Pole Zero Estimation

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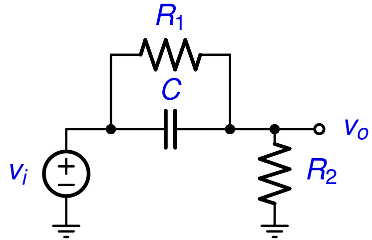
1st Order $H(s)$ with finite zero



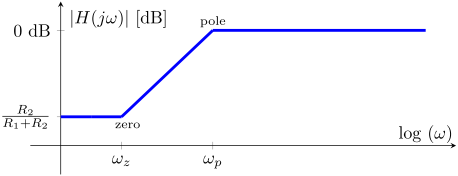
$$H(s) = \frac{R_2}{(\frac{1}{sC}||R_1) + R_2} \qquad (1)$$

$$H(s) = \left(\frac{R_2}{R_1 + R_2}\right)\left(\frac{1 + sCR_1}{1 + sC(R_1||R_2)}\right) \qquad (2)$$

- Zero at $s_z = -\frac{1}{CR_1}$ so $\omega_z = \frac{1}{CR_1}$
- Pole at $s_p = -\frac{1}{C(R_1||R_2)}$ so $\omega_p = \frac{1}{C(R_1||R_2)}$

1st Order $H(s)$ with finite zero

- Assume $R_1 \gg R_2$



- Above is a Bode Plot
  - Straight lines are drawn as an approximation to the actual transfer-function
- Not strictly a STC circuit since the finite zero (not at 0 or ∞) will change the step response

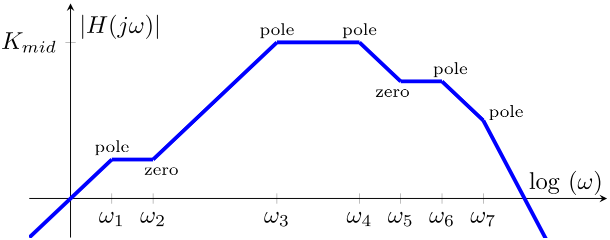
$H(s)$ with Multiple Real-Axis Poles/Zeros

- In RC circuits that have no feedback, the circuit will have all real-axis poles/zeros
- Bode Plot
  - Used when the poles/zeros are widely spaced apart
  - Draw straight lines between poles/zeros
  - As ω is increased...
    Poles reduce the slope in $|H(j\omega)|$
    Zeros increase the slope in $|H(j\omega)|$
- Example

$$H(s) = \frac{K_{mid} \times s(s+\omega_2)(1+s/\omega_5)}{(s+\omega_1)(s+\omega_3)(1+s/\omega_4)(1+s/\omega_6)(1+s/\omega_7)} \tag{3}$$

  - Has zero at dc and 2 zeros at ∞
    (since $n - m = 2$ where n/m is denominator/numerator order)
  - Here we have $\omega_1 < \omega_2 < \omega_3 < \omega_4 < \omega_5 < \omega_6 < \omega_7$

Bode Plot for $H(s)$ with Real-axis Poles/Zeros

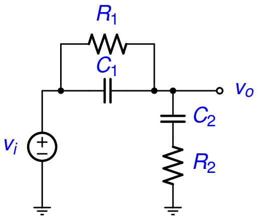


- Has a zero at dc
  - Since $|H(j\omega)| \rightarrow 0$ at 20dB/dec as $\omega \rightarrow 0$
- Has 2 zeros at ∞
  - Since $|H(j\omega)| \rightarrow 0$ at 40dB/dec as $\omega \rightarrow \infty$
- Midband gain, K_{mid}, occurs where

$$\omega_3 < \omega < \omega_4$$

Finding Zeros

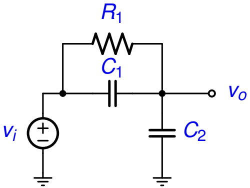
- Zeros are where the output goes to zero
- Can often be found easily by ...
  - Find values of s where ...
  - series impedance goes to ∞ (but not in shunt path)
  - shunt impedance goes to zero (but not in series path)
- Example 1



- Zero due to R_1 / C_1 going to ∞
- Zero due to R_2 / C_2 going to zero

Finding Zeros

- $\left(\frac{1}{R_1} + sC_1\right)^{-1} \to \infty \;\Rightarrow\; \frac{1}{R_1} + sC_1 = 0 \;\Rightarrow\; s = \frac{-1}{R_1 C_1}$
- $R_2 + \frac{1}{sC_2} = 0 \;\Rightarrow\; s = \frac{-1}{R_2 C_2}$
- So the above example has zeros frequencies
  $\omega_{z1} = \frac{1}{R_1 C_1}$ $\qquad \omega_{z2} = \frac{1}{R_2 C_2}$
- Example 2

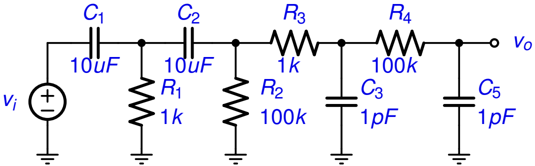


- Similar to above, $\omega_{z1} = \frac{1}{R_1 C_1}$ and there is no zero due to C_2
- At first glance, there is a zero at infinite freq due to C_2 however ...
  - At infinity freq, the series circuit also goes to zero impedance

Finding Poles

- Open/Short Circuit Time-Constant Estimation
- For use in systems with multiple real-axis poles
- In **any** system
  - The poles of the system **do not** depend on where the input is applied or the output is taken
    (not just real-axis poles systems)
- Quick method for estimating real-axis poles
  - Open/short circuit time constant estimation
  - Used by many designers to know what part of a circuit limits high or low frequency performance
  - Can quickly estimate pole locations
- Best explained with an example

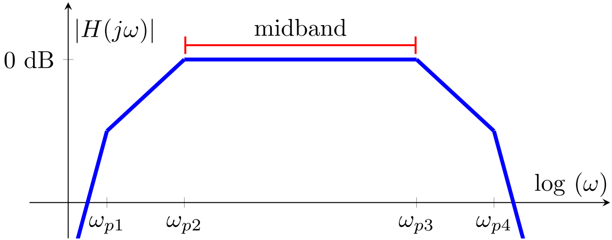
Open/Short Circuit Time-Constant Estimation



- $H(s) \equiv \frac{V_o}{V_i}$
- $H(s)$ has 2 zeros at dc due to C_1 and C_2
  - we call these Low Freq (LF) caps
- $H(s)$ has 2 zeros at ∞ due to C_3 and C_4
  - we call these High Freq (HF) caps
- For midband gain
  - Short LF caps and open HF caps
  - In above case, midband gain = 1

Open/Short Circuit Time-Constant Estimation

- The above example will result in the following Bode plot



- The LF caps result in the low freq poles, ω_{p1} and ω_{p2}
- The HF caps result in the high freq freq poles, ω_{p3} and ω_{p4}
- To estimate all the poles in the circuit
  - we estimate the pole due to each capacitor

Short Circuit Time-Constant Estimation

- Used to find pole estimations for each LF cap
- Zero all independent sources
- Open all HF caps
- For EACH LF cap, C_i
  - Short ALL other LF caps
  - Find R_{eqi} which is the equivalent resistance "seen" by C_i
  - $\omega_{pi} = \frac{1}{R_{eqi} C_i}$
- The low freq cutoff, ω_L is estimated to be

$$\omega_L \approx \sum_i \omega_{pi} \tag{4}$$

- If one ω_{pi} is MUCH larger than the others then...

$$\omega_L \approx \omega_{pi,MAX} \tag{5}$$

Short Circuit Time-Constant Estimation

- Above example
- C_1 (LF cap)
  - $R_{eq1} = R_1 || R_2 = 0.99\ \mathrm{k\Omega}$
  - $\omega_{p1} = \frac{1}{R_{eq1} C_1} = 101\ \mathrm{rad/s}$
- C_2 (LF cap)
  - $R_{eq2} = R_2 = 100\ \mathrm{k\Omega}$
  - $\omega_{p1} = \frac{1}{R_{eq2} C_2} = 1\ \mathrm{rad/s}$
- Since $\omega_{p1} \gg \omega_{p2}$

$$\omega_L \approx \omega_{p1} \approx 100\ \mathrm{rad/s}$$

Open Circuit Time-Constant Estimation

- Used to find pole estimations for each HF cap
- Zero all independent sources
- Short all LF caps
- For EACH HF cap, C_i
  - Open ALL other HF caps
  - Find R_{eqi} which is the equivalent resistance ”seen” by C_i
  - $\omega_{pi} = \frac{1}{R_{eqi} C_i}$
- The high freq cutoff, ω_H is estimated to be

$$\omega_H \approx \left(\sum_i \frac{1}{\omega_{pi}} \right)^{-1} = \omega_{p1} || \omega_{p2} || \omega_{p3} || \ \ldots \tag{6}$$

- If one ω_{pi} is MUCH smaller than the others then...

$$\omega_H \approx \omega_{pi,MIN} \tag{7}$$

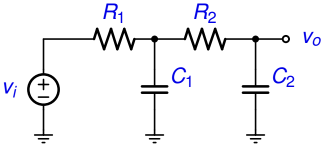
Open Circuit Time-Constant Estimation

- Above example
- C_3 (HF cap)
  - $R_{eq3} = R_3 = 1\ \mathrm{k\Omega}$
  - $\omega_{p3} = \frac{1}{R_{eq3}C_3} = 1e9\ \mathrm{rad/s}$
- C_4 (HF cap)
  - $R_{eq4} = R_3 + R_4 = 101\ \mathrm{k\Omega}$
  - $\omega_{p4} = \frac{1}{R_{eq4}C_4} = 10e6\ \mathrm{rad/s}$
- Since $\omega_{p4} \ll \omega_{p3}$

$$\omega_H \approx \omega_{p4} \approx 10e6\ \mathrm{rad/s}$$

Open Circuit Time-Constant Accuracy

- How accurate are pole estimates for the open circuit time-constant approach?
- Example



- Analysis gives

$$H(s) = \frac{1}{s^2(C_1 C_2 R_1 R_2) + s[C_2(R_1 + R_2) + C_1 R_1] + 1}$$

- Can find exact pole values by finding the roots of the denominator for various C and R values

Open Circuit Time-Constant Accuracy

- Fix value of caps to $C_1 = C_2 = 1.59\ \mathrm{pF}$
- Example 1: $R_1 = 1\ \mathrm{k}\Omega \qquad R_2 = 100\ \mathrm{k}\Omega$

	Actual	OCTC Estimate
f_{p2}	$990\mathrm{kHz}$	$990\mathrm{kHz}$
f_{p1}	$101\mathrm{MHz}$	$100\mathrm{MHz}$

- Example 2: $R_1 = 10\ \mathrm{k}\Omega \qquad R_2 = 100\ \mathrm{k}\Omega$

	Actual	OCTC Estimate
f_{p2}	$901\mathrm{kHz}$	$909\mathrm{kHz}$
f_{p1}	$11.1\mathrm{MHz}$	$10\mathrm{MHz}$

- Example 1: $R_1 = 30\ \mathrm{k}\Omega \qquad R_2 = 100\ \mathrm{k}\Omega$

	Actual	OCTC Estimate
f_{p2}	$723\mathrm{kHz}$	$769\mathrm{kHz}$
f_{p1}	$4.61\mathrm{MHz}$	$3.33\mathrm{MHz}$

Open Circuit Time-Constant Accuracy

- We see that as the poles get closer together
  - The pole estimates worsens
  - The estimate is better for the lower freq pole
  - The lower freq pole mainly determines ω_H so this is good
- In many circuits, the poles are more independent than the above example
  - May have gain stages between parts of the circuit
  - If the poles are independent, the estimate is exact.
  - So the above is a worse case example
- Strictly speaking, the open circuit time-constant approach was developed to only estimate ω_H
  - When all capacitor voltages are independent, it does a reasonable job of also showing where all the poles are.
  - If some capacitor voltages are dependent, then the number of poles equals the number of independent capacitor voltages.

Topics Covered

- Bode plots for real-axis poles/zeros
- Finding zeros
- Short circuit time constant estimation
  - To estimate low freq poles
- Open circuit time constant estimation
  - To estimate high freq poles