## Problem Set 4

Q1. Consider the PMOS differential amplifier shown below let $V_{t p}=-0.8 \mathrm{~V}$ and $k_{p}^{\prime} W / L=4 \mathrm{~mA} / \mathrm{V}^{2}$. Neglect channel-length modulation.

(a) For $V_{G 1}=V_{G 2}=0 V$, find $V_{o v}$ and $V_{G S}$ for each of M1 and M2. Also find $V_{S}, V_{D 1}$, and $V_{D 2}$.
(b) If the current source requires a minimum voltage of 0.5 V , find the input common-mode range.

Q2. Consider the differential amp below and let the two inputs $V_{G 2}=0 \mathrm{~V}$ and $V_{G 1}=v_{i d}$. For each case below, find $v_{i d}, V_{s}, V_{D 1}, V_{D 2}$ and $V_{D 2}-V_{D 1}$


$$
\begin{aligned}
& \mu_{n} C_{o x}=400 \mu \mathrm{~A} / \mathrm{V}^{2} \\
& W / L=12.5 \\
& V_{t}=0.5 \mathrm{~V} \\
& V_{D D}=1 \mathrm{~V} \\
& V_{S S}=1 \mathrm{~V}
\end{aligned}
$$

a) $I_{D 1}=I_{D 2}=100 \mu \mathrm{~A}$
b) $I_{D 1}=150 \mu \mathrm{~A}$ and $I_{D 2}=50 \mu \mathrm{~A}$
c) $I_{D 1}=200 \mu \mathrm{~A}$ and $I_{D 2}=0 \mathrm{~A}$
d) $I_{D 1}=50 \mu \mathrm{~A}$ and $I_{D 2}=150 \mu \mathrm{~A}$
e) $I_{D 1}=0 \mathrm{~A}$ and $I_{D 2}=200 \mu \mathrm{~A}$

You can make the assumption that $v_{i d} / 2 \ll V_{o v}$
Q3. Design the circuit below to obtain a dc voltage of +0.2 V at each of the drains of M1 and M2 when $V_{G 1}=V_{G 2}=0 V$. Operate all transistors at $V_{o v}=0.2 \mathrm{~V}$ and assume that for the process technology in which the circuit is fabricated, $V_{t n}=0.5 \mathrm{~V}$ and $\mu_{n} C_{o x}=250 \mu \mathrm{~A} / \mathrm{V}^{2}$. Neglect channellength modulation. Determine the values of $R, R_{D}$, and the $W / L$ ratios of M1, M2, M3, and M4. What is the input common-mode voltage range for your design?


Q4. An NMOS differential amplifier has a bias current of $I=400 \mu \mathrm{~A}$. The transistors have $V_{t}=0.5 \mathrm{~V}$, $W=20 \mu \mathrm{~m}, L=500 \mathrm{~nm}$ and $k_{n}^{\prime}=200 \mu \mathrm{~A} / \mathrm{V}^{2}$.
a) Determine $V_{G S}$ and $g_{m}$ in equilibrium state.
b) Determine $v_{I D}$ for full-current switching.
c) How should the bias current be adjuste such that the value $v_{I D}$ is doubled for full-current switching?

Q5. Design the the MOS differential amplifier below to operate at $V_{o v}=0.25 \mathrm{~V}$ and to provide a transconductance $g_{m}$ of $1 \mathrm{~mA} / \mathrm{V}$. Specify the W/L ratios and the bias current. The technology available provides $V_{t}=0.8 \mathrm{~V}$ and $\mu_{n} C_{o x}=100 \mu \mathrm{~A} / \mathrm{V}^{2}$.


Q6. It is required to design an NMOS differential amplifier to operate with a differential input voltage that can be as high as $v_{i d}=0.1 \mathrm{~V}$ while keeping the nonlinear term under the square root in Eq. (1) to a maximum of $50 \mathrm{e}-3$. A transconductance $g_{m}=1 \mathrm{~mA} / \mathrm{V}$ is needed. Find the required values of $V_{o v}, I, W / L$. Assume that the technology available has $\mu_{n} C o x=200 \mu \mathrm{~A} / \mathrm{V}^{2}$. What differential gain $A_{d}$ results when $R_{D}=10 \mathrm{k} \Omega$ ? Assume $\lambda=0$. What is the resulting output signal corresponding to $v_{i d}$ at its maximum value?

$$
\begin{equation*}
i_{D 1}=\frac{I}{2}+\left(\frac{I}{V_{o v}}\right)\left(\frac{v_{i d}}{2}\right) \sqrt{1-\left(\frac{v_{i d} / 2}{V_{o v}}\right)^{2}} \tag{1}
\end{equation*}
$$

Q7. Design a differential amplifier to operate with a supplies of $\pm 1 V$ and which dissipates no more than
$P=2 \mathrm{~mW}$ in its equilibrium state. Select $V_{o v}$ such that the differential voltage required to steer current from one-side to the other is $v_{i v}=0.4 \mathrm{~V}$. Assume the differential voltage gain is $A_{d}=5 \mathrm{~V} / \mathrm{V}$ and $k_{n}^{\prime}=400 \mu \mathrm{~A} / \mathrm{V}^{2}$. You can ignore the Early effect. Specify $I, R_{D}$, and $W / L$.

Q8. a) Draw the differential half-circuit for the amplifier below.
b) Derive an expression for the differential gain $A_{d}=V_{o d} / V_{i d}$ in terms of $g_{m}, R_{D}$, and $R_{S}$ (ignore the Early effect).
c) What is the gain if $R_{S}=0$ ?
d) What is the value of $R_{S}$ in terms of $1 / g_{m}$ that reduces the gain to half this value?

Q9. A design error has resulted in a gross mismatch in the circuit shown below. Specifically, M2 has twice the $\mathrm{W} / \mathrm{L}$ ratio of M 1 . If $v_{i d}$ is a small sine-wave signal, find:
(a) $I_{D 1}$ and $I_{D 2}$.
(b) $V_{o v}$ for each M1 and M2.
(c) The differential gain $A_{d}$ in terms of $R_{D}, I$, and $V_{o v}$.


Q10. For the differential amplifier shown in Q 2 , let M 1 and M 2 have $k_{p}^{\prime}(W / L)=4 \mathrm{~mA} / \mathrm{V}^{2}$, and assume that the current source has an output resistance of $30 \mathrm{k} \Omega$. Find $\left|V_{o v}\right|, g_{m},\left|A_{d}\right|,\left|A_{c m}\right|$, and the CMRR (in dB ) obtained with the output taken differentially. The drain resistances are known to have a mismatch of $2 \%$.

Q11. An NMOS differential pair operating at a bias current $I$ of $100 \mu \mathrm{~A}$ uses transistors for which $k_{n}^{\prime}=250 \mu \mathrm{~A} / \mathrm{V}^{2}$ and $W / L=10$. Find the three components of input offset voltage under the conditions that $\Delta R_{D} / R_{D}=5 \%, \Delta(W / L) /(W / L)=5 \%$, and $\Delta V_{t}=5 \mathrm{mV}$. In the worst case, what might
the total offset be? For the usual case of the three effects being independent, what is the offset likely to be?

Q12. An active-loaded NMOS differential amplifier operates with a bias current of $I=100 \mu \mathrm{~A}$. The NMOS transistors are operated at $V_{o v}=0.2 \mathrm{~V}$ and the PMOS devices at $\left|V_{o v}\right|=0.3 \mathrm{~V}$. The Early voltages are 20 V for the NMOS and 12 V for the PMOS transistors. Find $G_{m}, R_{o}$, and $A_{d}$. For what value of load resistance is the gain reduced by a factor or 2 ?

Q13. Consider the input stage of the CMOS op amp below with both inputs grounded. Assume that the two sides of the input stage are perfectly matched except that the threshold voltages of M3 and M4 have a mismatch $\Delta V_{t}$. Show that a current $g_{m 3} \Delta V_{t}$ appears at the output of the first stage. What is the corresponding input offset voltage? Use the parameters given below:

$$
\Delta V_{t}=2 \mathrm{mV}, I_{R E F}=90 \mu \mathrm{~A}, V_{t n}=0.7 \mathrm{~V}, V_{t p}=-0.8 \mathrm{~V}, \mu_{n} C_{o x}=160 \mu \mathrm{~A} / \mathrm{V}^{2}, \mu_{p} C_{o x}=40 \mu \mathrm{~A} / \mathrm{V}^{2}
$$

| Transistor | M1 | M2 | M3 | M4 | M5 | M6 | M7 | M8 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| W | $20 \mu \mathrm{~m}$ | $20 \mu \mathrm{~m}$ | $5 \mu \mathrm{~m}$ | $5 \mu \mathrm{~mm}$ | $40 \mu \mathrm{~m}$ | $10 \mu \mathrm{~m}$ | $40 \mu \mathrm{~m}$ | $40 \mu \mathrm{~m}$ |
| L | 800 nm | 800 nm | 800 nm | 800 nm | 800 nm | 800 nm | 800 nm | 800 nm |

Neglect Early voltage.


