Circuit Review

David Johns

Voltage

Voltage is the measure of the difference in the electric potential between 2 points. Voltage gives a measure of how strongly a charge will want to move between the 2 points. We define the units of voltage to be volts where 1 volt equals 1 joule per coulomb. In other words, in a static field where there is 1 volt between 2 points, 1 joule of energy will be needed to move 1 coulomb of charge.

Often the measure of voltage is referenced to a common ground in a circuit so that a voltage value can be given for a single node in the circuit.

The units of the voltage value is volts: $[V]$.

Current

Current is the measure of electron charge flowing through a conductor. Since electron charge is measured in coulombs, current is measured as coulombs per second that flow past a point. Since we use current measure so often, we define the units of columbs per second to be amps or ampere. Specifically, 1 amp equals 1 coulomb per second.

Since one electron has a charge of 1.60218×10^{-19} coulombs, a current of 1 amp implies that the number of electrons moving past a point is $1/(1.60218 \times 10^{-19}) = 6.24 \times 10^{18}$.

By convention, current is defined to go from a higher voltage to a lower voltage. In other words, current is defined to be in the opposite direction of electron flow.

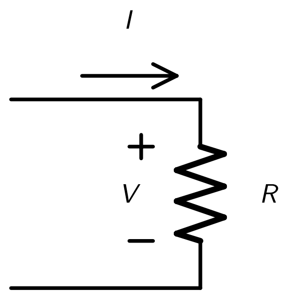
The units of the current value is amps: $[A]$.

Ohm's Law and Resistors

A resistor is an element that "resists" the flow of current. If the same voltage is placed across 2 resistors, there will be less current flow in the resistor with the larger resistor value.

The units of resistor value is Ohms: $[\Omega]$.

Ohm's law states the relationship between the current, I, through a resistor, R, and the voltage, V, across the resistor.



$$I = \frac{V}{R}$$

$$V = IR$$

It is important to get the correct polarity for the voltage relative to the direction of the current.

Power dissipation - Resistor

The power dissipated by the resistor, P, is given by the relationship

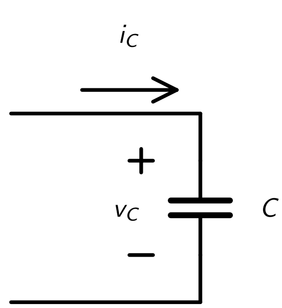
$$P = VI$$

which can also be written as

$$P = V^2/R = I^2R$$

Capacitors

Capacitors are an element that stores energy in its electric field.



The measure for the capacitance of a capacitor is farads. A capacitor of 1 farad will result in a 1V potential difference when 1 coulomb of charge is put into the capacitor. So we have the following relationship

$$q = Cv_C$$

where q is the charge on the capacitor. If we take the derivative with respect to time of the above equation (and assume C is a constant), we have

$$\frac{dq}{dt} = C\frac{dv_C}{dt}$$

and recognizing that $\frac{dq}{dt}$ is the current value, we have

$$i_C = C\frac{dv_C}{dt}$$

Now taking the Laplace transform of the above, we also have a similar relationship in the frequency domain

$$I_C(s) = sCV_C(s)$$

Since the above is in the same form as Ohm's law, we can see that the impedance of the capacitor, Z_C, is given by

$$Z_C = \frac{1}{sC}$$

The units of capacitance value is Farads: $[F]$.

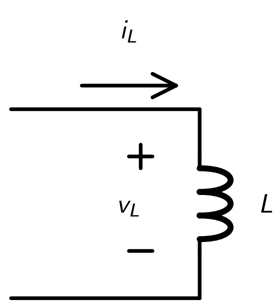
Power dissipation - Capacitor

A capacitor does not dissipate any power. Instead it stores the energy across it and releases that energy again depending on the situation. Defining the energy of the capacitor to be E_C, the energy stored in the capacitor's electric field is given by

$$E_C = \frac{Cv_C^2}{2}$$

Inductors

Inductors are an element that stores energy in its magnetic field.



The measure for the inductance of a inductor is henrys. An inductor of 1 henry will result in a 1V changing potential difference when a changing current of 1 amp is put into the inductor. So we have the following relationship

$$v_L = L\frac{di_L}{dt}$$

Now taking the Laplace transform of the above, we also have a similar relationship in the frequency domain

$$V_L(s) = sLI_L(s)$$

Since the above is in the same form as Ohm's law, we can see that the impedance of the inductor, Z_L, is given by

$$Z_L = sL$$

Power dissipation - Inductor

An inductor does not dissipate any power. Instead it stores the energy across it and releases that energy again depending on the situation. Defining the energy of the inductor to be E_L, the energy stored in the inductor's magnetic field is given by

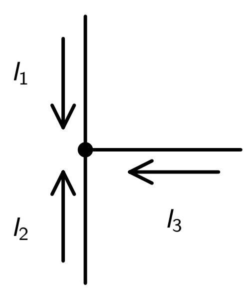
$$E_L = \frac{Li_L^2}{2}$$

The units of inductance value is Henrys: $[H]$.

KCL (Kirchhoff's Current Law)

Kirchhoff's current law (or KCL) states that the sum of all currents entering a node must equal zero.

Below is an example with 3 wires going into a node.



In this case, we have

$$I_1 + I_2 + I_3 = 0$$

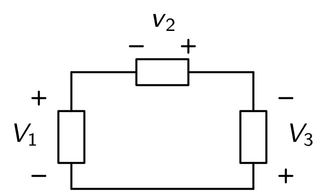
In the general case, if there are n currents flowing into a node, then

$$\sum_{k=1}^{n} I_k = 0$$

KVL (Kirchhoff's Voltage Law)

Kirchhoff's voltage law (or KVL) states that the sum of all voltages around a loop equal zero.

Below is an example with 3 elements around a loop. Each element might be a resistor, part of a transistor, capacitor, some other element or even multiple components.



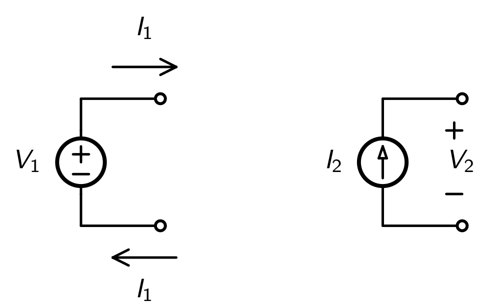
In this case, we have

$$V_1 + V_2 + V_3 = 0$$

In the general case, if there are n elements around a loop, then

$$\sum_{k=1}^{n} V_k = 0$$

Independent Sources



Above are 2 independent sources: a voltage source, V_1 and a current source, I_2. They are called independent sources since the value of their source (either V_1 or I_2) does not depend on the circuit that they are attached to.

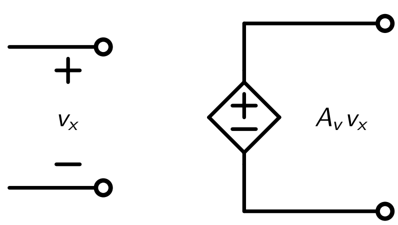
In both cases, power is delivered by the independent source to the circuit that is attached to the source. The power delivered by the source is equal to the voltage times the current.

Dependent Sources

There are 4 types of dependent sources:
1. VCVS (voltage controlled voltage source)
2. VCCS (voltage controlled current source)
3. CCVS (current controlled voltage source)
4. CCCS (current controlled current source)

The 2 common ones are the VCVS and VCCS whereas the remaining 2 (CCVS and CCCS) are rarely used but included here for completeness.

VCVS

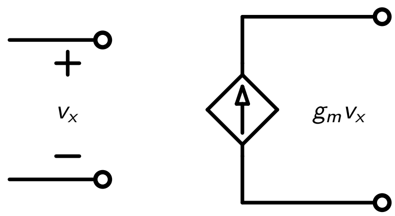


VCVS A_v units: $[V/V]$

(voltage controlled voltage source)

Above is a VCVS. The units for A_v are $[V/V]$ which is in effect a unitless quantity but is often shown this way for clarity.

VCCS

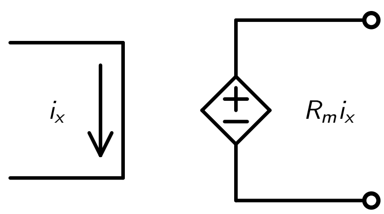


VCCS g_m units: $[A/V]$

(voltage controlled current source)

Above is a VCCS. The units for g_m are $[A/V]$ which is also equivalent to units of Ω^{-1}. g_m is referred to as a transconductance.

CCVS

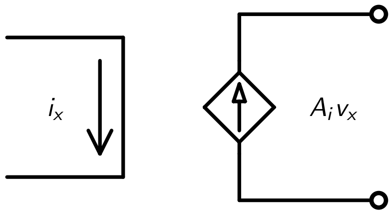


CCVS R_m units: $[V/A]$

(current controlled voltage source)

Above is a CCVS. The units for R_m are $[V/A]$ which is also equivalent to units of Ω. R_m is referred to as a transresistance.

CCCS



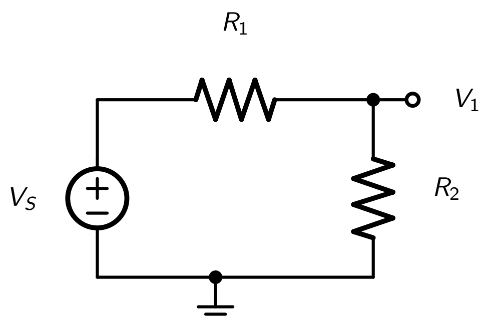
CCCS $\quad A_i$ units: $[A/A]$

(current controlled current source)

Above is a CCCS. The units for A_i are $[A/A]$ which is in effect a unitless quantity but is often shown this way for clarity.

Voltage Divider

It is often useful to find the relationship between 2 voltages in the circuit below (this circuit is called a voltage divider)



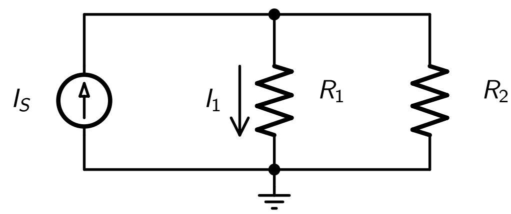
The relationship between V_S and V_1 is found to be

$$V_1 = \frac{R_2}{R_1 + R_2} V_S$$

This same equation can be used for 2 impedances instead of resistors. The extension is straighforward.

Current Divider

It is often useful to find the relationship between 2 currents in the circuit below (this circuit is called a current divider)



The relationship between I_S and I_1 is found to be

$$\boxed{I_1 = \frac{R_2}{R_1 + R_2} I_S}$$

This same equation can be used for 2 impedances instead of resistors.