MultiStage Amplifiers

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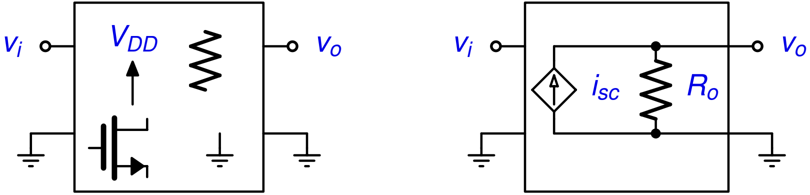
MultiStage Hand Analysis

- Why do hand analysis?
  - Gain insight into the circuit operation
  - If specs not met (gain, freq response, distortion, noise, power supply rejection, etc), what changes will improve circuit
- Good designers will make approximations since simulations give accurate results while hand analysis is used for insight.
- A common approx for hand analysis is to let $\lambda = 0$ when doing dc bias analysis or let the simulation determine the dc bias point.

Small Signal Gain Analysis

- Good method to find small signal voltage gain
  - Model circuit as Norton equivalent
  - Find output impedance, R_o, at output node
  - Find short circuit current, i_{sc} at output node as function of v_i
  - $v_o = i_{sc} R_o;\ i_{sc} = G_m v_i$
  - $v_o = G_m R_o v_i$

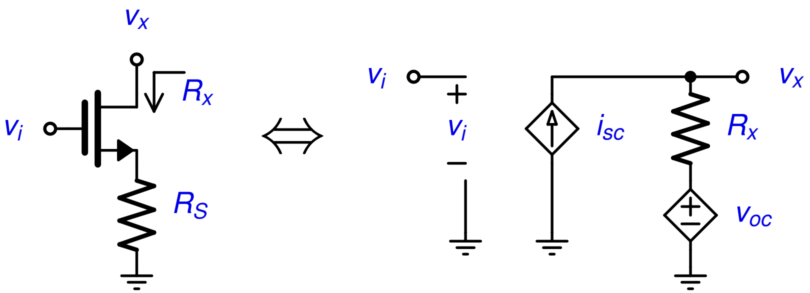


$i_{sc} = G_m v_i$

Small Signal Gain Analysis

- How to deal with larger multiple transistor circuits?
  - Using small-signal models is often complicated for hand analysis
  - Would like to do our analysis directly on the transistor level circuit
- Pre-calculate i_{sc} and R_o for 3 main 1 transistor amps
  1. Common-source amp
  2. Common-drain amp
  3. Common-gate amp
- We will use R_x instead of R_o
  - R_o is for node output impedance while R_x is for looking into transistor terminal
- Don't memorize formula... use a 1 page summary
- All transistors are assumed to be in the active region
- Both i_{sc} and v_{oc} are shown but ONLY 1 should be used (other set to zero)

Transistor Replacements

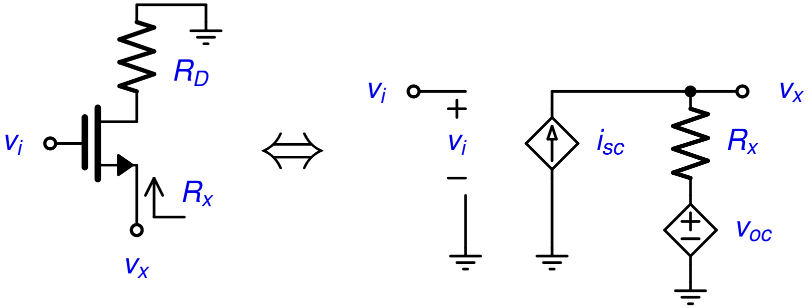


- **Common-source Amp**

$$i_{sc} = \frac{-g_m r_o}{r_o + (1 + g_m r_o) R_S} v_i \qquad R_x = r_o + (1 + g_m r_o) R_S$$

$$v_{oc} = -g_m r_o v_i$$

- If $g_m r_o \gg 1$, $i_{sc} \approx \frac{-1}{(1/g_m) + R_S} v_i$; $R_x \approx (1 + g_m R_S) r_o$
  $v_{oc} = -g_m r_o v_i$



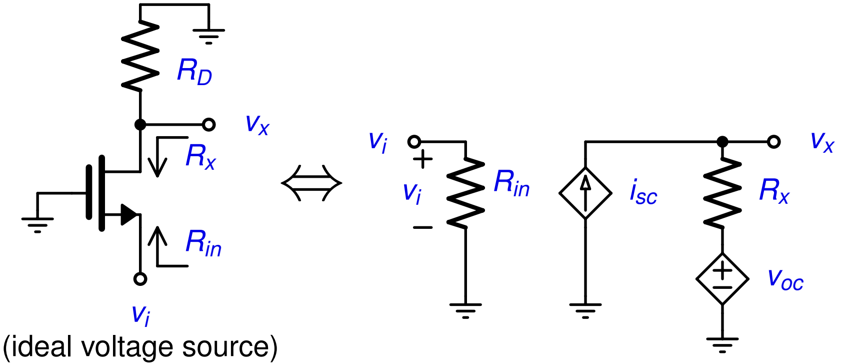
- **Common-drain Amp**

  $i_{sc} = \frac{g_m r_o}{r_o + R_D} v_i$ $\qquad R_x = \frac{r_o + R_D}{(1 + g_m r_o)}$

  $v_{oc} = \frac{g_m r_o}{(1 + g_m r_o)} v_i$

- If $g_m r_o \gg 1$, $v_{oc} \approx v_i$; $R_x \approx \frac{1}{g_m} + \frac{R_D}{g_m r_o}$
  $i_{sc} = \frac{g_m r_o}{r_o + R_D} v_i$

Transistor Replacements



- **Common-gate Amp**

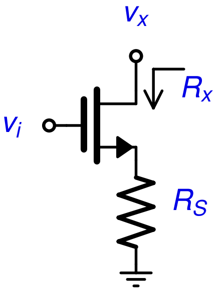
  $i_{sc} = \frac{(1+g_m r_o)}{r_o} v_i$ $R_x = r_o$

  $v_{oc} = (1 + g_m r_o) v_i$ $R_{in} = \frac{r_o + R_D}{(1+g_m r_o)}$

- If $g_m r_o \gg 1$, $i_{sc} \approx g_m v_i$; $R_{in} \approx \frac{1}{g_m} + \frac{R_D}{g_m r_o}$; $v_{oc} \approx g_m r_o v_i$; $R_x = r_o$

Transistor Replacements - summary

common-source

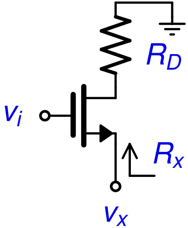


$$i_{sc} = \frac{-g_m r_o}{r_o + (1+g_m r_o) R_S} v_i$$

$$v_{oc} = -g_m r_o v_i$$

$$R_x = r_o + (1 + g_m r_o) R_S$$

common-drain

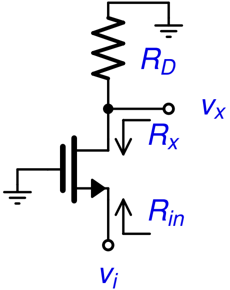


$$i_{sc} = \frac{g_m r_o}{r_o + R_D} v_i$$

$$v_{oc} = \frac{g_m r_o}{(1+g_m r_o)} v_i$$

$$R_x = \frac{r_o + R_D}{(1+g_m r_o)}$$

common-gate



$$i_{sc} = \frac{(1+g_m r_o)}{r_o} v_i$$

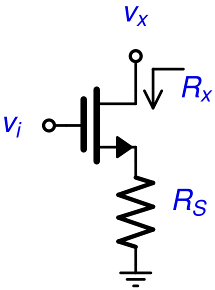
$$v_{oc} = (1 + g_m r_o) v_i$$

$$R_x = r_o$$

$$R_{in} = \frac{r_o + R_D}{(1+g_m r_o)}$$

Transistor Replacements - summary - $g_m r_o \gg 1$

common-source

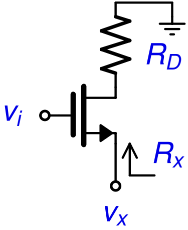


$i_{sc} \approx \frac{-1}{(1/g_m)+R_S} v_i$

$v_{oc} = -g_m r_o v_i$

$R_x \approx (1 + g_m R_S) r_o$

common-drain

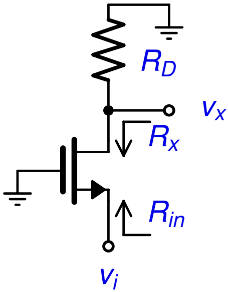


$i_{sc} = \frac{g_m r_o}{r_o+R_D} v_i$

$v_{oc} \approx v_i$

$R_x \approx \frac{1}{g_m} + \frac{R_D}{g_m r_o}$

common-gate



$i_{sc} \approx g_m v_i$

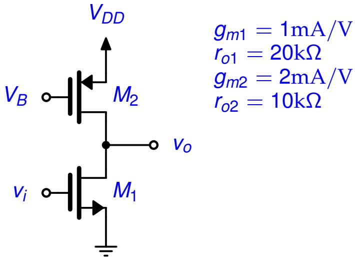
$v_{oc} \approx g_m r_o v_i$

$R_x = r_o$

$R_{in} \approx \frac{1}{g_m} + \frac{R_D}{g_m r_o}$

Example 1

- Common-source (exact solution)



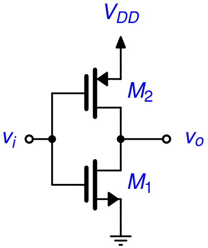
- Define R_{D1}/R_{D2} to be the impedance looking into drain of M_1/M_2
- $R_{D1} = r_{o1} + (1 + g_{m1}r_{o1})R_S$ and $R_S = 0$
- $R_{D1} = r_{o1}$
- Similarly $R_{D2} = r_{o2}$

Example 1

- $R_o = R_{D1}||R_{D2} = r_{o1}||r_{o2} = 6.67\mathrm{k}\Omega$
- M_1 is a common-source amp so
- $i_{sc} = \frac{-g_{m1}r_{o1}}{r_{o1}+(1+g_{m1}r_{o1})R_S} = -g_{m1}v_i$ since $R_S = 0$
- $v_o = i_{sc}R_o = -g_{m1}R_o v_i$
- $vo/vi = -g_{m1}R_o = -6.67V/V$
- $R_{out} = R_o = 6.67\mathrm{k}\Omega$
- Common-source (approx solution)
  - For this example, we obtain the same answer if we make approximations assuming $g_m r_o \gg 1$

Example 2

- Dual Common-source (exact solution)



$g_{m1} = 1\text{mA/V}$
$r_{o1} = 20\text{k}\Omega$
$g_{m2} = 2\text{mA/V}$
$r_{o2} = 10\text{k}\Omega$

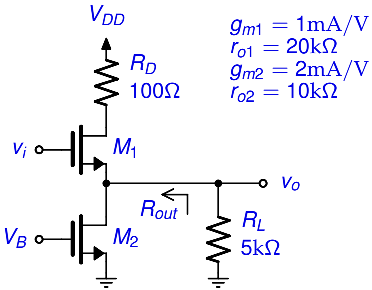
- Here, the dc value is adjusted such that both M_1 and M_2 are active.
- $R_o = 6.67\text{k}\Omega$ is the same as the previous example
- For i_{sc}, we combine each of the isc currents for M_1/M_2

Example 2

- Recall, the small-signal models are the same for PMOS and NMOS and as a result each i_{sc} is the current that comes OUT of the drains of each transistor
- $i_{sc} = i_{sc1} + i_{sc2} = -g_{m1} v_i - g_{m2} v_i$
- $v_o / v_i = -(g_{m1} + g_{m2}) \times R_o = -20\mathrm{V}/\mathrm{V}$

Example 3

- Common-drain (exact solution)



- $R_{out} = R_{S1}||R_{D2}$
- $R_o = R_{out}||R_L$
- $R_{D2} = r_{o2} = 10\mathrm{k}\Omega$; $R_{S1} = \frac{r_{o1}+R_D}{(1+g_{m1}r_{o1})} = 957\Omega$

Example 3

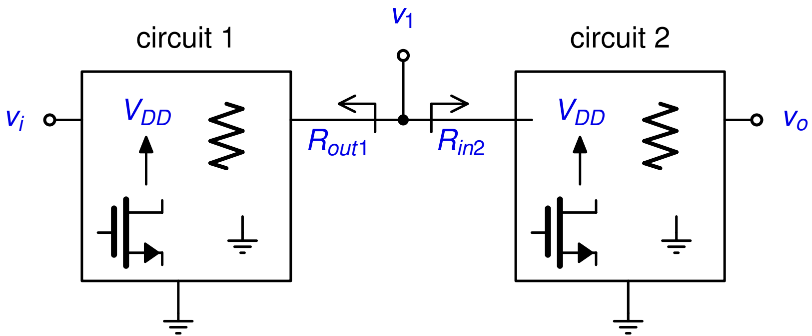
- Note when $R_D = 0$
  - $R_{S1} = 952\Omega$ which is only an increase of 5Ω even though a 100Ω resistor put in series with M_1.
- $R_{out} = R_{S1}||R_{D2} = 874\Omega$
- $R_o = R_{out}||R_L = 744\Omega$
- For i_{sc} we have $i_{sc} = G_m v_i$ where
- $G_m = (g_{m1} r_{o1})/(r_{o1} + R_D) = 995\mu\mathrm{A}/\mathrm{V}$
- $v_o/v_i = G_m \times R_o = 0.74\mathrm{V}/\mathrm{V}$
- Although this voltage gain is less than one, the purpose of this amplifier is to have a low output impedance so it can drive more current into a load resistance

Example 3 - Approx Solution

- $R_{S1} = (1/g_{m1}) + R_D/(g_{m1} r_{o1}) = 1005\Omega$
- $R_{D2} = r_{o2} = 10\mathrm{k}\Omega$
- $v_{oc} = v_i$
- v_o node is a resistor divider node
- $v_o = \frac{(R_{D2}||R_L)}{(R_{D2}||R_L)+R_{S1}} v_{oc} = \frac{3.33k}{3.33k+1.005k} v_i$
- $vo/vi = 0.768\mathrm{V/V}$
- which is only a 4% difference (of course, this depends on how large $g_m r_o$ is greater than 1.

Cascade of Circuits

- Need a method to deal with a cascade of circuits



- Direct Approach
- Cascade Approach

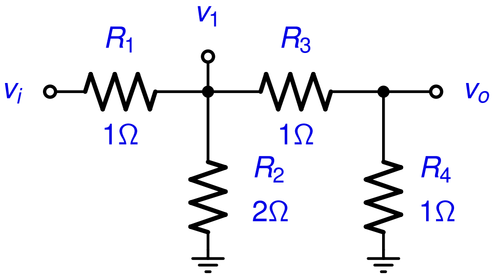
Direct Approach

- Find i_{sc} and R_o at v_o
- $v_o = i_{sc} R_o = G_m R_o v_i$
- Can be difficult to directly find i_{sc}
  - One way is to short v_o AND then find v_1'
    then USE IDEAL v_1' and find i_{sc}
  - v_1' is voltage at v_1 with v_o shorted to ground
- R_o is not too difficult using the transistor replacement formulae
- Main disadvantage
  - Gives no info about v_1
  - v_1' is not the same as v_1

Cascade Approach

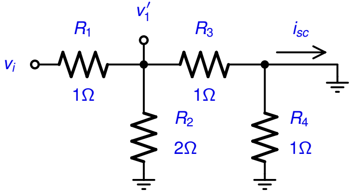
- First find $v_1 = i_{sc1} R_{o1} = G_{m1} R_{o1} v_i$
  - i_{sc1} is the short circuit current seen at v_1 due to v_i
  - R_{o1} is the impedance to ground seen at v_1
    $R_{o1} = R_{out1} || R_{in2}$
- Next USE IDEAL v_1 source and
  - Find $v_o = i_{sc2} R_{o2} = G_{m2} R_{o2} v_1$
    i_{sc2} is the short circuit current at v_o for IDEAL v_1
    R_{o2} is the impedance to ground at v_o for $v_1 = 0$
    (in other words for an ideal v_1)
- Combine the above 2 results
  $v_o / v_i = (v_1 / v_i) \times (v_o / v_1)$
- This approach gives you the intermediate gain as well as the overall gain

Cascade Example 1



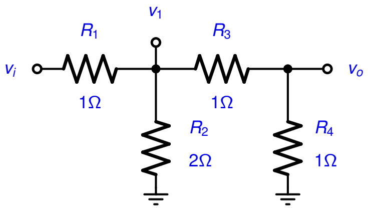
- This example is best solved using resistor divider approach
  - Used it here to demonstrate the direct and cascade approaches

Cascade Example 1 - Direct Approach



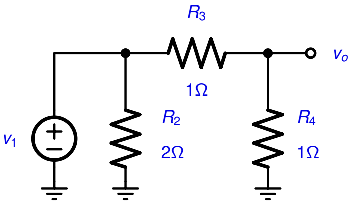
- $i'_{sc1} = v_i/R_1 = v_i;\ R'_{o1} = R_1||R_2||R_3 = (2/5)\Omega$
- $v_1' = i'_{sc1}R'_{o1} = (2/5)v_i$
- $i_{sc} = v_1'/R_3 = (2/5)v_i$
- $R_o = R_4||(R_3 + (R_1||R_2)) = (5/8)\Omega$
- $v_o = i_{sc}R_o = (2/5)(5/8)v_i = (1/4)v_i$

Cascade Example 1 - Cascade Approach



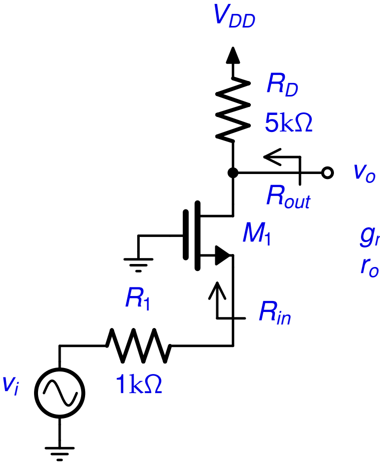
- $i_{sc1} = v_i/R_1 = v_i$
- $R_{o1} = R_1||(R_2||(R_3 + R_4)) = (1/2)\Omega$
- $v_1 = i_{sc1}R_{o1} = (1/2)v_i$

Cascade Example 1 - Cascade Approach



- $i'_{sc} = v_1/R_3 = v_1$
- $R'_{o1} = R_3||R_4 = (1/2)\Omega$
- $v_o = i'_{sc1}R'_{o1} = (1/2)v_1$
- $v_o/v_i = (v_1/v_i) \times (v_o/v_1) = (1/4)v_i$
- This approach gave us the intermediate gain v_1/v_i

Cascade Example 2 - Common-Gate Amp



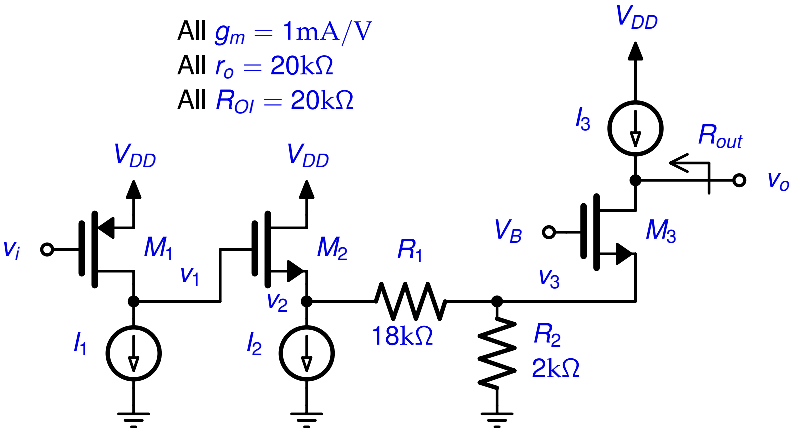
$g_{m1} = 1\mathrm{mA/V}$
$r_{o1} = 20\mathrm{k}\Omega$

- Find $v_o/v_i,\ R_{in},\ R_{out}$

Cascade Example 2 - Common-Gate Amp

- $R_{in} = (r_{o1} + R_D)/(1 + g_{m1} r_{o1}) = 1.19\mathrm{k}\Omega$
- Let R_{D1} be the impedance seen looking into the drain of M_1
  $R_{D1} = r_{o1} + (1 + g_{m1} r_{o1}) R_1 = 41\mathrm{k}\Omega$
- $R_{out} = R_{D1} || R_D = 41\mathrm{k} || 5\mathrm{k} = 4.46\mathrm{k}\Omega$
- Let v_1 be the voltage at the source of M_1
- $v_1/v_i = R_{in}/(R_{in} + R_1) = 0.5435\mathrm{V/V}$
- $i'_{sc} = G'_{m1} v_1$ where $G'_{m1} = (1 + g_{m1} r_{o1})/r_{o1} = 1.05\mathrm{mA/V}$
- $R'_o = R_D || r_{o1} = 4\mathrm{k}\Omega$
- $v_o/v_1 = G'_{m1} R'_o = 4.2\mathrm{V/V}$
- $v_o/v_i = v_1/v_i \times v_o/v_1 = 2.283\mathrm{V/V}$

Cascade Example 3 - MultiStage



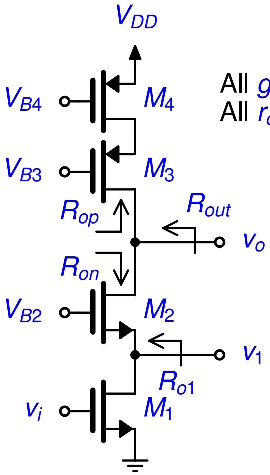
- Find $v_o/v_i,\ R_{out}$
- R_{OI} is the impedance of a current source

Cascade Example 3 - MultiStage

- $v_1/v_i = -g_{m1}(r_{o1}||R_{OI1}) = -10\mathrm{V}/\mathrm{V}$
- $v_{oc2} = K_2 v_1$ where $K_2 = g_{m2}r_{o2}/(1 + g_{m2}r_{o2}) = 0.9524\mathrm{V}/\mathrm{V}$
- $R_{x2} = (1/g_{m2})||r_{o2} = 0.9524\mathrm{k}\Omega$
- Define R_{s3} looking into source of M_3 and R_{1L} looking into left side of R_1
- $R_{s3} = (r_{o3} + R_{OI3})/(1 + g_{m3}r_{o3}) = 1.905\mathrm{k}\Omega$
- $R_{1L} = R_1 + (R_2||R_{s3}) = 18.98\mathrm{k}\Omega$
- We now have a resistive divider at node v_2
- $v_2/v_1 = K_2(R_{1L}||R_{OI2})/(R_{1L}||R_{OI2} + R_{x2}) = 0.8675\mathrm{V}/\mathrm{V}$
- $v_3/v_2 = (R_{s3}||R_2)/((R_{s3}||R_2) + R_1) = 51.41\mathrm{mV}/\mathrm{V}$

Cascade Example 3 - MultiStage

- $i'_{sco} = G'_{mo} v_3$ where $G'_{mo} = (1 + g_{m3} r_{o3})/r_{o3} = 1.05\mathrm{mA/V}$
- $R'_o = R_{Ol3} || r_{o3} = 10\mathrm{k\Omega}$
- $v_o/v_3 = G'_{mo} R'_o = 10.5\mathrm{V/V}$
- $v_o/v_i = (v_1/v_i)(v_2/v_1)(v_3/v_2)(v_o/v_3) = -4.683\mathrm{V/V}$
- For R_{out} define R_{1R} looking into right side of R_1
- $R_{1R} = R_1 + (R_{Ol2} || R_{x2}) = 18.91\mathrm{k\Omega}$
- Define R_{d3} looking into the drain of M_3
- $R_{d3} = r_{o3} + (1 + g_{m3} r_{o3})(R_2 || R_{1R}) = 57.98\mathrm{k\Omega}$
- $R_{out} = R_{Ol3} || R_{d3} = 14.87\mathrm{k\Omega}$



All $g_m = 1\mathrm{mA/V}$
All $r_o = 20\mathrm{k}\Omega$

- Find $v_o/v_i,\ R_{out},\ v_1/v_i,\ R_{o1}$

Cascade Example 4 - Cascode Amp

- First we can R_{out} and R_{o1}
- $R_{op} = r_{o4} + (1 + g_{m3} r_{o3}) r_{o4} = 440\text{k}\Omega$
- $R_{on} = r_{o1} + (1 + g_{m2} r_{o2}) r_{o1} = 440\text{k}\Omega$
- $R_{out} = R_{op} || R_{on} = 220\text{k}\Omega$
- Define R_{s2} looking into source of M_2
- $R_{s2} = (r_{o2} + R_{op})/(1 + g_{m2} r_{o2}) = 21.9\text{k}\Omega$
- $R_{o1} = R_{s2} || r_{o1} = 10.45\text{k}\Omega$

Cascade Example 4 - Cascode Amp

- Find v_1/v_i
- $i_{sc1} = G_{m1} v_i$ where $G_{m1} = -g_{m1} = -1\mathrm{mA/V}$
- $v_1/v_i = G_{m1} R_{o1} = -10.45\mathrm{V/V}$
- Find v_o/v_1
- $R'_o = R_{op} || r_{o3} = 19.13\mathrm{k\Omega}$ (v_1 set to ground)
- $i'_{sco} = G'_{mo} v_1$ where $G'_{mo} = (1 + g_{m3} r_{o3})/r_{o3} = 1.05\mathrm{mA/V}$
- $v_o/v_1 = G'_{mo} R'_o = 20.09\mathrm{V/V}$
- Combine the above 2 results for v_o/v_i
- $v_o/v_i = (v_1/v_i)(v_o/v_1) = -210\mathrm{V/V}$

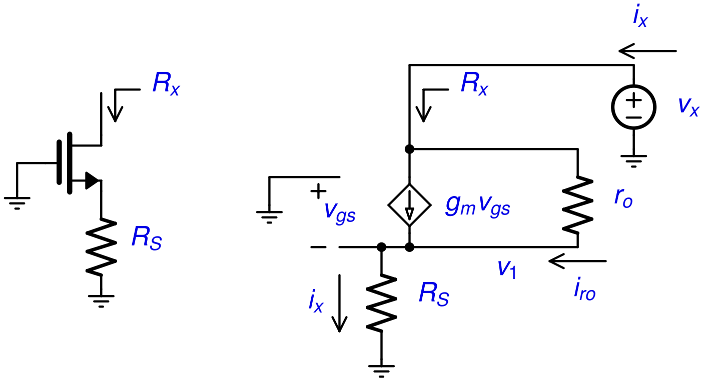
Cascade Example 4 - Cascode Amp

- We can do rapid approx analysis assuming $g_m r_o \gg 1$
- $R_{op} \approx g_{m3} r_{o3} r_{o4} = 400\mathrm{k}\Omega$
- $R_{on} \approx g_{m2} r_{o2} r_{o1} = 400\mathrm{k}\Omega$
- $R_{out} = R_{op} || R_{on} \approx 200\mathrm{k}\Omega$
- $R_{s2} \approx (1/g_{m2}) + R_{op}/(g_{m2} r_{o2}) = 21\mathrm{k}\Omega$
- $R_{o1} = R_{s2} || r_o 1 = 10.2\mathrm{k}\Omega$
- $v_1/v_i = -g_{m1} R_{o1} = 10.2\mathrm{V/V}$
- For i_{sco} we assume all $r_o \to \infty$ so $i_{sco} = -g_{m1} v_i$
- $v_o/v_i = -g_{m1} R_{out} = 210\mathrm{V/V}$
- This approach is typically "good enough" for hand analysis

Topics Covered

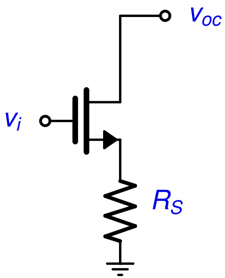
- Benefits of hand analysis
  - Gain insight
  - Understand what changes will improve circuit
  - Only rough approximations needed
  - Often let $\lambda = 0$ for dc bias analysis or rely on simulation
- Small signal gain analysis
- Pre-calculate i_{sc} and R_o for 3 main amps
- Transistor replacements
- How to deal with cascade of circuits
  - Direct approach
  - Cascade approach
- Multi-transistor circuit examples

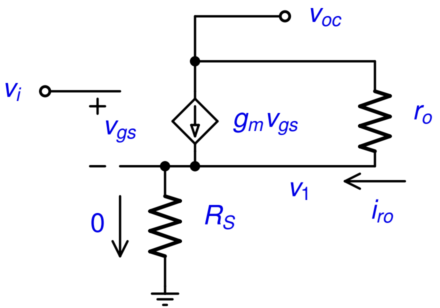
Common-Source R_x Equivalent



- $v_{gs} = 0 - i_x R_S; \qquad i_{ro} = (v_x - i_x R_S)/r_o$
- $i_x = i_{ro} + g_m v_{gs} = (v_x - i_x R_S)/r_o + g_m(-i_x R_S)$
- $i_x(1 + g_m r_o + R_S/r_o) = v_x/r_o \qquad R_x \equiv v_x/i_x$
- $R_x = r_o + (1 + g_m r_o) R_S$

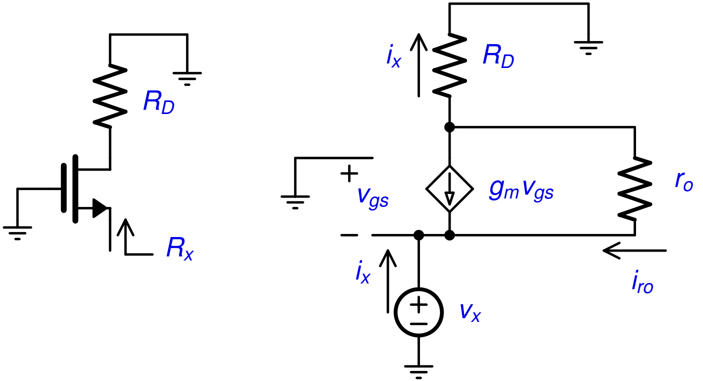
Common-Source i_{sc} and v_{oc} Equivalent





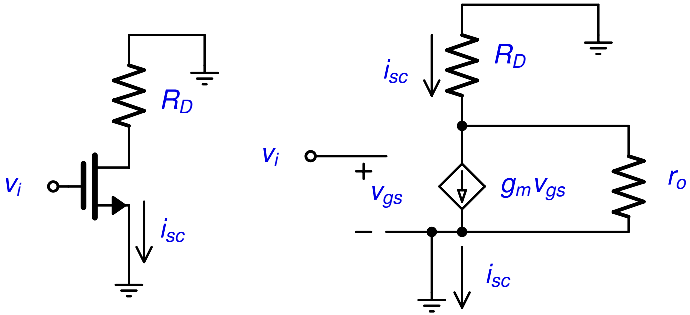
- Since v_{oc} node is open, $I_{RS} = 0\mathrm{A}$; $v_1 = 0\mathrm{V}$; $v_{gs} = v_i$
- $i_{ro} = -g_m v_i$; $v_{oc} = i_{ro} r_o = -g_m r_o v_i$
- $i_{sc} = v_{oc}/R_x = (-g_m r_o v_i)/(r_o + (1 + g_m r_o)R_S)$

Common-Drain R_x Equivalent



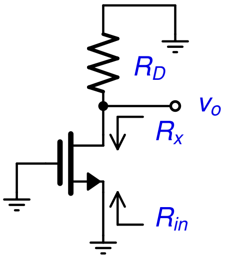
- $v_{gs} = -v_x; \qquad i_x = -g_m v_{gs} + (v_x - i_x R_D)/r_o$
- $i_x r_o = g_m r_o v_x + v_x - i_x R_D; \qquad i_x(r_o + R_D) = v_x(1 + g_m r_o)$
- $R_x \equiv v_x/i_x = (r_o + R_D)/(1 + g_m r_o)$

Common-Drain i_{sc} and v_{oc} Equivalent



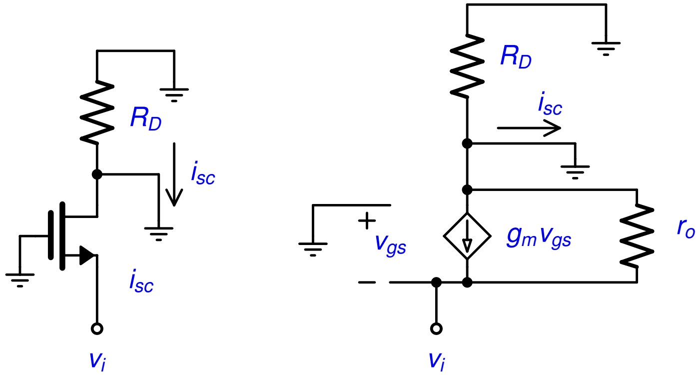
- $v_{gs} = v_i;\qquad i_{sc} = g_m v_{gs} + (-i_{sc} R_D - 0)/r_o$
- $i_{sc}(r_o + R_D) = g_m r_o v_i \qquad i_{sc} = ((g_m r_o)/(r_o + R_D))v_i$
- $v_{oc} = i_{sc} \times R_x = ((g_m r_o)/(1 + g_m r_o))v_i$

Common-Gate R_x and R_{in} Equivalent



- R_{in} same as looking into the source of a common-drain amp
  $R_{in} = (r_o + R_D)/(1 + g_m r_o)$
- R_x same as single transistor current mirror
  $R_x = r_o$

Common-Gate i_{sc} and v_{oc} Equivalent



- $v_{gs} = -v_i \qquad i_{sc} = -g_m v_{gs} + v_i/r_o$
- $i_{sc} = (g_m + 1/r_o)v_i = ((1 + g_m r_o)/r_o)v_i$
- $v_{oc} = i_{sc} \times R_x = (1 + g_m r_o)v_i$