## University of Toronto

## Final Exam - Online

Date - Dec 17, 2020: 6:30pm
Duration - 3 hrs (submit by $9: 30 \mathrm{pm}$ )
ECE 331 - Analog Electronics

> Lecturer - D. Johns

- Open book (Equation sheet is on the last page for convenience)
- Unless otherwise stated, assume $g_{m} r_{o} \gg 1$
- Notation: 15 e 3 is equivalent to $15 \times 10^{3}$
- Grading indicated by [].
- Upload your solutions as a single pdf file
- At the beginning of the first page of your solution, write

Last Name; First Name; Student Number; SEED Number:
(SEED number is shown below)

| Question | 1 | 2 | 3 | 4 | 5 | 6 | Total |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Points: | 6 | 6 | 6 | 6 | 6 | 6 | 36 |
| Score: |  |  |  |  |  |  |  |

## Grading Table

SEED: 01

Q1. Consider the amplifier stage shown below and only consider the shown capacitors.

[3]
(a) Find the small-signal dc gain $v_{o} / v_{i}$ ?

## Solution

$r_{s 1}=1 / g_{m 1}=1 /(800 e-6)=1.25 \mathrm{k} \Omega ; r_{s 2}=1 / g_{m 2}=1 /(800 e-6)=1.25 \mathrm{k} \Omega$
Find the short-circuit current at $v_{o}$ relative to $v_{i}$ assuming all $r_{o} \rightarrow \infty$.
$i_{s c} / v_{i}=1 /\left(r_{s 1}+r_{s 2}\right)=1 /((1.25 e 3)+(1.25 e 3))=400 \mu \mathrm{~A} / \mathrm{V}$
Find the output impedance at $v_{o}$
$R_{o}=r_{o 3}\left\|\left(\left(1+g_{m 2} * r_{s 1}\right) * r_{o 2}\right)=(40 e 3)\right\|((1+(800 e-6) *(1.25 e 3)) *(40 e 3))=26.67 \mathrm{k} \Omega$ The gain is given by

$$
\begin{equation*}
v_{o} / v_{i}=i_{s c} / v_{i} * R_{o}=(400 e-6) *(26.67 e 3)=10.67 \mathrm{~V} / \mathrm{V} \tag{3}
\end{equation*}
$$

(b) Find the pole frequency at node $v_{1}$ in $\mathrm{rad} / \mathrm{s}$ ?

## Solution

Define $R_{x}$ to be the impedance seen looking into the source of $M_{2}$
$R_{x}=r_{s 2}+r_{o 3} /\left(g_{m 2} * r_{o 2}\right)=(1.25 e 3)+(40 e 3) /((800 e-6) *(40 e 3))=2.5 \mathrm{k} \Omega$
The impedance to gnd seen at node $v_{1}$ is given by
$R_{v 1}=r_{s 1}\left\|R_{x}=(1.25 e 3)\right\|(2.5 e 3)=833.3 \Omega$
The pole freq is $\omega_{p 1}=1 /\left(R_{v 1} * C_{1}\right)=1 /((833.3) *(1.4 e-12))=857.1 \mathrm{Mrad} / \mathrm{s}$

Q2. Consider feedback amp shown below where the input is a current source, $I_{S}$ with a parallel resistance of $R_{S}$.

[3] (a) Find $L, A_{L \infty}$ and $A_{C L} .\left(\right.$ Assume $\left.A_{L 0}=0\right)$

## Solution

Define $R_{x}$ to be the impedance looking into the source of $M_{1}$
$R_{x}=1 / g_{m 1}+R_{1} /\left(g_{m 1} * r_{o 1}\right)=1 /(800 e-6)+(10 e 3) /((800 e-6) *(22 e 3))=1.818 \mathrm{k} \Omega$
Define $R_{y}$ to be the impedance at the $v_{o}$ node with the loop broken
$R_{y}=r_{o 2}\left\|R_{2}\right\|\left(R_{3}+R_{S} \| R_{x}\right)=(20 e 3)\|(12 e 3)\|((10 e 3)+(12 e 3) \|(1.818 e 3))=4.552 \mathrm{k} \Omega$
Starting at the gate of $M_{2}$ (node $v_{2}$ ) and going around the loop, we have

$$
\begin{aligned}
& v_{o} / v_{2}=-g_{m 2} * R_{y}=-(1.4 e-3) *(4.552 e 3)=-6.372 \mathrm{~V} / \mathrm{V} \\
& v_{1} / v_{o}=\left(R_{S} \| R_{x}\right) /\left(R_{S} \| R_{x}+R_{3}\right)=((12 e 3) \|(1.818 e 3)) /((12 e 3) \|(1.818 e 3)+(10 e 3))=0.1364 \mathrm{~V} / \mathrm{V} \\
& v_{2} / v_{1}=g_{m 1} *\left(r_{o 1} \| R_{1}\right)=(800 e-6) *((22 e 3) \|(10 e 3))=5.5 \mathrm{~V} / \mathrm{V} \\
& L=-v_{o} / v_{2} * v_{1} / v_{o} * v_{2} / v_{1}=-(-6.372) *(0.1364) *(5.5)=4.779 \\
& A_{L \infty}=-R_{3}=-(10 e 3)=-10 \mathrm{k} \Omega \\
& A_{C L}=A_{L \infty} *(L /(1+L))=(-10 e 3) *((4.779) /(1+(4.779)))=-8.27 \mathrm{k} \Omega
\end{aligned}
$$

(b) Find $R_{\text {in }}$ and $R_{\text {out }}$

## Solution

For $R_{\text {out }}, R_{\text {out }}^{\prime}=R_{y}=4.552 \mathrm{k} \Omega$ (from above) is the output resistance with the loop broken $L_{S}=0$ and $L_{O}=L$

$$
R_{o u t}=R_{o u t}^{\prime} *\left(1+L_{S}\right) /\left(1+L_{O}\right)=(4.552 e 3) *(1+(0)) /(1+(4.779))=787.6 \Omega
$$

For $R_{i n}$, define $R_{i n 2}$ to be the input resistance that INCLUDES $R_{S}$
$R_{i n 2}^{\prime}$ is the input resistance (including $R_{S}$ ) with the loop broken
$R_{i n 2}^{\prime}=R_{S}\left\|\left(R_{3}+r_{o 2} \| R_{2}\right)\right\| R_{x}=(12 e 3)\|((10 e 3)+(20 e 3) \|(12 e 3))\|(1.818 e 3)=1.448 \mathrm{k} \Omega$
$R_{i n 2}=R_{i n 2}^{\prime} *\left(1+L_{S}\right) /\left(1+L_{O}\right)=(1.448 e 3) *(1+(0)) /(1+(4.779))=250.6 \Omega$
Since $R_{i n 2}=R_{S} \| R_{\text {in }}$ we have

$$
R_{i n}=1 /\left(1 / R_{i n 2}-1 / R_{S}\right)=1 /(1 /(250.6)-1 /(12 e 3))=255.9 \Omega
$$

Q3. Consider the amplifier stage shown below where the input/output characteristic for the class B output stage (from $v_{2}$ to $v_{o}$ ) is shown. For this class B output stage, the gain is $k=0.9$ for $\left|v_{2}\right|>V_{x}$ until $\left|v_{o}\right|$ reaches $V_{\max }$ while the dead-band region results in $v_{o}=0$ for $\left|v_{2}\right|<V_{x}$. The gain of the opamp is $A_{v}=8 \mathrm{~V} / \mathrm{V}$.



$$
V_{\max }=4.5 \mathrm{~V}
$$

$$
V_{x}=0.65 \mathrm{~V}
$$

(a) What is the dead-band region for $v_{i}$ to $v_{o}$ ?

## Solution

The dead-band region is reduced by the gain of the amplifier (the class-B output stage gain does not affect the dead-band region).

$$
\begin{equation*}
V_{x}^{\prime}=V_{x} / A_{v}=(0.65) /(8)=81.25 \mathrm{mV} \tag{2}
\end{equation*}
$$

(b) What is the gain outside the dead-band region but before the output reaches $V_{\max }$ ?

## Solution

The gain outside the dead-band region and before $v_{o}=V_{\max }$ is given by
$k^{\prime}=L /(1+L)$ where $L$ is the loop gain in the region given by
$L=A_{v} * k=(8) *(0.9)=7.2$
$k^{\prime}=L /(1+L)=(7.2) /(1+(7.2))=0.878 \mathrm{~V} / \mathrm{V}$
(c) What is the value of $v_{i}$ when the output just reaches $V_{\max }$ ?

## Solution

Outside the dead-band region, $v_{o}$ is given by the equation $v_{o}=\left(v_{i}-V_{x}^{\prime}\right) k^{\prime}$ where $V_{x}^{\prime}$ is the new dead-band region and $k^{\prime}$ is the closed-loop gain Setting $v_{o}$ equal to $V_{\max }=4.5 \mathrm{~V}$ and solving for $v_{i}$, we have

$$
v_{i}=\left(V_{\max } / k^{\prime}\right)+V_{x}^{\prime}=((4.5) /(0.878))+(81.25 e-3)=5.206 \mathrm{~V}
$$

Q4. Assume an opamp is ideal with the transfer-function,
$A(s)=\frac{k_{d c}}{\left(1+s / \omega_{p 1}\right)\left(1+s / \omega_{p 2}\right)\left(1+s / \omega_{p 3}\right)}$
The straight-line Bode phase plot for the amplifier is shown below and the dc gain is given by $k_{d c}=1.5 e 3$
Assume the poles are widely spaced apart.

(a) Find the values for $\omega_{p 1}, \omega_{p 2}$, and $\omega_{p 3}$ in $\mathrm{rad} / \mathrm{s}$

## Solution

The pole frequencies can be found by looking at the straight-line Bode phase plot.
Assuming the poles are widely spaced apart,
the phase at where the poles occur are at $-45^{\circ},-135^{\circ},-225^{\circ}$
Therefore, the pole locations are

$$
\begin{aligned}
\omega_{p 1} & =1 \mathrm{rad} / \mathrm{s} \\
\omega_{p 2} & =1 \mathrm{krad} / \mathrm{s} \\
\omega_{p 3} & =100 \mathrm{krad} / \mathrm{s}
\end{aligned}
$$

[4] (b) Ignoring the effect of the highest frequency pole, if this amplifier is used in a non-inverting configuration, what is the smallest dc closed-loop amplifier gain that will result in a phase margin of $65^{\circ}$ ? (use the actual $A(s)$ and NOT the straight-line Bode plot).

## Solution

The loop gain is $A(s) \beta$ resulting in $L(s)=A(s) \beta=\frac{k_{d c} \beta}{\left(1+s / \omega_{p 1}\right)\left(1+s / \omega_{p 2}\right)}$
( $\omega_{p 3}$ is ignored so it is set to $\infty$ in $A(s)$ )
$\angle L(j \omega)=-\operatorname{atan}\left(\omega / \omega_{p 1}\right)-\operatorname{atan}\left(\omega / \omega_{p 2}\right)$
$P M=\angle L\left(j \omega_{1}\right)-\left(-180^{\circ}\right)$ where $\omega_{1}$ is defined as $\left|L\left(j \omega_{1}\right)\right|=1$
We know that for $\mathrm{PM}=65^{\circ}$, the unity gain freq $\omega_{1}$ will have the relationship $\omega_{p 1} \ll \omega_{1}<\omega_{p 2}$ so reconizing that $\omega_{1} / \omega_{p 1} \gg 1$, we can make the approximation $\operatorname{atan}\left(\omega_{1} / \omega_{p 1}\right) \approx 90^{\circ}$ leading to
$P M=90^{\circ}-\operatorname{atan}\left(\omega_{1} / \omega_{p 2}\right)=65^{\circ}$
$\omega_{1}=\omega_{p 2} * \tan (((90-\mathrm{PM}) / 180) * \pi)=(1 e 3) * \tan (((90-(65)) / 180) *(3.142))=466.3 \mathrm{rad} / \mathrm{s}$
and we now find $\beta$ by making use of $\left|L\left(j \omega_{1}\right)\right|=1$
$\beta=\left(\omega_{1} / \omega_{p 1}\right) *\left(\sqrt{1+\left(\omega_{1} / \omega_{p 2}\right)^{2}}\right) / k_{d c}=((466.3) /(1)) *\left(\sqrt{1+((466.3) /(1 e 3))^{2}}\right) /(1.5 e 3)=0.343$
The closed-loop gain, $A_{c l}$ is given by $A_{c l}=A_{0} /\left(1+A_{0} \beta\right)$ where $A_{0}=k_{d c}$ resulting in the min $A_{c l}$ given by
$A_{c l}=k_{d c} /\left(1+k_{d c} * \beta\right)=(1.5 e 3) /(1+(1.5 e 3) *(0.343))=2.91 \mathrm{~V} / \mathrm{V}$
(we could also have used $A_{c l} \approx 1 / \beta$ since $k_{d c} \beta \gg 1$ )
The minimum closed-loop gain that results in $\mathrm{PM}=65^{\circ}$ is $A_{c l}=2.91 \mathrm{~V} / \mathrm{V}$

Q5. Consider the CMOS push-pull output stage shown below. The size for the NMOS output transistor has $W_{1}=90 \mu \mathrm{~m}$ and $L_{1}=90 \mathrm{~nm}$ while the PMOS output transistor has $W_{7}=180 \mu \mathrm{~m}$ and $L_{7}=105 \mathrm{~nm}$. It is desired that $I_{Q}=800 \mu \mathrm{~A}$ while $I_{1}=I_{2}=66.67 \mu \mathrm{~A}$.

(a) Find the width and length for $M_{2}, M_{3}, M_{8}, M_{9}$ so that the desired currents are obtained.

## Solution

$M_{2}$ and $M_{3}$ are matched and $M_{8}$ and $M_{9}$ are matched so we just need the sizes of $M_{2}$ and $M_{8}$. The length of $M_{2}$ should match the length of $M_{1}$ so $L_{2}=L_{1}=(90 e-9)=90 \mathrm{~nm}$
We also have a current mirror between $M_{2}$ and $M_{1}$ when $v_{o}=0$ given by $I_{Q}=I_{D 1}=\left(W_{1} / W_{2}\right) *\left(I_{1} / 2\right)$ resulting in

$$
W_{2}=\left(\left(I_{1} / 2\right) / I_{Q}\right) * W_{1}=(((66.67 e-6) / 2) /(800 e-6)) *(90 e-6)=3.75 \mu \mathrm{~m}
$$

The length of $M_{8}$ should match the length of $M_{7}$ so $L_{8}=L_{7}=(105 e-9)=105 \mathrm{~nm}$
We also have a current mirror between $M_{8}$ and $M_{7}$ when $v_{o}=0$ given by $I_{Q}=I_{D 7}=\left(W_{7} / W_{8}\right) *\left(I_{2} / 2\right)$ resulting in

$$
W_{8}=\left(\left(I_{2} / 2\right) / I_{Q}\right) * W_{7}=(((66.67 e-6) / 2) /(800 e-6)) *(180 e-6)=7.5 \mu \mathrm{~m}
$$

[3] (b) When simulating this power amp, it is found that the gain of the NMOS error amp is too large and should be reduced to improve the stability of the circuit. What changes can be made to the circuit to reduce the error amp gain by a factor of 2 while keeping $I_{Q}$ unchanged?

## Solution

The gain of the error amp is $A_{v}=g_{m 5} *\left(r_{o 5} \| r_{o 3}\right)$. To reduce the gain, we can either reduce $g_{m 5}$ or the output impedances. It is generally preferable to reduce $g_{m}$ (as there are 2 output impedances involved and also, you would need either more current or a shorter length but the length of $M_{3}$ has to match the length of $M_{1}$ ).
To reduce $g_{m 5}$ by a factor of 2 , you need to reduce the $W_{5}$ by a factor of 4 (due to the square root relationship assuming the current remains unchanged). Of course, $W_{4}$ would also be reduced by a factor of 4 .
Reduce both $W_{4}$ and $W_{5}$ by a factor of 4

Q6. Consider a class AB BJT output stage as shown below with an output load, $R_{L}=30 \Omega$ and a maximum amplitude of $\pm 12 \mathrm{~V}$ (limited by the input swing). The power transistors ( $Q_{N}$ and $Q_{P}$ ) both have $I_{S, \text { pow }}=200 \mathrm{fA}$ and $\beta_{\text {pow }}=50$ while the bias transistor $\left(Q_{1}\right)$ has $I_{S}=20 \mathrm{fA}$ and $\beta_{1}=180$. Assume $V_{T}=25 \mathrm{mV}$

[2] (a) Assuming a quiescent current (in the power transistors) of 12 mA , find the 3 values of the input voltage corresponding to when the output is either $-12 \mathrm{~V}, 0 \mathrm{~V}$ or 12 V .

## Solution

Define $\alpha_{\text {pow }}=\beta_{\text {pow }} /\left(\beta_{\text {pow }}+1\right)=(50) /((50)+1)=0.9804$
Define $I_{S \alpha, \text { pow }}=I_{S, p o w} / \alpha_{\text {pow }}=204 \mathrm{fA}$
The quadratic equation for class-AB BJT amplifiers is ...
$i_{n}^{2}-i_{L} i_{n}-I_{Q}^{2}=0$

When $i_{L}=0, i_{n}=I_{Q}$
When $i_{L} \gg I_{Q}$, we can ignore the $I_{Q}^{2}$ term leading to
$i_{n} \approx i_{L}$
In addition, we can find the pnp emitter current from
$i_{p}=I_{Q}^{2} / i_{n}$ which is found from the equation $i_{n} i_{p}=I_{Q}^{2}$
To find the input voltage, we make use of
$v_{o}=v_{i}+V_{E B, P}$ leading to $v_{i}=v_{o}-V_{E B, P}$
We need to find the 3 values of $V_{E B, P}$ which are found from the $i_{p}$ values using...
$V_{E B, p}=V_{T} * \ln \left(i_{p} /\left(I_{S \alpha}\right)\right)$

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\(v_{o}=0 \mathrm{~V}:\)
\(i_{L}=0\) leading to \(i_{p}=I_{Q}=(12 e-3)=12 \mathrm{~mA}\)
\(V_{E B, p}=V_{T} * \log \left(i_{p} / I_{S \alpha, \text { pow }}\right)=(25 e-3) * \log ((12 e-3) /(204 e-15))=0.6199 \mathrm{~V}\)
    \(v_{i}=v_{o}-V_{E B, p}=(0)-(0.6199)=-0.6199 \mathrm{~V}\)
\(v_{o}=12 \mathrm{~V}:\)
\(i_{L}=v_{o} / R_{L}=(12) /(30)=0.4 \mathrm{~A}\) leading to \(i_{n} \approx i_{L}=0.4 \mathrm{~A}\)
\(i_{p}=I_{Q}^{2} / i_{n}=(12 e-3)^{2} /(0.4)=360 \mu \mathrm{~A}\)
\(V_{E B, p}=V_{T} * \log \left(i_{p} / I_{S \alpha, p o w}\right)=(25 e-3) * \log ((360 e-6) /(204 e-15))=0.5323 \mathrm{~V}\)
    \(v_{i}=v_{o}-V_{E B, p}=(12)-(0.5323)=11.47 \mathrm{~V}\)
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\(v_{o}=-12 \mathrm{~V}:\)
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$v_{o}=-12 \mathrm{~V}:$
$i_{L}=v_{o} / R_{L}=(-12) /(30)=-0.4$ A leading to $i_{p} \approx-i_{L}=0.4 \mathrm{~A}$ (by symmetry)
$i_{L}=v_{o} / R_{L}=(-12) /(30)=-0.4$ A leading to $i_{p} \approx-i_{L}=0.4 \mathrm{~A}$ (by symmetry)
$V_{E B, p}=V_{T} * \log \left(i_{p} / I_{S \alpha, \text { pow }}\right)=(25 e-3) * \log ((0.4) /(204 e-15))=0.7076 \mathrm{~V}$
$V_{E B, p}=V_{T} * \log \left(i_{p} / I_{S \alpha, \text { pow }}\right)=(25 e-3) * \log ((0.4) /(204 e-15))=0.7076 \mathrm{~V}$
$v_{i}=v_{o}-V_{E B, p}=(-12)-(0.7076)=-12.71 \mathrm{~V}$

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    \(v_{i}=v_{o}-V_{E B, p}=(-12)-(0.7076)=-12.71 \mathrm{~V}\)
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(b) Design the bias circuit for a quiescent current (in the power transistors) of 12 mA and a minimum current of 1.2 mA through the Vbe multiplier circuit.

## Solution

Define $V_{o, \max }=12 \mathrm{~V}$ as the maximum peak output voltage
Define $I_{\text {min }}=1.2 \mathrm{~mA}$ as the min current through Vbe multiplier
$I_{B}$ is sized by choosing $I_{B}$ to be $I_{\min }+I_{B n, \max }$ where $I_{B n, \max }$ is the largest current into the base of $Q_{N}$.
$I_{B n, \max }=\left(V_{o, \max } / R_{L}\right) *\left(1 /\left(\beta_{\text {pow }}+1\right)\right)=((12) /(30)) *(1 /((50)+1))=7.843 \mathrm{~mA}$

$$
I_{B}=I_{\min }+I_{B n, \max }=(1.2 e-3)+(7.843 e-3)=9.043 \mathrm{~mA}
$$

When $I_{\text {min }}$ is going to the Vbe multiplier, we choose to let half flow through $R_{2}$ and half flow through $I_{C 1}$ of $Q_{1}$ (this is a reasonable design choice).
As more current flows into the Vbd multiplier, $I_{C 1}$ will absorb most of the extra current
So $I_{R}=I_{\min } / 2=(1.2 e-3) / 2=600 \mu \mathrm{~A}$

We have from part(a) that for $v_{o}=0$, and $I_{Q}=12 \mathrm{~mA}$ then $V_{E B, p}=0.6199 \mathrm{~V}$ and due to matching, $V_{B E, n}=V_{E B, p}=0.6199 \mathrm{~V}$ leading to
$V_{B B}=V_{B E, n}+V_{E B, p}=(0.6199)+(0.6199)=1.24 \mathrm{~V}$
Define $\alpha_{1}=\beta_{1} /\left(\beta_{1}+1\right)=(180) /((180)+1)=0.9945$
Define $I_{S \alpha, 1}=I_{S} / \alpha_{1}=20.11 \mathrm{fA}$
(since $\beta_{1}$ is so large, we could have ignored the effect of $\alpha_{1}$ here)

At $v_{o}=0, i_{n}=I_{Q}$ resulting in $Q_{N}$ base current given by
$I_{B n, Q}=I_{Q} /\left(\beta_{\text {pow }}+1\right)=(12 e-3) /((50)+1)=235.3 \mu \mathrm{~A}$
$I_{C 1}=I_{B}-I_{R}-I_{B n, Q}=(9.043 e-3)-(600 e-6)-(235.3 e-6)=8.208 \mathrm{~mA}$
$V_{b e 1}=V_{T} * \log \left(I_{C 1} / I_{S \alpha, 1}\right)=(25 e-3) * \log ((8.208 e-3) /(20.11 e-15))=0.6684 \mathrm{~V}$
$R_{1}=V_{b e 1} / I_{R}=(0.6684) /(600 e-6)=1.114 \mathrm{k} \Omega\left(\right.$ Assuming $\left.I_{B 1} \approx 0\right)$
$V_{B B}=V_{b e 1}+\left(V_{b e 1} / R_{1}\right) * R_{2}=V_{b e 1} *\left(1+R_{2} / R_{1}\right)$
$R_{2}=\left(\left(V_{B B} / V_{b e 1}\right)-1\right) * R_{1}=(((1.24) /(0.6684))-1) *(1.114 e 3)=952.5 \Omega$
$R_{1}=1.114 \mathrm{k} \Omega ; R_{2}=952.5 \Omega$

## Equation Sheet

Constants: $k=1.38 \times 10^{-23} \mathrm{~J} \mathrm{~K}^{-1} ; q=1.602 \times 10^{-19} \mathrm{C} ; V_{T}=k T / q \approx 26 \mathrm{mV}$ at $300 \mathrm{~K} ; \epsilon_{0}=8.85 \times 10^{-12} \mathrm{Fm}^{-1}$; $k_{o x}=3.9 ; C_{o x}=\left(k_{o x} \epsilon_{0}\right) / t_{o x} ; \omega=2 \pi f$
NMOS: $k_{n}=\mu_{n} C_{o x}(W / L) ; V_{t n}>0 ; v_{D S} \geq 0 ; V_{o v}=V_{G S}-V_{t n}$ (triode) $v_{D S} \leq V_{o v} ; v_{D}<v_{G}-V_{t n} ; i_{D}=k_{n}\left(V_{o v} v_{D S}-\left(v_{D S}^{2} / 2\right)\right)$
(active) $v_{D S} \geq V_{o v} ; i_{D}=0.5 k_{n} V_{o v}^{2}\left(1+\lambda v_{D S}\right) ; g_{m}=k_{n} V_{o v}=2 I_{D} / V_{o v}=\sqrt{2 k_{n} I_{D}} ; r_{s}=1 / g_{m}$;
$r_{o}=L /\left(\left|\lambda^{\prime}\right| I_{D}\right)$
PMOS: $k_{p}=\mu_{p} C_{o x}(W / L) ; V_{t p}<0 ; v_{S D} \geq 0 ; V_{o v}=V_{S G}-\left|V_{t p}\right|$
(triode) $v_{S D} \leq V_{o v} ; v_{D}>v_{G}+\left|V_{t p}\right| ; i_{D}=k_{p}\left(V_{o v} v_{S D}-\left(v_{S D}^{2} / 2\right)\right)$
(active) $v_{S D} \geq V_{o v} ; i_{D}=0.5 k_{p} V_{o v}^{2}\left(1+|\lambda| v_{S D}\right) ; g_{m}=k_{p} V_{o v}=2 I_{D} / V_{o v}=\sqrt{2 k_{p} I_{D}} ; r_{s}=1 / g_{m} ;$ $r_{o}=L /\left(\left|\lambda^{\prime}\right| I_{D}\right)$
BJT: (active) $i_{C}=I_{S} e^{\left(v_{B E} / V_{T}\right)}\left(1+\left(v_{C E} / V_{A}\right)\right) ; g_{m}=\alpha / r_{e}=I_{C} / V_{T} ; r_{e}=V_{T} / I_{E} ; r_{\pi}=\beta / g_{m} ; r_{o}=\left|V_{A}\right| / I_{C} ;$ $i_{C}=\beta i_{B} ; i_{E}=(\beta+1) i_{B} ; \alpha=\beta /(\beta+1) ; i_{C}=\alpha i_{E} ; R_{b}=(\beta+1)\left(r_{e}+R_{E}\right) ; R_{e}=\left(R_{B}+r_{\pi}\right) /(\beta+1)$

$v_{o c} \approx v_{i}$
$R_{x} \approx 1 / g_{m}+R_{D} /\left(g_{m} r_{o}\right)$
$v_{o} / v_{i} \approx g_{m}\left(r_{o} \| R_{D}\right)$
Diff Pair: $A_{d}=g_{m} R_{D} ; A_{C M}=-\left(R_{D} /\left(2 R_{S S}\right)\right)\left(\Delta R_{D} / R_{D}\right) ; A_{C M}=-\left(R_{D} /\left(2 R_{S S}\right)\right)\left(\Delta g_{m} / g_{m}\right)$;
$V_{O S}=\Delta V_{t} ; V_{O S}=\left(V_{O V} / 2\right)\left(\Delta R_{D} / R_{D}\right) ; V_{O S}=\left(V_{O V} / 2\right)(\Delta(W / L) /(W / L))$
Large signal: $i_{D 1}=(I / 2)+\left(I / V_{o v}\right)\left(v_{i d} / 2\right)\left(1-\left(v_{i d} / 2 V_{o v}\right)^{2}\right)^{1 / 2}$
1st order: step response $y(t)=Y_{\infty}-\left(Y_{\infty}-Y_{0+}\right) e^{-t / \tau}$;
unity gain freq for $T(s)=A_{M} /\left(1+\left(s / \omega_{3 d B}\right)\right)$ for $A_{M} \gg 1 \Rightarrow \omega_{t} \simeq\left|A_{M}\right| \omega_{3 d B}$
Freq: for real axis poles/zeros $T(s)=k_{d c} \frac{\left(1+s / z_{1}\right)\left(1+s / z_{2}\right) \ldots\left(1+s / z_{m}\right)}{\left(1+s / \omega_{1}\right)\left(1+s / \omega_{2}\right) \ldots\left(1+s / \omega_{n}\right)}$
OTC estimate $\omega_{H} \simeq 1 /\left(\sum \tau_{i}\right) ;$ dominant pole estimate $\omega_{H} \simeq 1 /\left(\tau_{\text {max }}\right)$
STC estimate $\omega_{L} \simeq \sum 1 / \tau_{i} ;$ dominant pole estimate $\omega_{L} \simeq 1 /\left(\tau_{\text {min }}\right)$
Miller: $Z_{1}=Z /(1-K) ; Z_{2}=Z /(1-1 / K)$
Mos caps: $C_{g s}=(2 / 3) W L C_{o x}+W L_{o v} C_{o x} ; \quad C_{g d}=W L_{o v} C_{o x} ; \quad C_{d b}=C_{d b 0} / \sqrt{1+V_{d b} / V_{0}}$;
$\omega_{t}=g_{m} /\left(C_{g s}+C_{g d}\right) ;$ for $C_{g s} \gg C_{g d} \Rightarrow f_{t} \simeq\left(3 \mu V_{o v}\right) /\left(4 \pi L^{2}\right)$
Feedback: $A_{f}=A /(1+A \beta) ; x_{i}=(1 /(1+A \beta)) x_{s} ; d A_{f} / A_{f}=(1 /(1+A \beta)) d A / A ; \omega_{H f}=\omega_{H}(1+A \beta) ; \omega_{L f}=$ $\omega_{L} /(1+A \beta)$;
Loop Gain $L \equiv-s_{r} / s_{t} ; A_{f}=A_{\infty}(L /(1+L))+d /(1+L) ; Z_{\text {port }}=Z_{p^{\circ}}\left(\left(1+L_{S}\right) /\left(1+L_{O}\right)\right): P M=$ $\angle L\left(j \omega_{t}\right)+180 ; G M=-\left|L\left(j \omega_{180}\right)\right|_{d b}$;
Pole splitting $\omega_{p 1}^{\prime} \simeq 1 /\left(g_{m} R_{2} C_{f} R_{1}\right) ; \omega_{p 2}^{\prime} \simeq\left(g_{m} C_{f}\right) /\left(C_{1} C_{2}+C_{f}\left(C_{1}+C_{2}\right)\right)$
Pole Pair: $s^{2}+\left(\omega_{o} / Q\right) s+\omega_{o}^{2} ; Q \leq 0.5 \Rightarrow$ real poles; $Q>1 / \sqrt{2} \Rightarrow$ freq resp peaking
Power Amps: Class A : $\eta=(1 / 4)\left(\hat{V}_{O} / I R_{L}\right)\left(\hat{V}_{O} / V_{C C}\right) ;$ Class B : $\eta=(\pi / 4)\left(\hat{V_{O}} / V_{C C}\right) ; P_{D N \_m a x}=V_{C C}^{2} /\left(\pi^{2} R_{L}\right)$;
Class AB : $i_{n} i_{p}=I_{Q}^{2} ; I_{Q}=\left(I_{S} / \alpha\right) e^{V_{B B} /\left(2 V_{T}\right)} ; i_{n}^{2}-i_{L} i_{n}-I_{Q}^{2}=0$
2-stage opamp: $\omega_{p 1} \simeq\left(R_{1} G_{m 2} R_{2} C_{c}\right)^{-1} ; \quad \omega_{p 2}=G_{m 2} / C_{2} ; \omega_{z}=\left(C_{c}\left(1 / G_{m 2}-R\right)\right)^{-1}$;
$S R=I / C_{c}=\omega_{t} V_{\text {ov } 1} ;$ will not SR limit if $\omega_{t} \hat{V}_{O}<S R$

