov} = V_{GS} - V_{tn} = 0.3\mathrm{V}$
- $I_D = 0.5\mu_n C_{ox}(W/L)V_{ov}^2 = 216\mu\mathrm{A}$
- $V_D = V_{DD} - I_D R_D = 1.784\mathrm{V}$
  - $V_{DS} > V_{ov}$ so M_1 is in active region
- Small signal parameters ...
- $g_m = 2I_D/V_{ov} = 1.44\mathrm{mA/V}$
- $r_o = L/(\lambda_n' I_D) = 18.52\mathrm{k\Omega}$

Example 1 - Small Signal Circuit

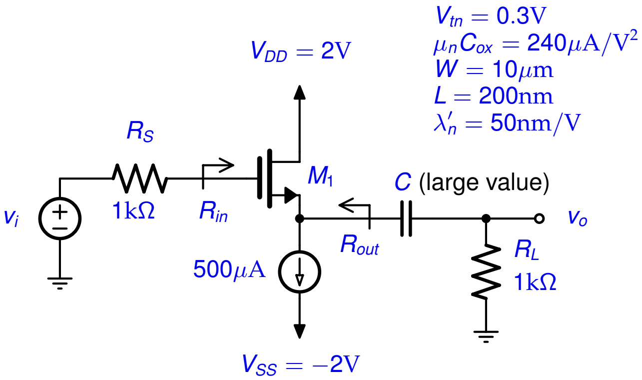
Small signal circuit



- Independent sources have been set to zero
- $v_o = -g_m v_{gs}(R_D||r_o)$
- $v_{gs} = v_i$
- $v_o/v_i = -g_m(R_D||r_o)$
- $v_o/v_i = -1.366\mathrm{V}/\mathrm{V}$
- So a change in v_i by $10\mathrm{mV}$ would result in a $-13.66\mathrm{mV}$ change in v_o

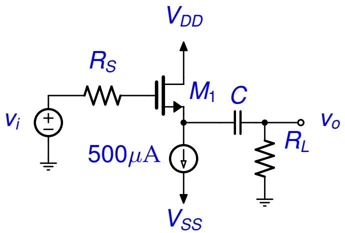
Example 2

- Find the small signal gain v_o/v_i, R_{in}, R_{out} for the circuit shown below.



- Capacitor is a "large value"
  - an open circuit for dc bias analysis
  - a short circuit for small signal analysis (assumes v_i is an ac signal, cap impedance can be ignored)

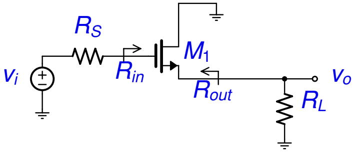
Example 2



- First we find the dc operating point...
- $v_i = 0$ and $I_D = 500\mu\mathrm{A}$
- $I_D = \mu_n C_{ox}(W/L)V_{ov}^2$
- $V_{ov} = 0.2887\mathrm{V}$
- $V_S = -0.5887\mathrm{V}$
- $V_{DS} > V_{ov}$ so M_1 is in active region
- Small signal values ...
- $g_m = 2I_D/V_{ov} = 3.464\mathrm{mA/V}$
- $r_s = 1/g_m = 288.7\Omega$
- $r_o = L/(\lambda_n' I_D) = 8\mathrm{k}\Omega$

Example 2

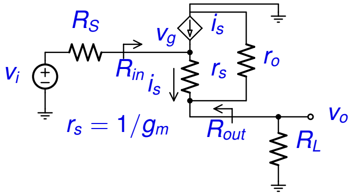
Small signal circuit



- All independent sources set to zero including current source
- Also short capacitor C for small signal analysis
- Now substitute the T-model for M_1

Example 2

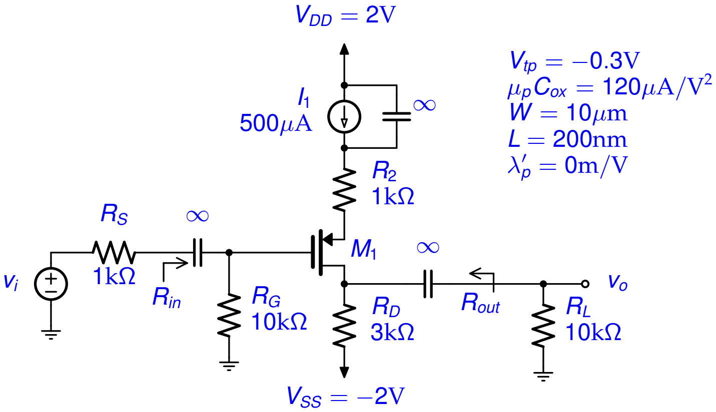
Small signal circuit



- $v_g = \left(\frac{R_{in}}{R_{in}+R_S}\right) v_i$
- $R_{in} \to \infty$ since gate current is zero
- $v_g = v_i$
- Resistor divider between v_g and v_o
- $v_o = \left(\frac{R_L||r_o}{(R_L||r_o)+r_s}\right) v_g$
- $v_o/v_i = \frac{R_L||r_o}{(R_L||r_o)+r_s} = 0.7549\mathrm{V}/\mathrm{V}$
- To find R_{out}, set all independent sources to zero
  - set $v_i = 0$ so $v_g = 0$
- $R_{out} = r_s||r_o = 278.6\Omega$

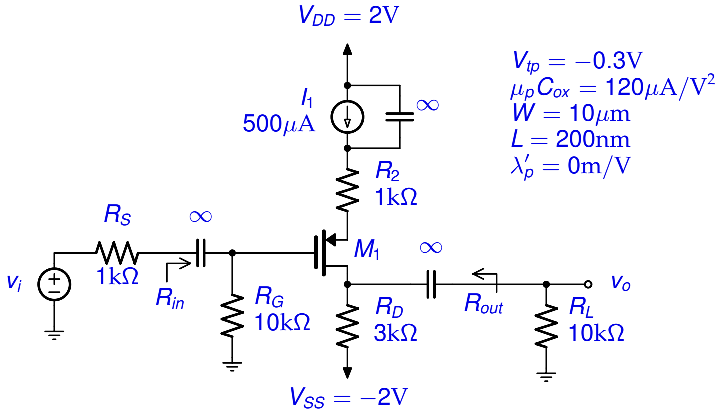
Example 3

- Find the small signal gain v_o/v_i for the circuit shown below.



Example 3

- Find the small signal gain v_o/v_i for the circuit shown below.



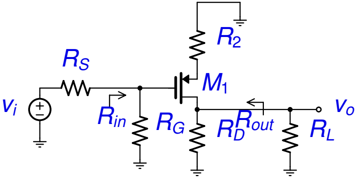
- Caps have value of ∞ showing they are large values

Example 3

- For dc analysis, we set $v_i = 0$ and find ...
- $I_D = 500\mu\mathrm{A}$
- $v_{ov} = 0.4082\mathrm{V}$
- $V_S = 0.7082\mathrm{V}$
- $V_D = -0.5\mathrm{V}$ so $V_{SD} = 1.2082\mathrm{V}$
- Since $V_{SD} > V_{ov}$, M_1 in active region
- $g_m = 2I_D/V_{ov} = 2.449\mathrm{mA/V}$
- $r_s = 1/g_m = 408.2\Omega$
- $r_o = L/(|\lambda'_p|I_D) \to \infty$ since $\lambda_p = 0$

Example 3

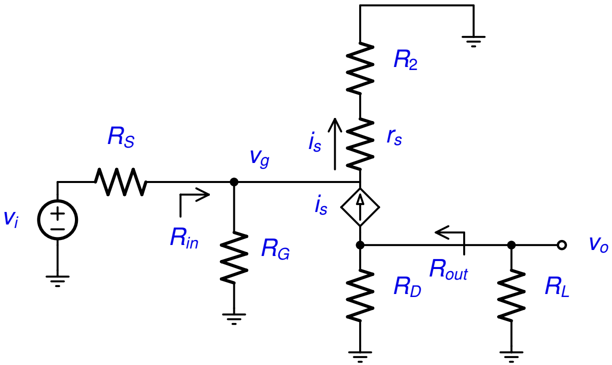
Small signal circuit



- $v_g = \left(\frac{R_{in}}{R_{in}+R_S}\right) v_i$

Example 3

- Small signal circuit with T-model for M_1



- Since the gate current is zero, $R_{in} = R_G$

Example 3

- For R_{out}, set $v_i = 0$ which results in $i_s = 0$ so ...
- $R_{out} = R_D = 3\text{k}\Omega$
- For v_o/v_i, first find v_g/v_i which is
- $v_g/v_i = R_G/(R_G + R_S) = 0.9091\text{V}/\text{V}$
- Now find, v_o/v_g ...

$$v_o = -i_s(R_D||R_L)$$
$$i_s = v_g/(r_s + R_S)$$
$$v_o/v_g = -(R_D||R_L)/(r_s + R_S) = -1.639\text{V}/\text{V}$$

- Combining with v_g/v_i ...

$$\frac{v_o}{v_i} = \frac{v_o}{v_g}\frac{v_g}{v_i} = -1.49\text{V}/\text{V}$$

Topics Covered

- What is small-signal analysis?
- Small-signal models (cutoff/triode/active regions)
  - Transconductance, g_m
  - Finite output impedance, r_o
  - Standard model and T-model
- Small-signal analysis examples