Thevenin/Norton Equivalent

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Thevenin's theorem is used to replace a multiple element circuit with a single voltage source and resistor while Norton's theorem is used to replace a multiple element circuit with a single current source and resistor.

Specifically, Thevenin's theorem states:

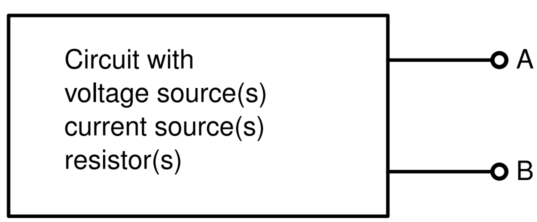
"Any linear electrical network containing only voltage sources, current sources and resistances can be replaced at terminals A–B by an equivalent combination of a voltage source V_{oc} in a series connection with a resistance R_o."

Norton's theorem is almost identical except that the replacement is a single current source I_{sc} in parallel with a resistor. The resistor R_o is the same value for both the Thevenin and Norton equivalent circuits.

The voltage and current sources may be either independent or dependent sources. An independent source is one where the value does not depend on any anything else in the circuit. A dependent source is a source where the value of the source changes based on a voltage or current somewhere in the circuit.

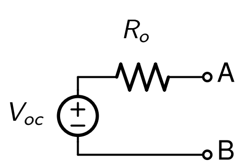
Consider a linear circuit shown below with a port A-B as shown.

Original Circuit

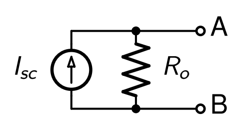


Independent and/or dependent sources

The above circuit can be replaced with either the Thevenin Equivalent or the Norton Equivalent shown below.



Thevenin Equivalent



Norton Equivalent

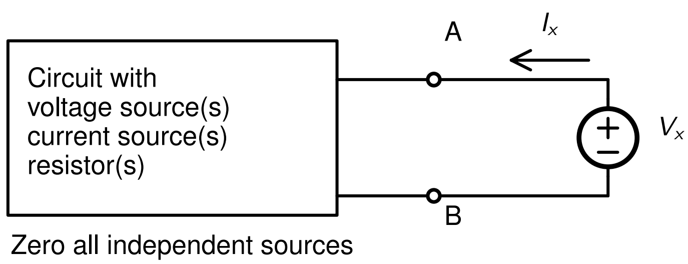
If more circuit elements are attached to port A-B, the Thevenin and Norton equivalent circuits will behave exactly the same as the original circuit.

Since either of the above equivalent circuits can replace the original circuit, one might ask, ”is it better to use one instead of the other?”. The short answer is that either will work however, the more complete answer is that if you are doing hand analysis, you will get a better intuitive understanding of the circuit if you use the one best suited for the load that is attached to the circuit.

For example, if the original circuit is attached to a large resistive load, then the Thevenin equivalent circuit should be used for analysis. If the original circuit is attached to a small resistve load, then the Norton equivalent circuit will give better intuitive understanding of the circuit. In this case, small and large are relative terms and are relative to the Thevenin/Norton output resistance, R_o.

Finding R_o

To find the value of R_o, an arbitrary voltage V_x can be applied to port A-B as shown below



The Thevenin/Norton resistance is then

$$R_o = \frac{V_x}{I_x}$$

When finding I_x, all independent sources should be set to zero. This is equivalent to shorting all independent voltage sources and opening all independent current sources. However, dependent sources remain in the circuit while finding I_x.

Finding V_{oc}

To find the value of V_{oc}, find the open-circuit voltage at port A-B. In other words, disconnect any other elements connected to port A-B and find the voltage from B to A.

Finding I_{sc}

To find the value of I_{sc}, find the short-circuit current at port A-B. In other words, short port A-B and find the short-circuit current flowing through the shorting wire from A to B.

Relationship between V_{oc} and I_{sc}

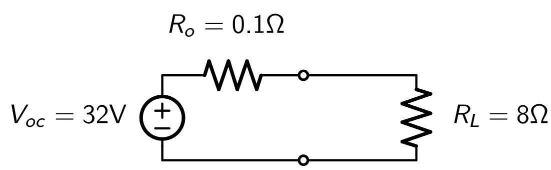
Since the Thevenin equivalent and Norton equivalent are both equal to the original circuit, they too must be equivalent. As a result,

$$I_{sc} = \frac{V_{oc}}{R_o}$$

Example 1

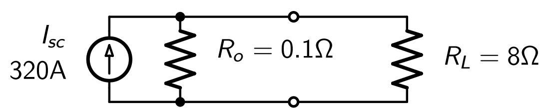
A 32V audio amplifier has a 0.1Ω ohm output resistance and is driving a 8Ω ohm loudspeaker. Show the Thevenin and Norton Equivalent models and explain why one is not intuitively satisfying.

The Thevenin equivalent model is shown below.



Thevenin Equivalent

The Norton equivalent model is shown below.

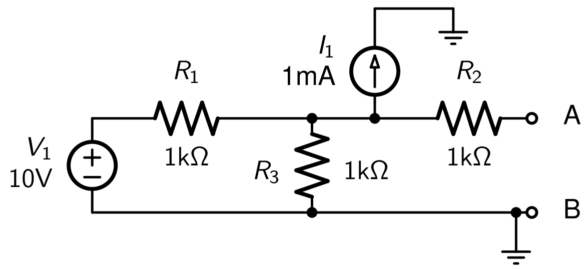


Norton Equivalent

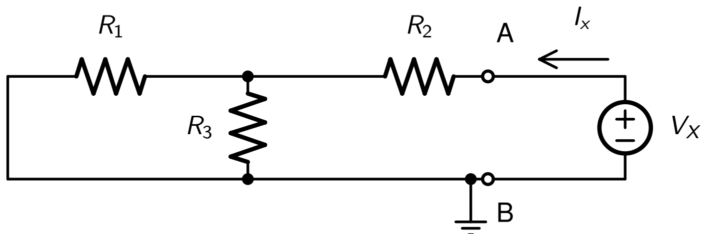
The Norton equivalent model is not intuitively satisfying since it includes a 320A current source and may give the false impression that some wires may have to be able to carry 320A of current. In fact, almost all that current goes through R_o and there is no large amount of 320A current anywhere in the amplifier.

Example 2

Find the Thevenin and Norton Equivalent circuits for the circuit below



To find R_o, we set $V_1 = 0$ and $I_1 = 0$ resulting in the circuit below.



We could do nodal analysis but in this case, it is clear that the resistance looking into port A-B is R_2 in series with R_1 in parallel with R_3 so

$$R_o = R_2 + R_1||R_3 = (1e3) + (1e3)||(1e3) = 1.5\text{k}\Omega$$

To find V_{oc}, we use superposition since the circuit is linear and we have 2 independent sources, V_1 and I_1. Find V_{ocv} due to V_1 with $I_1 = 0$. Next, find V_{oci} due to I_1 with $V_1 = 0$. Finally, add the two results to find V_{oc}.

$$V_{ocv} = (R_3/(R_1 + R_3)) * V_1 = ((1e3)/((1e3) + (1e3))) * (10) = 5\text{V}$$

$$V_{oci} = (R_1||R_3) * (-I_1) = ((1e3)||(1e3)) * (-(1e-3)) = -0.5\text{V}$$

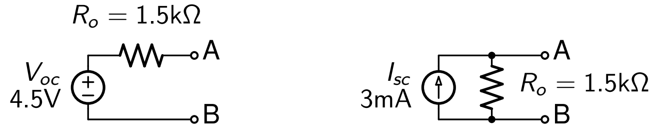
Note that the voltage across R_2 is zero since no current flows across R_2 when the port is open.

$$V_{oc} = V_{ocv} + V_{oci} = (5) + (-0.5) = 4.5\text{V}$$

To find I_{sc} we could calculate the short circuit output current but since we already know V_{oc} and R_o, we can find I_{sc} as

$$I_{sc} = V_{oc}/R_o = (4.5)/(1.5e3) = 3\text{mA}$$

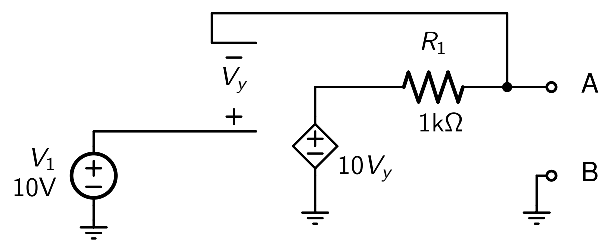
So the equivalent circuits are ...



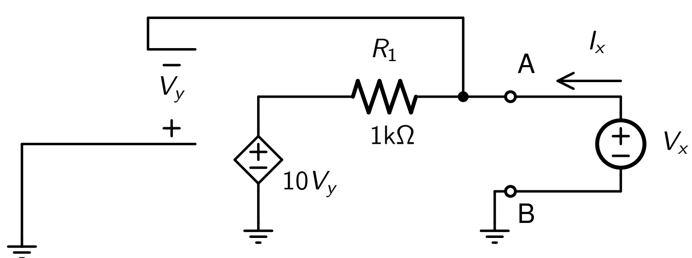
Thevenin Equivalent Norton Equivalent

Example 3

Find the Thevenin and Norton Equivalent circuits for the circuit below



To find R_o, we set $V_1 = 0$ and have the following circuit

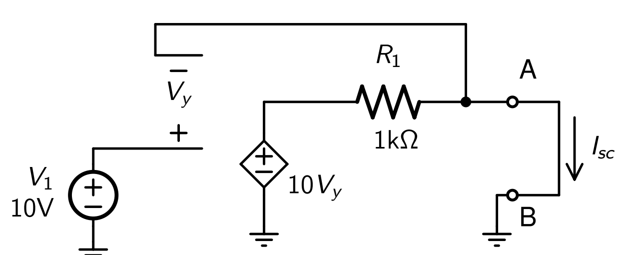


We have $V_y = -V_x$ which leads to

$$I_x = \frac{V_x - (10V_y)}{R_1} = \frac{V_x + 10V_x}{R_1} = \frac{11V_x}{R_1}$$

$$R_o = V_x/I_x = \frac{R_1}{11} = 90.91\Omega$$

To find I_{sc}, we use the following circuit



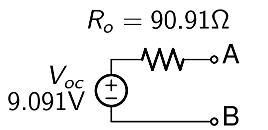
$$V_y = V_1 = (10) = 10\text{V}$$

$$I_{sc} = \frac{10V_y - 0}{R_1} = 0.1\text{A}$$

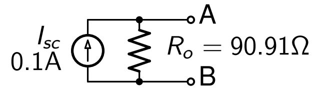
And we can find V_{oc} from

$$V_{oc} = I_{sc} * R_o = (0.1) * (90.91) = 9.091\text{V}$$

So the equivalent circuits are ...



Thevenin Equivalent



Norton Equivalent