

## LOOP GAIN ANALYSIS

FOR ANALYSIS OF FEEDBACK SYSTEMS

THERE ARE GENERALLY 3 THINGS  
TO FIND IN FEEDBACK CIRCUITS

1) CLOSED-LOOP GAIN OR TRANSFER  
FUNCTION

2) PORT IMPEDANCE (INPUT OR  
OUTPUT IMPEDANCE)

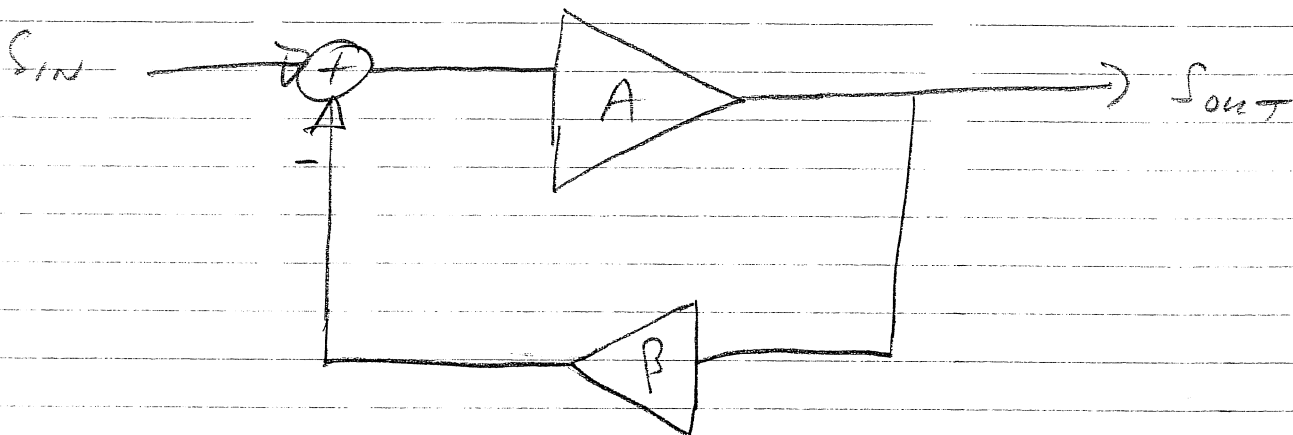
3) STABILITY ANALYSIS

IS THE SYSTEM STABLE

AND HOW STABLE IS IT?

CLOSED-LOOP GAIN

IDEAL CASE



A, B ARE UNIDIRECTIONAL

$S_{IN} \Rightarrow$  INPUT VOLTAGE OR CURRENT

$S_{OUT} \Rightarrow$  OUTPUT VOLTAGE OR CURRENT

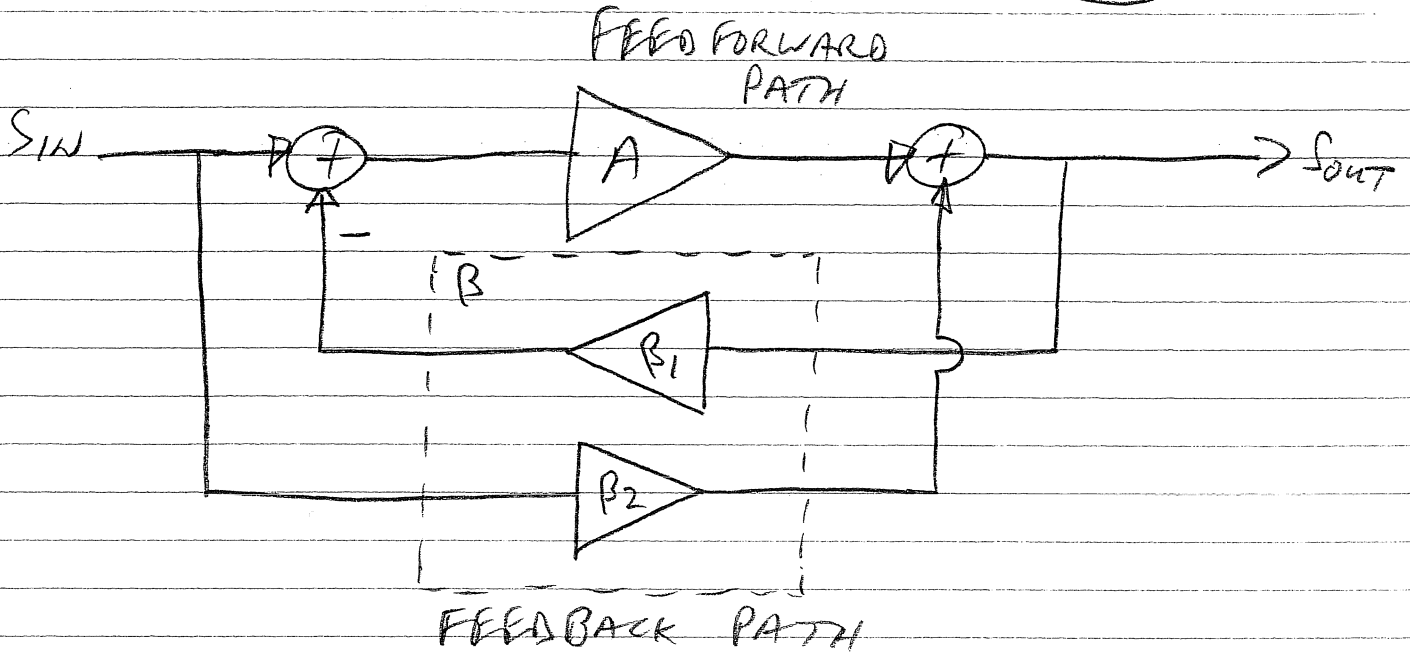
HOWEVER, IN PRACTICE

GENERALLY A IS UNIDIRECTIONAL

WHILE B IS BIDIRECTIONAL

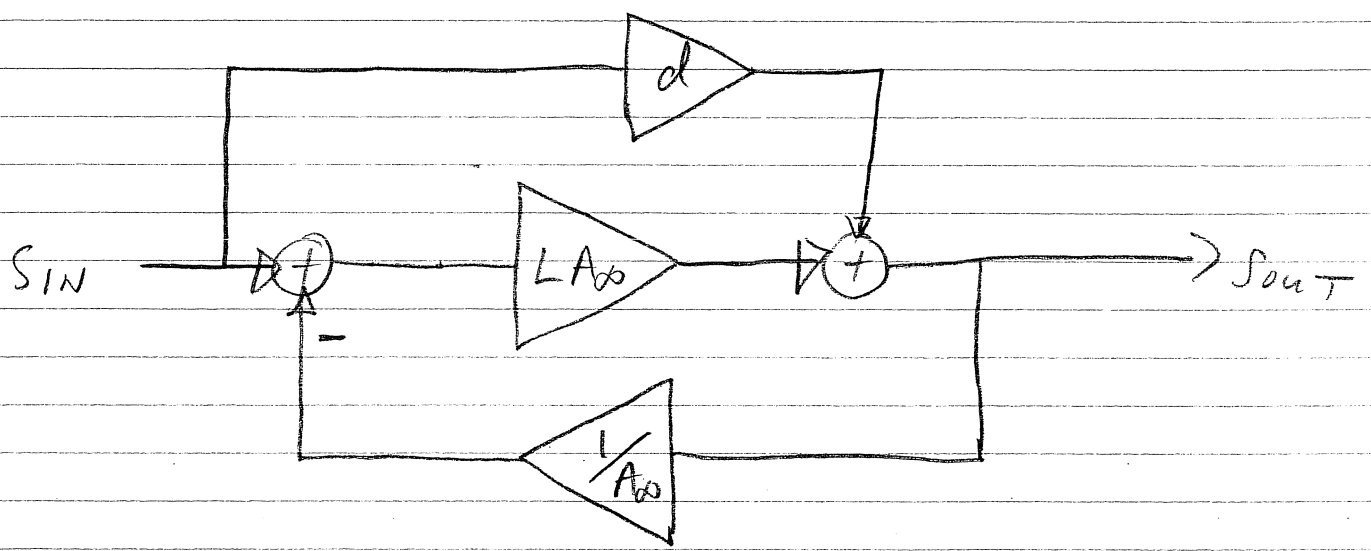
LEADING TO ...

L63



THE ABOVE CAN BE RE-ARRANGED  
AND DEFINE  $A_{\infty} \equiv \frac{1}{\beta_1}$   $d \equiv \beta_2$   $L \equiv A\beta_1$

CALLED ASYMPTOTIC GAIN MODEL



(LGA)

CAN SHOW THE CLOSED-LOOP GAIN,  $A_{CL}$  IS EQUAL TO

$$A_{CL} \equiv \frac{S_{OUT}}{S_{IN}} = A_{\infty} \left( \frac{L}{1+L} \right) + d \left( \frac{1}{1+L} \right)$$

WHERE

$L$  IS LOOP-GAIN WHEN  $S_{IN} = 0$

$$A_{\infty} \equiv \frac{S_{OUT}}{S_{IN}} \Big|_{L \rightarrow \infty}$$

$$d \equiv \frac{S_{OUT}}{S_{IN}} \Big|_{L \rightarrow 0}$$

SO TO FIND  $A_{CL} \Rightarrow$  FIND  $L, A_{\infty}, d$

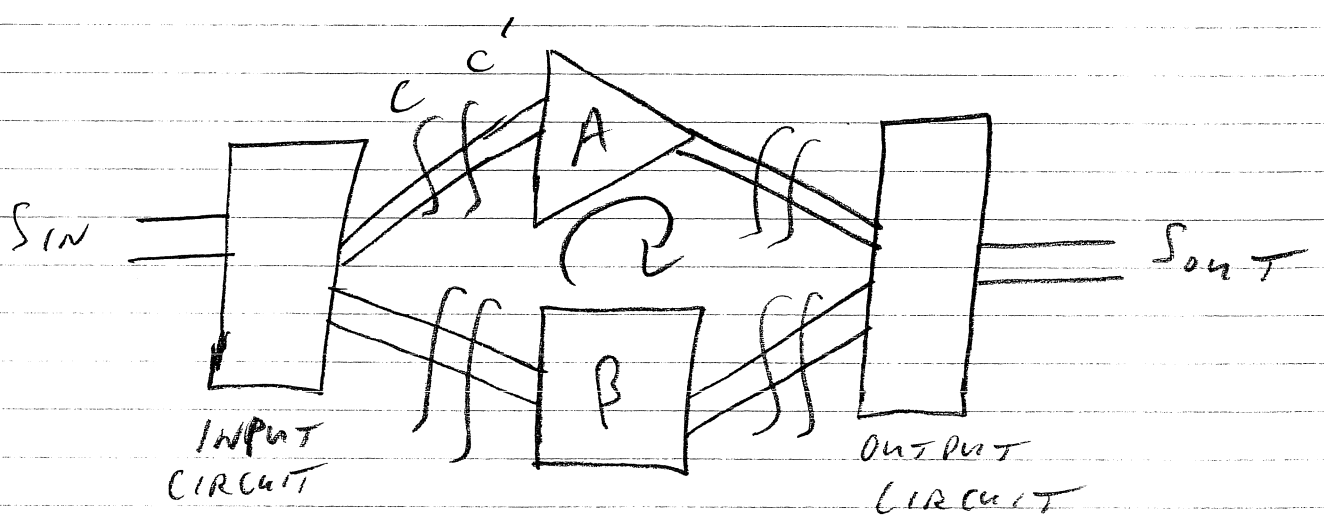
LOOP GAIN: L


=> SET  $S_{IN} = 0$

=> BREAK FEEDBACK LOOP  
FIND IMPEDANCE AT BREAK POINT  
↓ TERMINATE LOOP WITH THIS IMPEDANCE

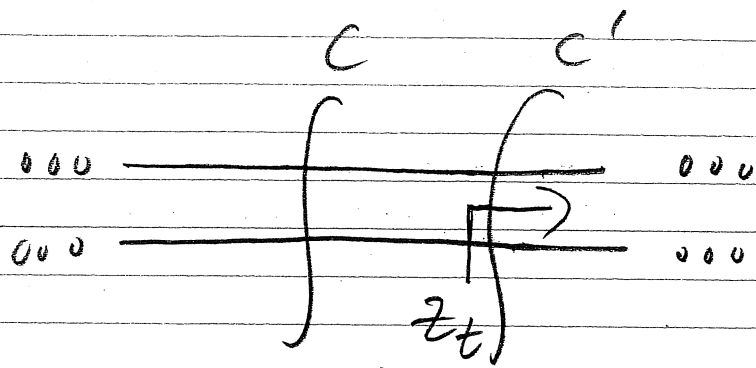
=> INSERT TEST SIGNAL EITHER  $V_t$  OR  $I_t$   
AND FIND  $V_r$  OR  $I_r$

$L \equiv -\frac{V_r}{V_t}$  OR  $L \equiv -\frac{I_r}{I_t}$

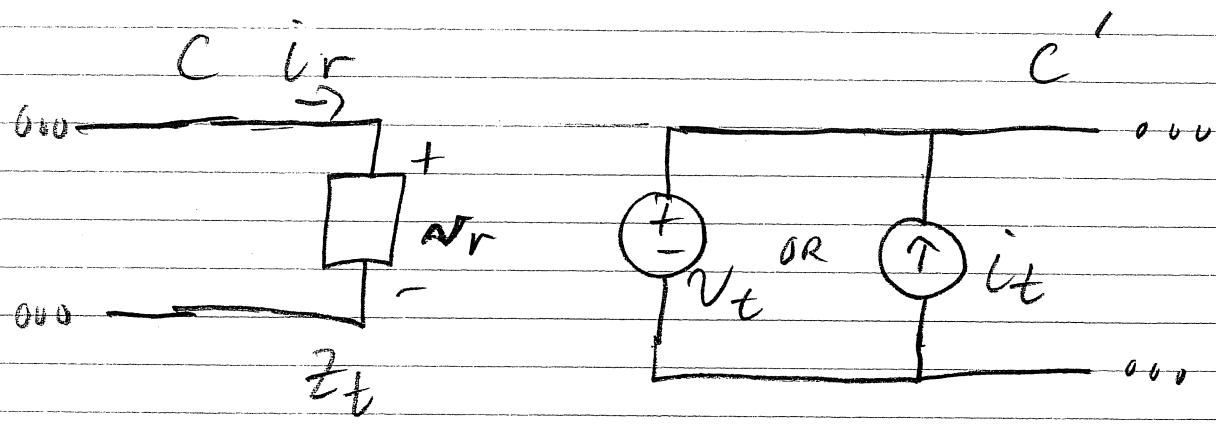


BREAK AT ONE OF 

L66



DIRECTION OF FEEDBACK

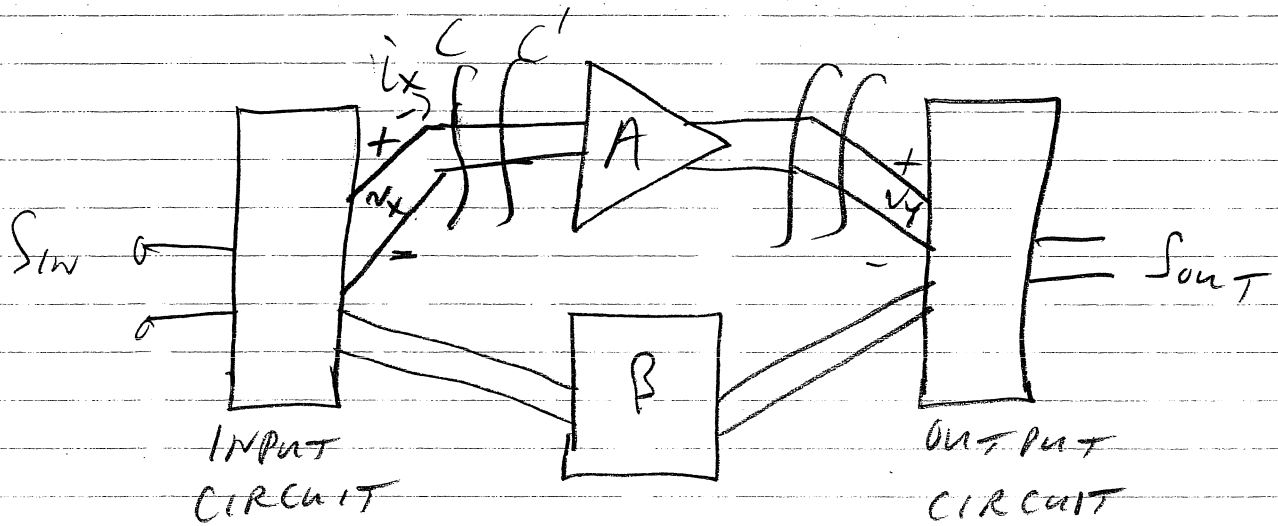


$$L \equiv -\frac{v_r}{v_t} \quad \text{OR} \quad L \equiv -\frac{i_r}{i_t}$$

CAN BREAK LOOP ANYWHERE  
 USUALLY FIND A SPOT WHERE  
 $Z_t$  IS EASILY CALCULATED

$L > 0$  FOR NEGATIVE FEEDBACK

ASYMPTOTIC GAIN,  $A_{\infty}$



BREAK AT ONE OF  $\Sigma\Sigma$

INSERT IDEAL VOLTAGE AMP, GAIN  $A_v$

LET  $A_v \rightarrow \infty \Rightarrow L \rightarrow \infty$

DUE TO FEEDBACK  $v_y$  IS FINITE

SO IF  $A_v \rightarrow \infty \Rightarrow v_x \rightarrow 0$   
&  $i_x \rightarrow 0$

WE HAVE

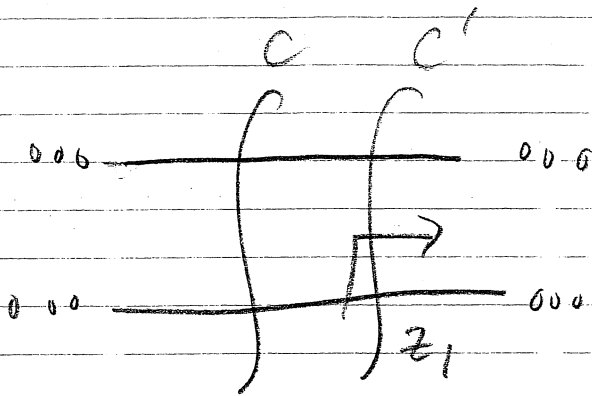
$$A_{\infty} \equiv \frac{S_{out}}{S_{in}} \Big|_{L \rightarrow \infty \Rightarrow v_x \rightarrow 0, i_x \rightarrow 0}$$

USE RESULT THAT  $v_x \rightarrow 0$  &  $i_x \rightarrow 0$

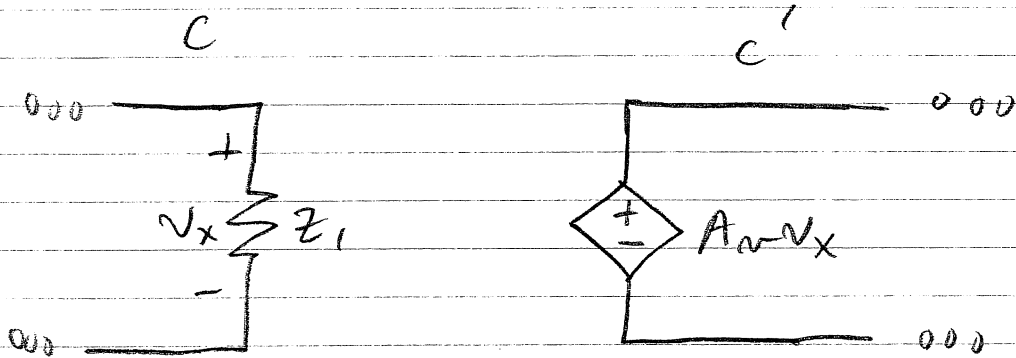
TO FIND  $A_{\infty}$

LG7A

$A_{\infty}$



DIRECTION OF  
FEEDBACK



LET  $A_v \rightarrow \infty$



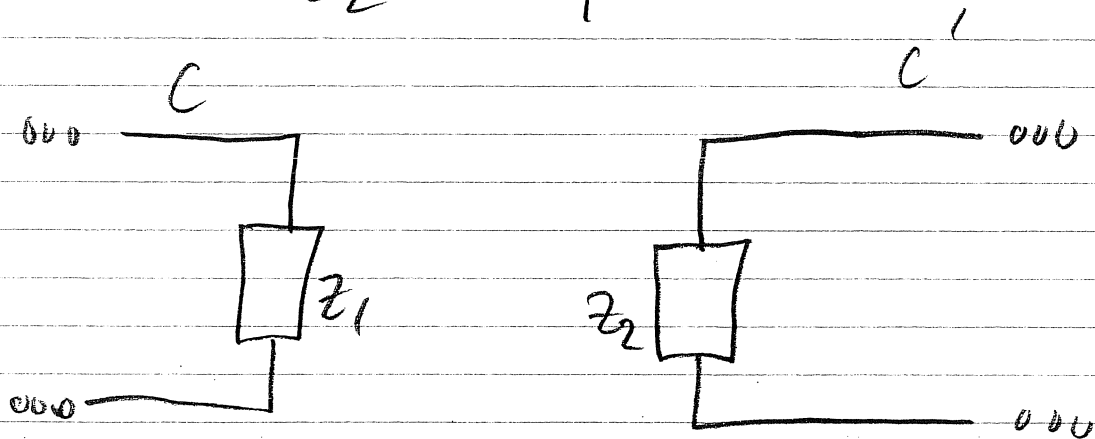
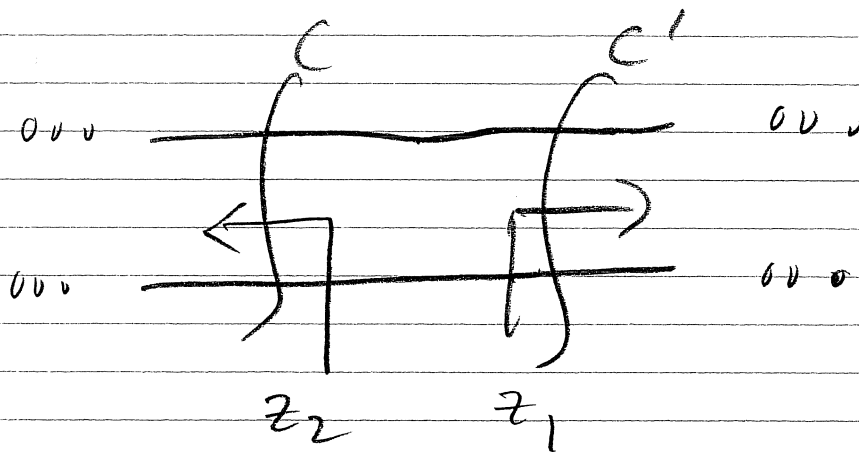
FEED THROUGH TERM,  $d$

(168)

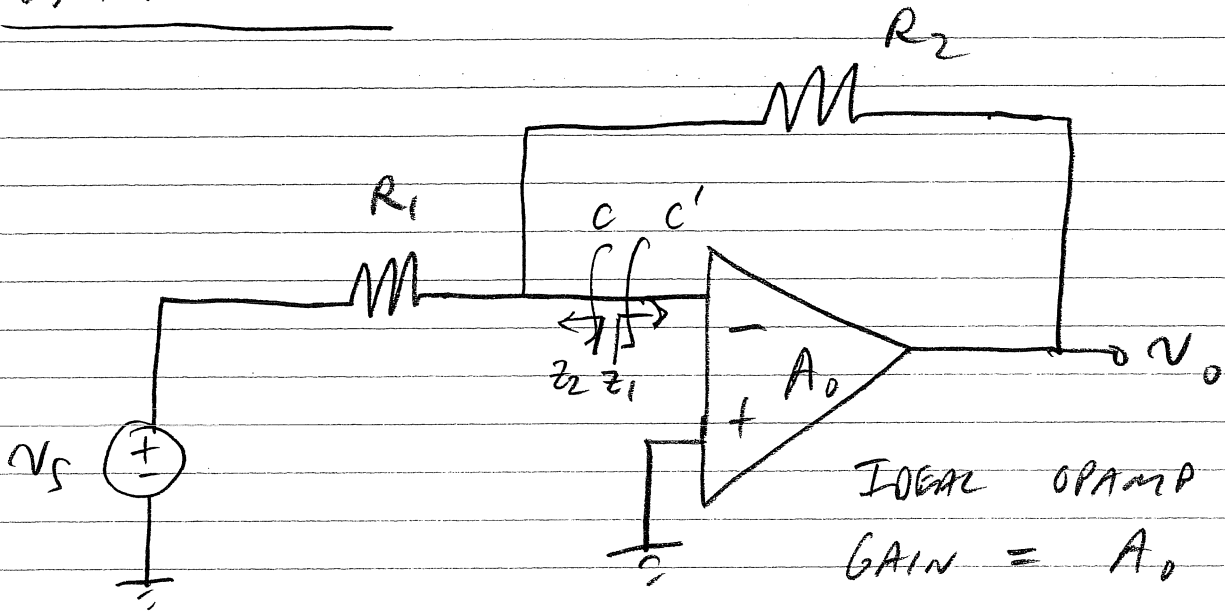
BREAK IN FEED THROUGH PATH  
(SAME AS  $A_{\rho}$  BREAK)

✦ ADD APPROPRIATE TERMINATIONS  
(WHICH CAUSES  $L \rightarrow 0$ )

$$d \equiv \frac{S_{DUT}}{S_{IN}} \Big|_{L \rightarrow 0}$$

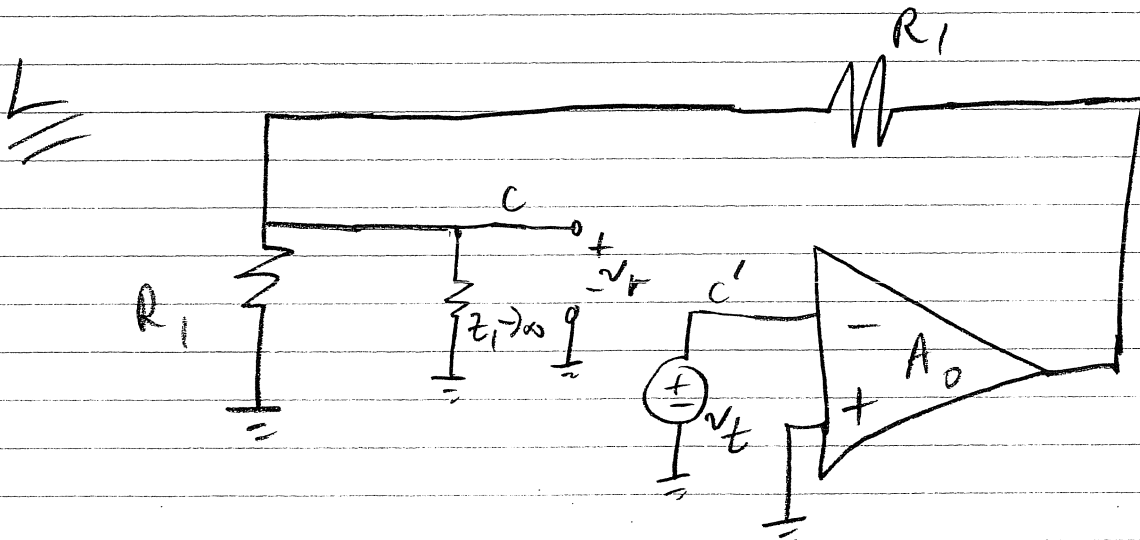


EXAMPLE 1



$$z_1 \rightarrow \infty$$

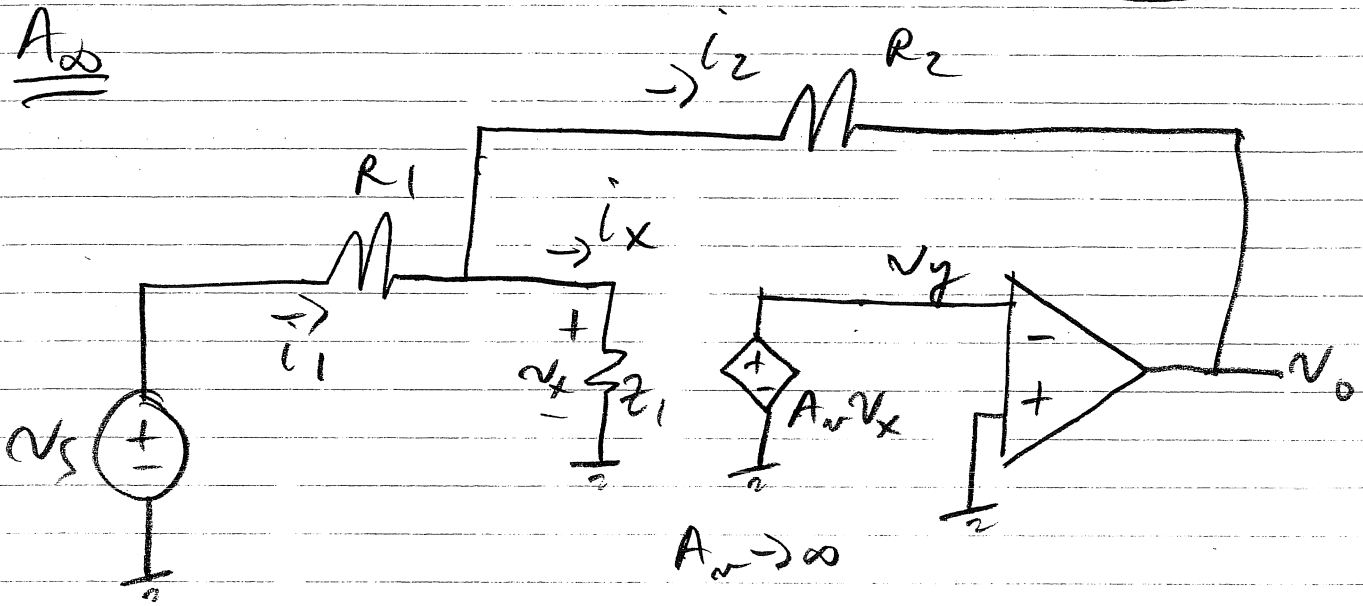
$$z_2 = R_1 \parallel R_2$$



$$L \equiv -\frac{v_r}{v_t} = A_0 \left( \frac{R_1}{R_1 + R_2} \right)$$

LG10

$A_{\infty}$



FEEDBACK  $\Rightarrow v_o$  FINITE  $\Rightarrow v_y$  FINITE

$\Rightarrow v_x = 0 \Rightarrow i_x = 0$

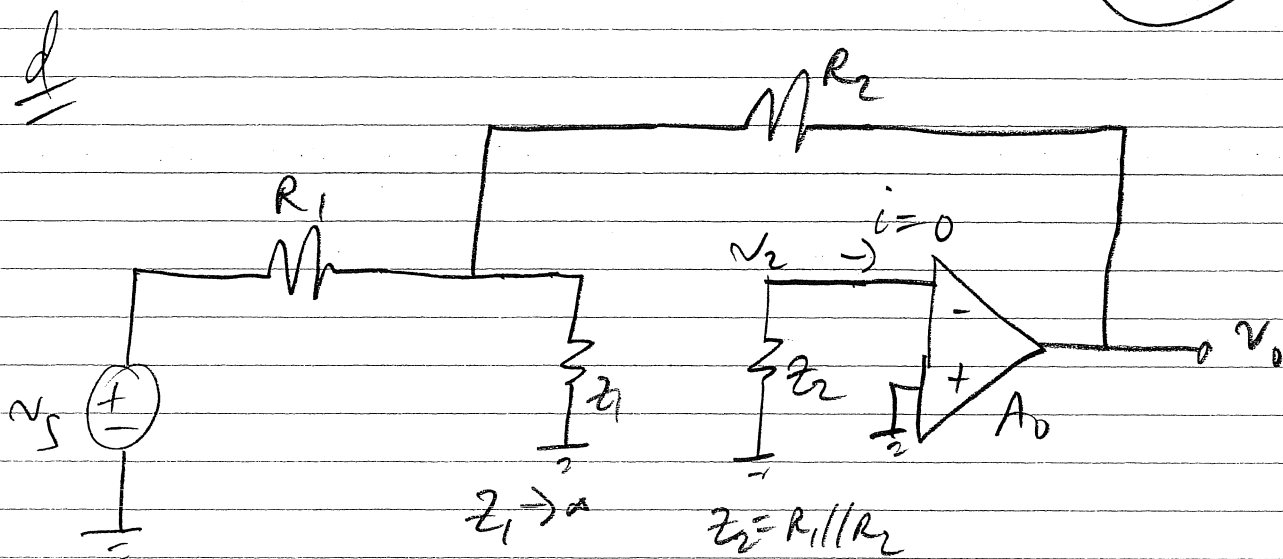
$$i_1 = \frac{v_s}{R_1} \Rightarrow i_2 = i_1$$

$$v_o = 0 - i_2 R_2 = 0 - v_s \left( \frac{R_2}{R_1} \right)$$

$$\frac{v_o}{v_s} = - \frac{R_2}{R_1} \quad A_{\infty} \equiv \frac{v_o}{v_s} \Big|_{A_v \rightarrow \infty}$$

$$A_{\infty} = - \frac{R_2}{R_1}$$

LG11



SINCE  $i = 0 \Rightarrow v_2 = 0 \Rightarrow v_o = 0$

$$d \equiv \frac{v_o}{v_s} \Big|_{L \rightarrow 0} = 0$$

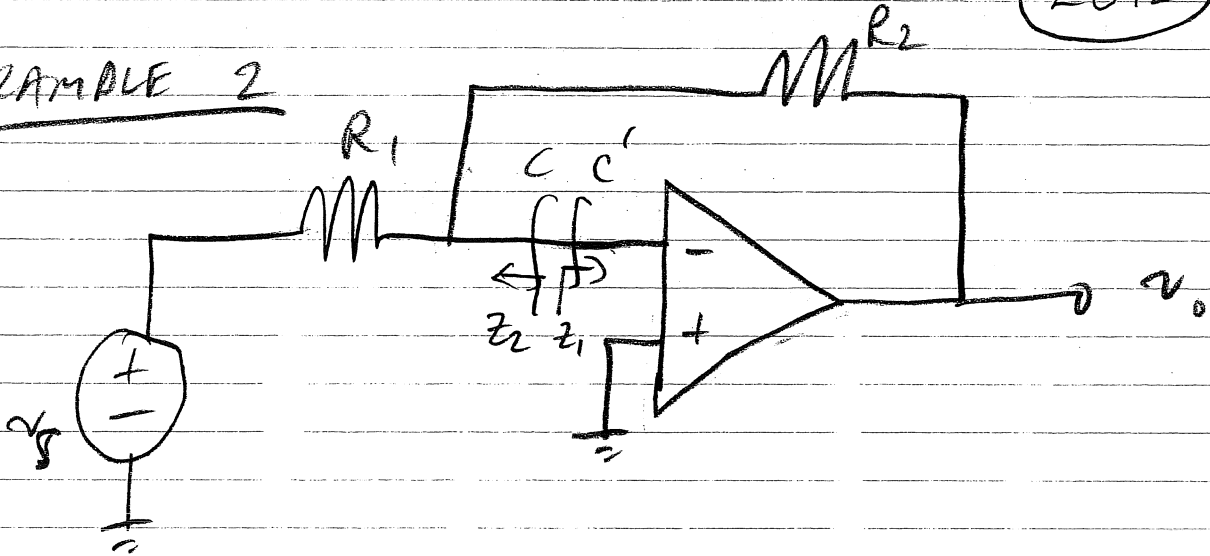
$$A_{CL} = A_o \left( \frac{L}{1+L} \right) + d \left( \frac{1}{1+L} \right)$$

$$A_{CL} = -\left( \frac{R_2}{R_1} \right) \left( \frac{A_o \left( \frac{R_1}{R_1 + R_2} \right)}{1 + A_o \left( \frac{R_1}{R_1 + R_2} \right)} \right)$$

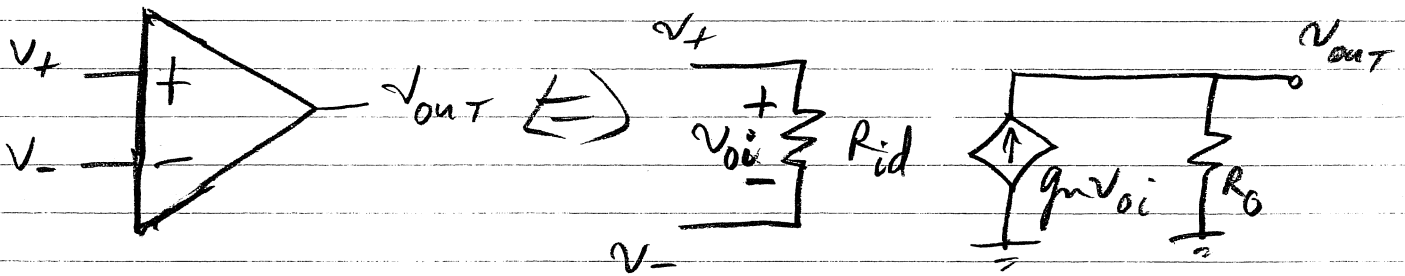
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EXAMPLE 2



WITH OPAMP MODEL

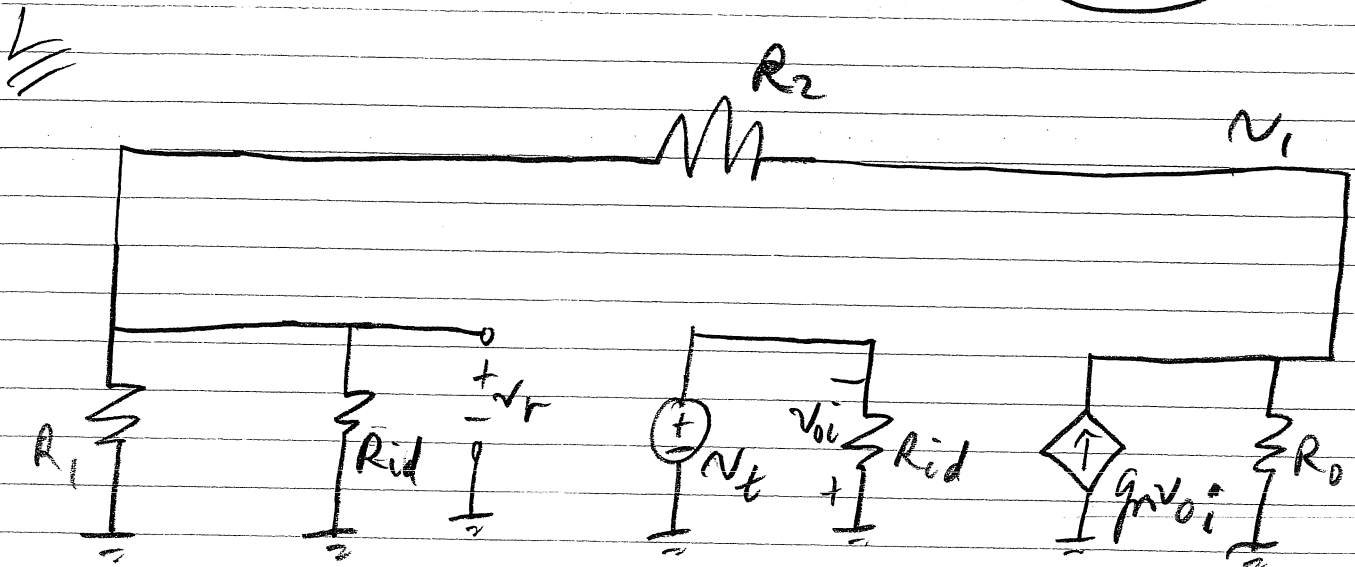


So

$$z_1 = R_{id}$$

$$z_2 = R_1 \parallel (R_2 + R_o)$$

L613



$$V_{oi} = -V_t$$

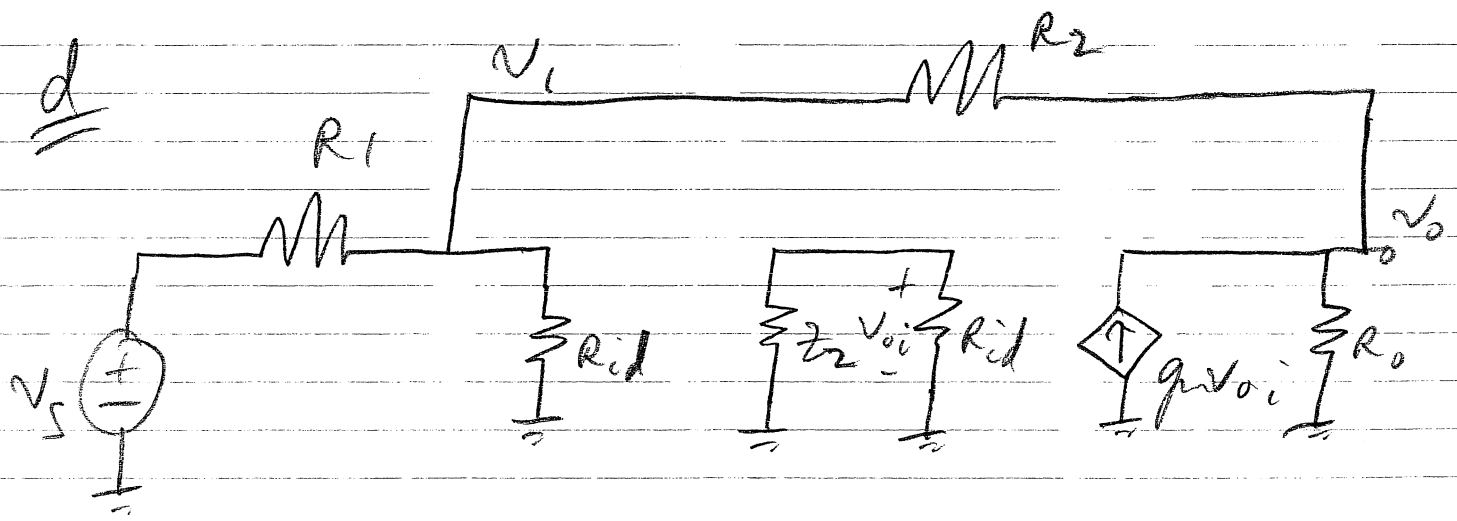
$$V_1 = g_m V_{oi} [R_0 \parallel [R_2 + (R_1 \parallel R_{id})]]$$

$$V_r = \frac{R_1 \parallel R_{id}}{(R_1 \parallel R_{id}) + R_2} V_1$$

$$L \equiv -\frac{V_r}{V_t} = \frac{g_m R_0 (R_1 \parallel R_{id})}{R_0 + R_2 + (R_1 \parallel R_{id})}$$

NOTE  $L > 0$  AS EXPECTED

A<sub>d</sub> AS BEFORE  $A_d = -\left(\frac{R_2}{R_1}\right)$



$$v_{oi} = 0 \Rightarrow g_m v_{oi} = 0$$

$$v_i = \frac{(R_{id} \parallel (R_2 + R_o))}{[R_{id} \parallel (R_2 + R_o)] + R_1} v_s$$

$$v_o = \left(\frac{R_o}{R_o + R_2}\right) v_i$$

$$d \equiv \frac{v_o}{v_s} \Big|_{L \rightarrow 0} = \frac{[R_{id} \parallel (R_2 + R_o)] (R_o)}{([R_{id} \parallel (R_2 + R_o)] + R_1) (R_o + R_2)}$$

$$A_{CL} = A_d \left(\frac{L}{1+L}\right) + d \left(\frac{1}{1+L}\right)$$

(LG15)

IF  $R_2 = R_1 = 1k$

$$R_{id} = 1k$$

$$g_m = 20 \text{ mA/V}$$

$$R_o = 100k$$

$$L = 9.85$$

$$A_{\infty} = -1 \quad d = 0.493$$

$$A_{CL} = A_{\infty} \left( \frac{L}{1+L} \right) + d \left( \frac{1}{1+L} \right)$$

$$= -0.862$$

NOTE

IF  $d$  IGNORED  $A_{CL} \approx A_{\infty} \left( \frac{L}{1+L} \right) = -0.908$

$\approx 5\%$  ERROR

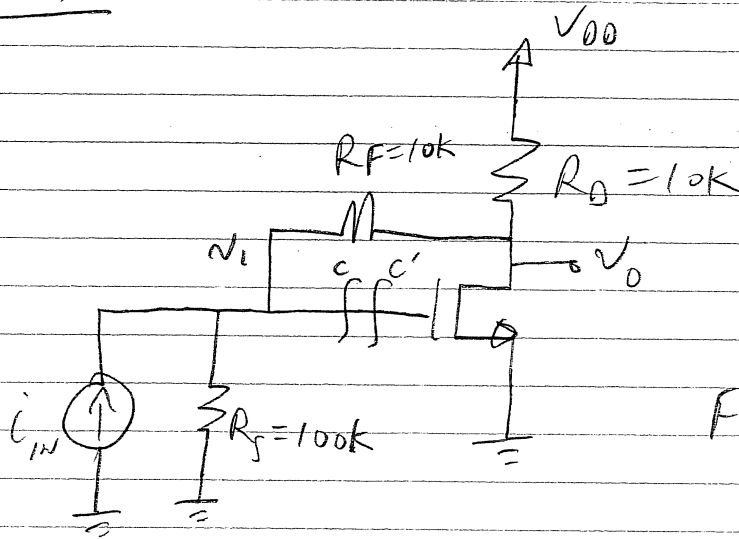
IN MANY CASES CAN IGNORE  $d$

BUT IN SOME CASES  $d$  VERY IMPORTANT



Ex 3

LG16



$$g_m = 1 \text{ mA/V}$$

$$r_o = 50 \text{ k}$$

$$\text{FIND } A_{CL} \equiv \frac{v_o}{i_{IN}}$$

$$L = g_m [R_o \parallel r_o \parallel (R_F + R_S)] \left( \frac{R_S}{R_S + R_F} \right)$$
$$= 7.04 \text{ V/V}$$

$$A_{\infty} = -R_F = -10 \text{ k}$$

$$d \equiv \left. \frac{v_o}{i_{IN}} \right|_{L \rightarrow 0} \Rightarrow v_i = i_{IN} (R_S \parallel [R_F + (R_o \parallel r_o)])$$
$$v_i = i_{IN} (15.5 \text{ k})$$

$$v_o = \left( \frac{R_o \parallel r_o}{(R_o \parallel r_o) + R_F} \right) v_i = 0.4545 v_i$$

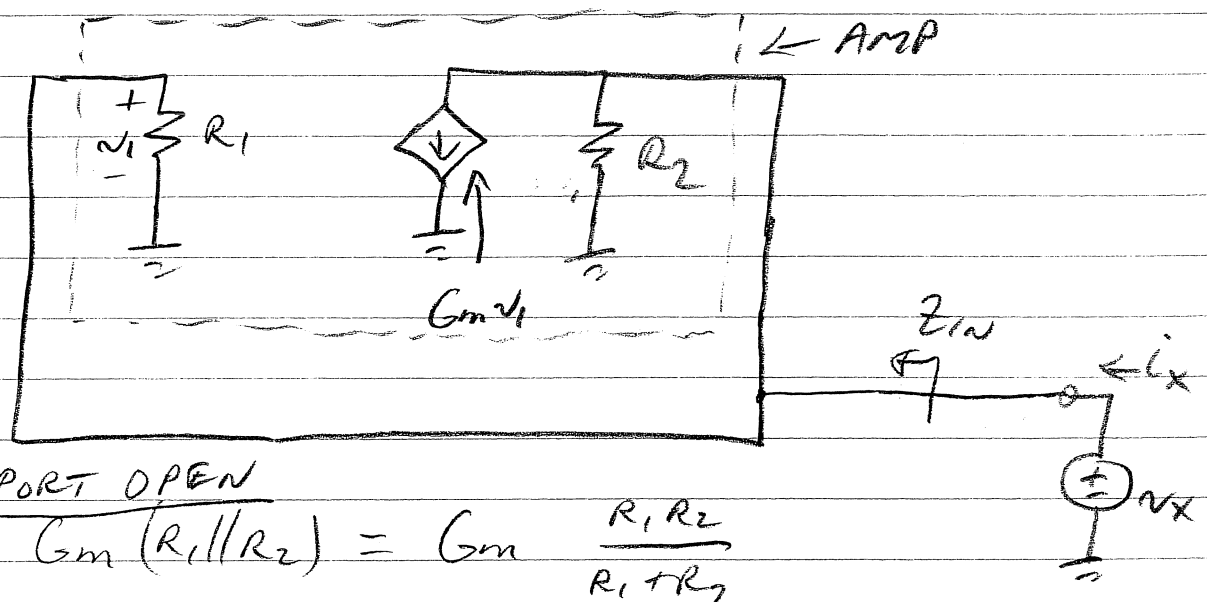
$$d = \frac{v_o}{i_{IN}} = 7.05 \text{ k}$$

$$A_{CL} = A_{\infty} \left( \frac{L}{1+L} \right) + d \left( \frac{1}{1+L} \right) = (-10) \left( \frac{7.04}{8.04} \right) + 7.05 \left( \frac{1}{8.04} \right)$$

$$A_{CL} = -7.88 \text{ V/A}$$

FINDING CLOSED-LOOP INPUT/OUTPUT IMPEDANCE

CONSIDER PORT IMPEDANCE (SHUNT)



WITH PORT OPEN

$$L_0 = G_m (R_1 || R_2) = G_m \frac{R_1 R_2}{R_1 + R_2}$$

$$G_m = \left( \frac{R_1 + R_2}{R_1 R_2} \right) L_0$$

$$Z_{in} \equiv \frac{v_x}{i_x}$$

$$i_x = \frac{v_x}{R_1} + \frac{v_x}{R_2} + G_m v_x$$

$$= v_x \left( \frac{1}{R_1} + \frac{1}{R_2} + \left( \frac{R_1 + R_2}{R_1 R_2} \right) L_0 \right)$$

$$= v_x \left( \frac{R_1 + R_2 + (R_1 + R_2) L_0}{R_1 R_2} \right)$$

LG7A

$$i_x = \left( \frac{R_1 + R_2}{R_1 R_2} \right) (1 + L_0) v_x$$

$$Z_{IN} \equiv \frac{v_x}{i_x} = \frac{R_1 R_2}{(R_1 + R_2) (1 + L_0)} = \frac{R_1 \parallel R_2}{(1 + L_0)}$$

DEFINE  $Z_{IN}^0 = R_1 \parallel R_2$

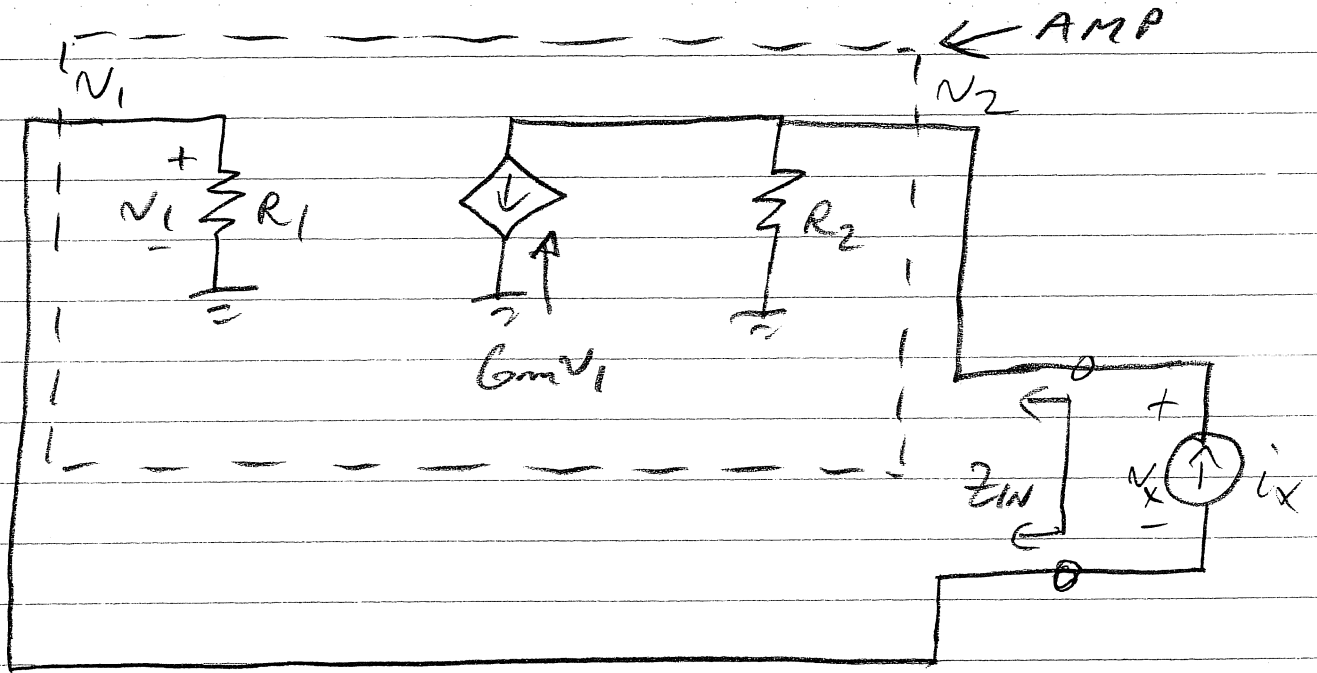
IMPEDANCE LOOKING INTO PORT WHEN  
FEEDBACK LOOP BROKEN INSIDE AMP  
( $L=0$ )

$$Z_{IN} = \frac{Z_{IN}^0}{(1 + L_0)}$$

$L_0$  IS LOOP GAIN WITH PORT OPEN

LG17B

CONSIDER PORT IMPEDANCE (SERIES)



WITH PORT SHORTED  $\Rightarrow L_S = G_m (R_1 \parallel R_2)$

$$G_m = \left( \frac{R_1 + R_2}{R_1 R_2} \right) L_S$$

$$Z_{IN} \equiv \frac{V_X}{i_X}$$

$$V_1 = -i_X R_1$$

$$V_2 = i_X R_2 - G_m V_1 R_2$$

$$= i_X R_2 + i_X G_m R_1 R_2$$

$$= i_X R_2 + i_X (R_1 + R_2) L_S$$

$$V_x = V_2 - V_1$$

$$V_x = i_x (R_2 + (R_1 + R_2)L_5 + R_1)$$

$$Z_{IN} = \frac{V_x}{i_x} = R_1 + R_2 + (R_1 + R_2)L_5$$

$$Z_{IN} = (R_1 + R_2)(1 + L_5)$$

DEFINE  $Z_{IN}^0 = (R_1 + R_2)$

IMPEDANCE LOOKING INTO PORT WHEN  
FEEDBACK LOOP IS BROKEN INSIDE AMP  
( $L=0$ )

$$Z_{IN} = Z_{IN}^0 (1 + L_5)$$

$L_5$  IS LOOP GAIN WITH PORT SHORTED

LG7D

# FINDING CLOSED-LOOP INPUT/OUTPUT IMPEDANCE IN GENERAL

FOR A PORT  $Z_P$

$$Z_P = Z_{P^0} \left[ \frac{1 + L_S}{1 + L_O} \right]$$

$L_S \Rightarrow$  LOOP GAIN WITH PORT SHORTED

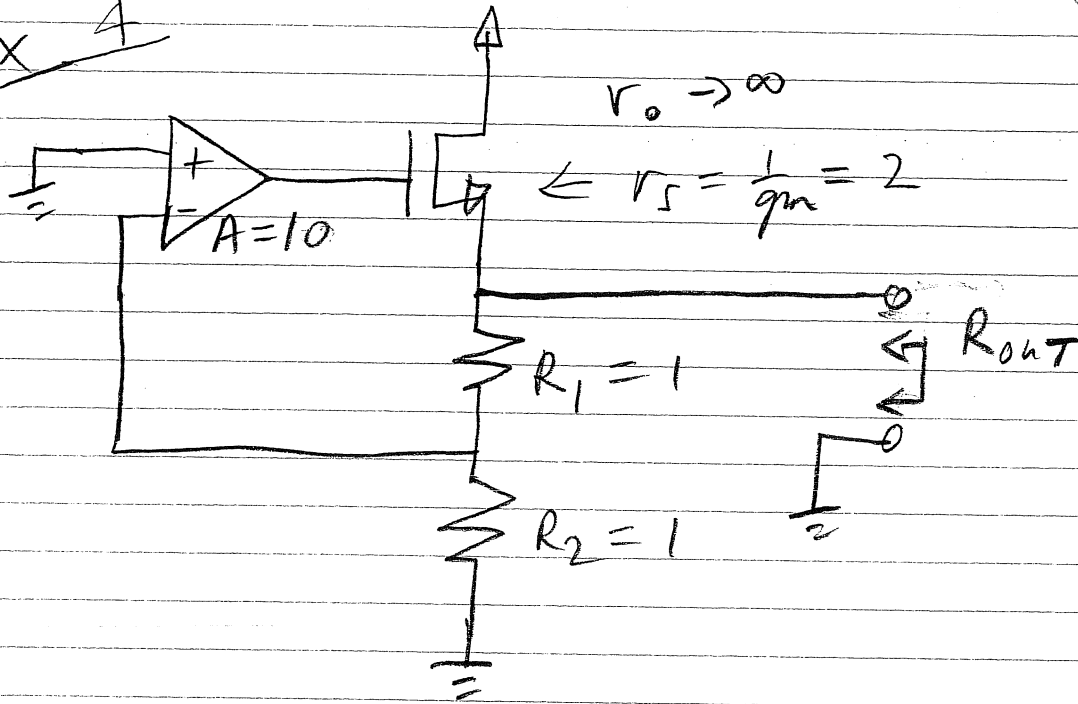
$L_O \Rightarrow$  LOOP GAIN WITH PORT OPENED

$Z_{P^0} \Rightarrow$  PORT IMPEDANCE WITH LOOP  
BROKEN INSIDE AMP

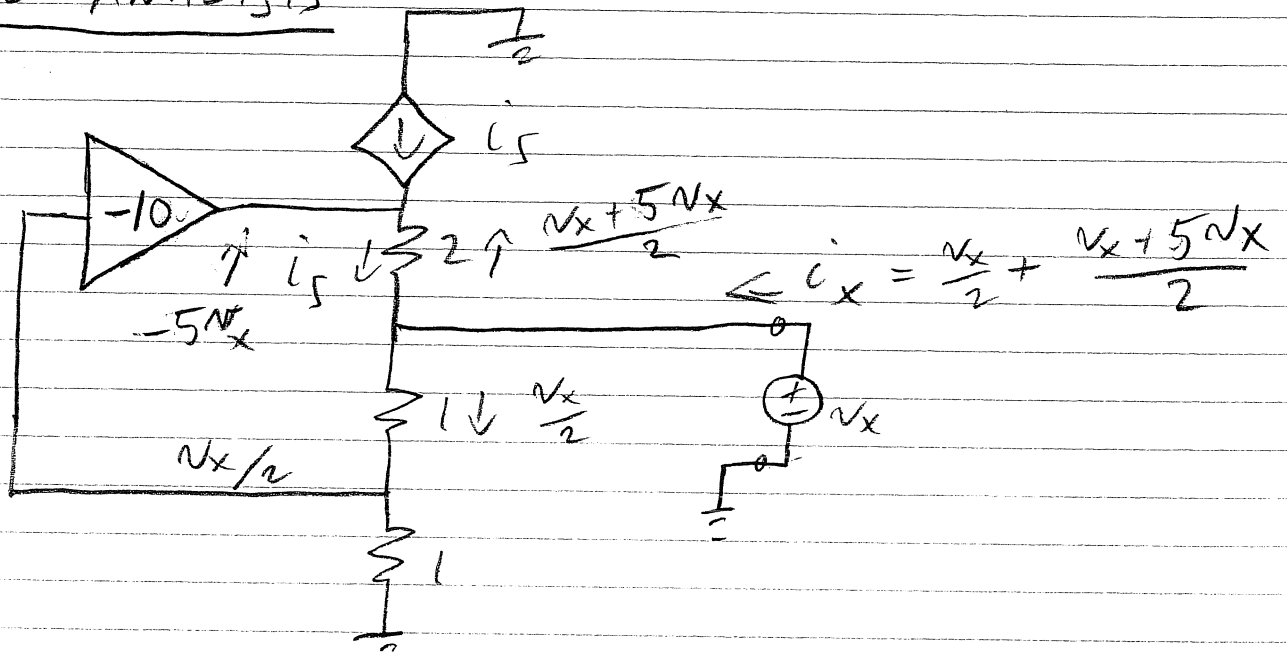
CAN USUALLY MOVE PORT CLOSER  
TO FEEDBACK SUCH THAT ONE  
OF  $L_S$  OR  $L_O$  IS ZERO

AND OTHER IS  $L$  (THE LOOP GAIN)

EX 4



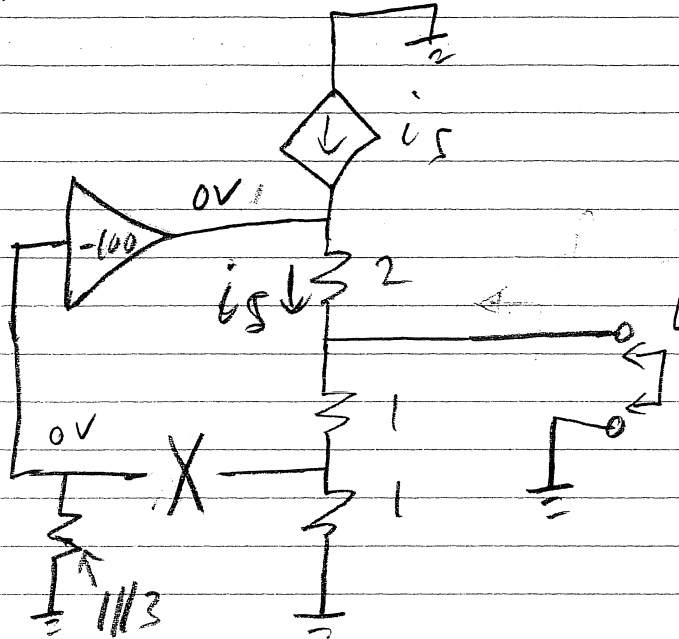
FULL ANALYSIS



$$R_{out} \equiv \frac{v_x}{i_x} = \left( \frac{1}{2} + \frac{1}{2} + \frac{5}{2} \right)^{-1} = \frac{2}{7}$$

LG19

### LG ANALYSIS



$$L = \left(\frac{1}{2}\right)\left(\frac{1}{2}\right)(10) = \frac{5}{2}$$

$$R_1(L=0) = 2 \parallel 2 = 1 \Omega$$

$$L_S = 0$$

$$L_O = L$$

$$R_{out} = R_1(L=0) \left[ \frac{1+L_S}{1+L_O} \right] = (1) \left( \frac{1+0}{1+\frac{5}{2}} \right)$$

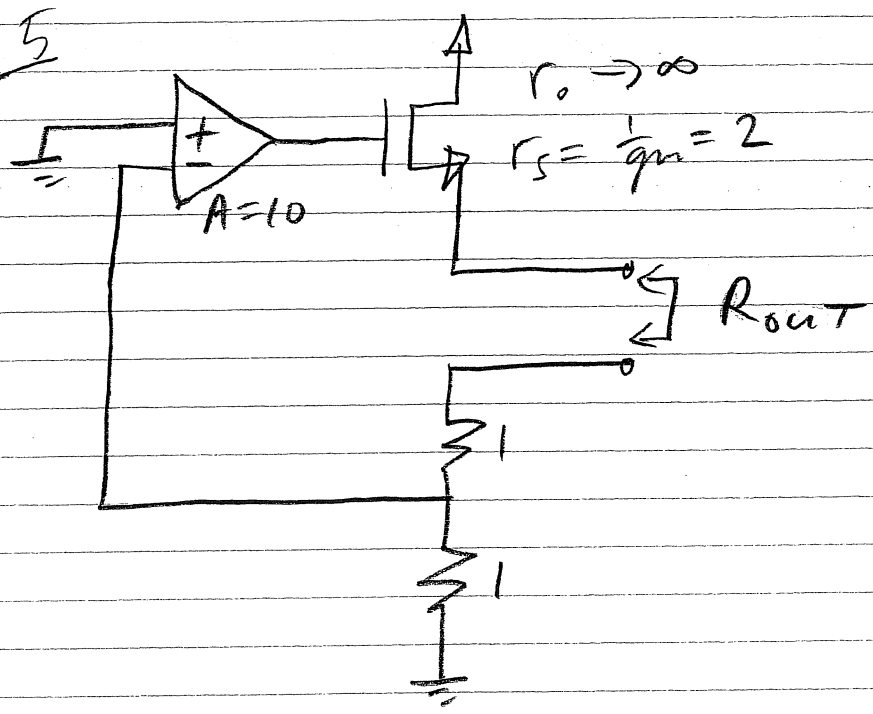
$$R_{out} = \frac{2}{7} \checkmark$$

7/11

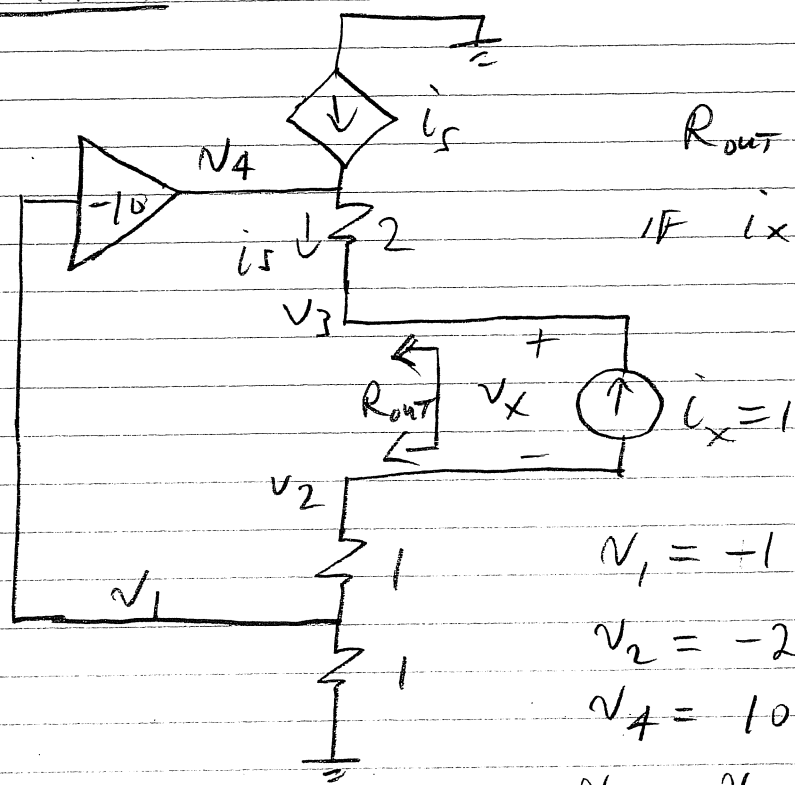


LG20

EX 5



FULL-ANALYSIS



$$R_{out} = \frac{v_x}{i_x}$$

$$\text{If } i_x = 1 \Rightarrow R_{out} = v_x$$

$$v_1 = +1 \text{ V}$$

$$v_2 = -2 \text{ V}$$

$$v_4 = 10 \text{ V}$$

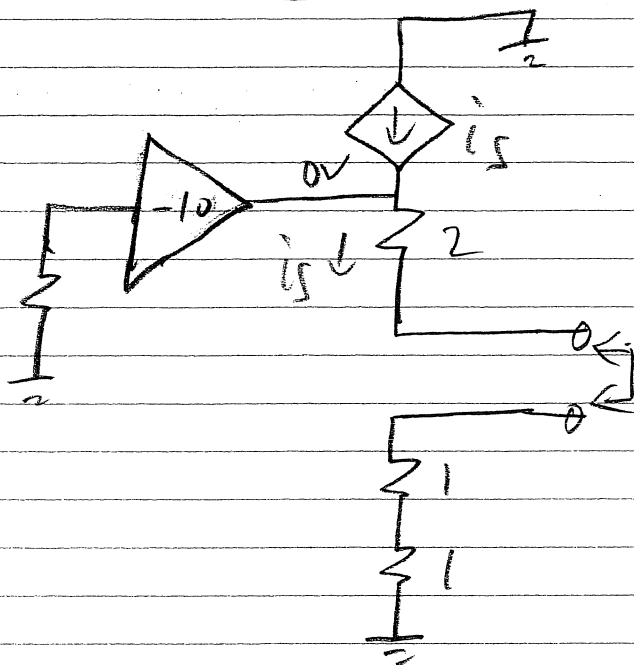
$$v_3 = v_4 + 2i_s = 10 + 2 = 12 \text{ V}$$

$$v_x = v_3 - v_2 = 14 \text{ V}$$

$$R_{out} = \underline{\underline{14 \Omega}}$$

(L621)

# LG ANALYSIS



$$L = \frac{5}{2}$$
$$L_S = L = \frac{5}{2}$$
$$L_0 = 0$$

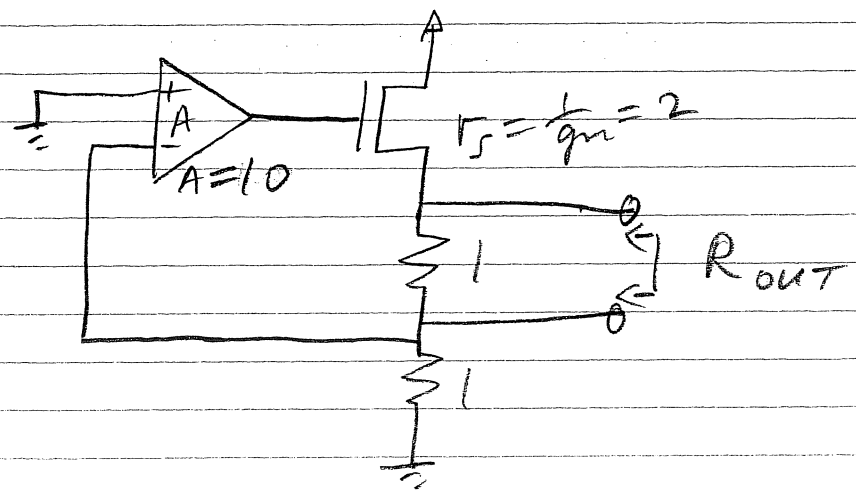
$$R_1(L=0) = 2 + 2 = 4$$

$$R_{out} = R_1(L=0) \left[ \frac{1+L_S}{1+L_0} \right] = (4) \left( \frac{1+\frac{5}{2}}{1+0} \right)$$

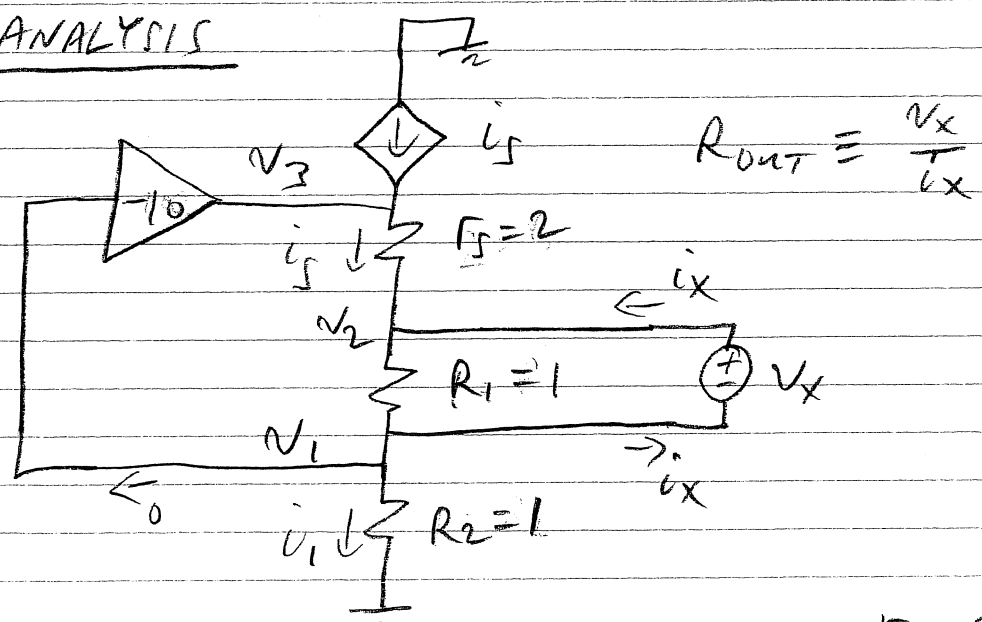
$$= 14 \Omega \checkmark$$

EX 6

A CASE WHERE NEITHER  $L_S$  OR  $L_O$  EQUALS 0



FULL ANALYSIS



$i_s = i_1 \Rightarrow v_3 - v_2 = 2v_1$       SINCE  $r_s = 2R_2$

$v_1 + v_x + 2v_1 = -10v_1 \Rightarrow v_x = -13v_1$       (1)

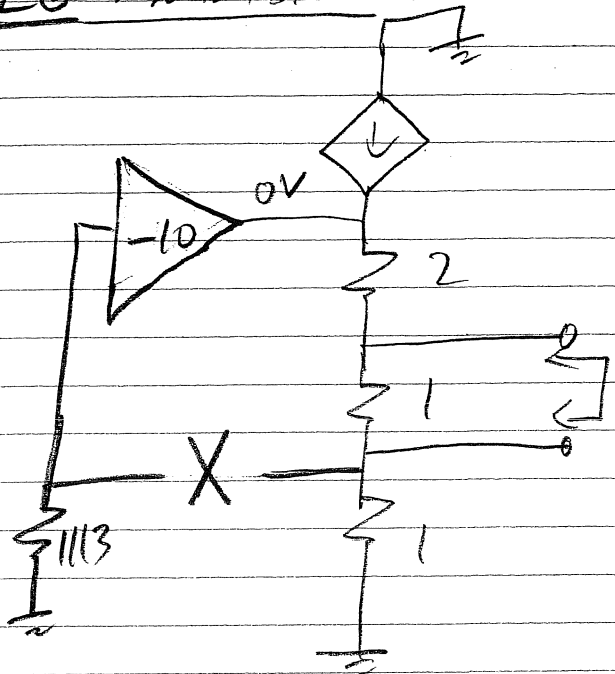
$\frac{v_x}{R_1} = i_x + \frac{v_1}{R_2} \Rightarrow v_x = i_x + v_1 \Rightarrow v_1 = v_x - i_x$       (2)

PUT (2) INTO (1)  $\Rightarrow 14v_x = 13i_x \Rightarrow R_{out} = \frac{v_x}{i_x}$

$R_{out} = \frac{13}{14}$

LG23

LG ANALYSIS



$$R_1(L=0) = 1 \parallel 3 = \frac{3}{4}$$

$$L_0 = L = \frac{5}{2}$$

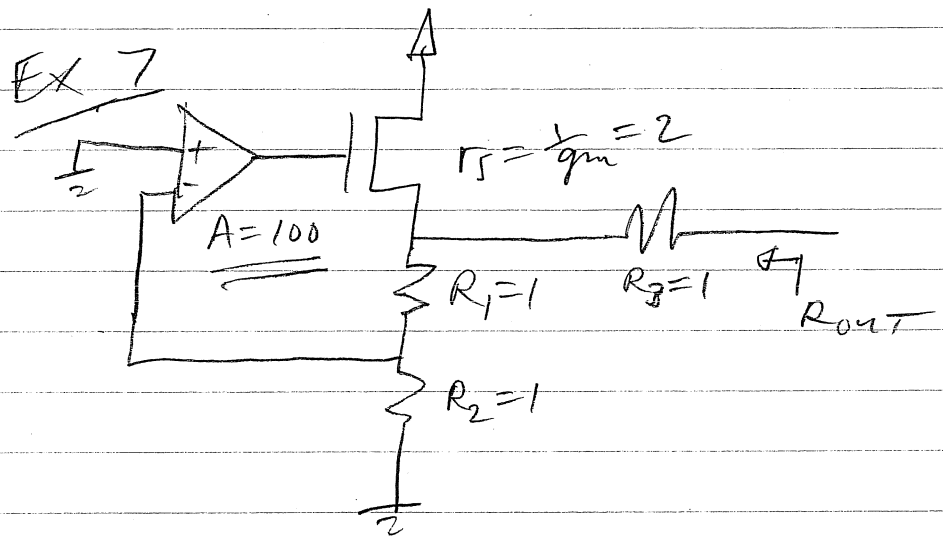
$$L_5 = \left(\frac{1}{3}\right)(10) = \frac{10}{3}$$

$$R_{out} = R_1(L=0) \left[ \frac{1+L_5}{1+L_0} \right] = \left(\frac{3}{4}\right) \left( \frac{1+\frac{10}{3}}{1+\frac{5}{2}} \right) = 13$$

$$R_{out} = \left(\frac{3}{4}\right) \left( \frac{\left(\frac{13}{3}\right)}{\left(\frac{7}{2}\right)} \right) = \frac{13}{14} \Omega \quad \checkmark$$

LG24

WHAT IF PORT IMPEDANCE TO BE FOUND WHERE EXTRA RESISTANCE NOT IN FEEDBACK LOOP EXISTS?



CAN USE LG ANALYSIS DIRECTLY

$$R_i(L=0) = R_3 + [r_s \parallel (R_1 + R_2)] = 2 \Omega$$

$$L_0 = L = 25 \quad L_s = \left( \frac{R_3 \parallel (R_1 + R_2)}{R_3 \parallel (R_1 + R_2) + r_s} \right) \left( \frac{R_2}{R_2 + R_1} \right) A$$

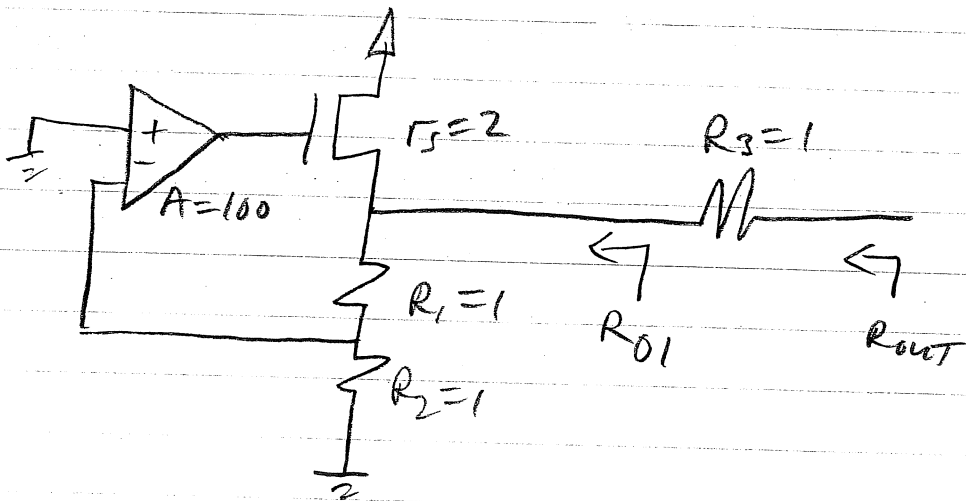
$$= 12.5$$

$$R_{out} = R_i(L=0) \left[ \frac{1 + L_s}{1 + L_0} \right] = 2 \left[ \frac{1 + 12.5}{1 + 25} \right]$$

$$= 1.0385$$

HOWEVER EASIER TO FIND IMPEDANCE  
CLOSER TO FEEDBACK LOOP & THEN  
 INCLUDE  $R_3$

LG25



$$R_{01} = 1 \left[ \frac{1+L_5}{1+L_0} \right] = 1 \left[ \frac{1+0}{1+25} \right] = \frac{1}{26}$$

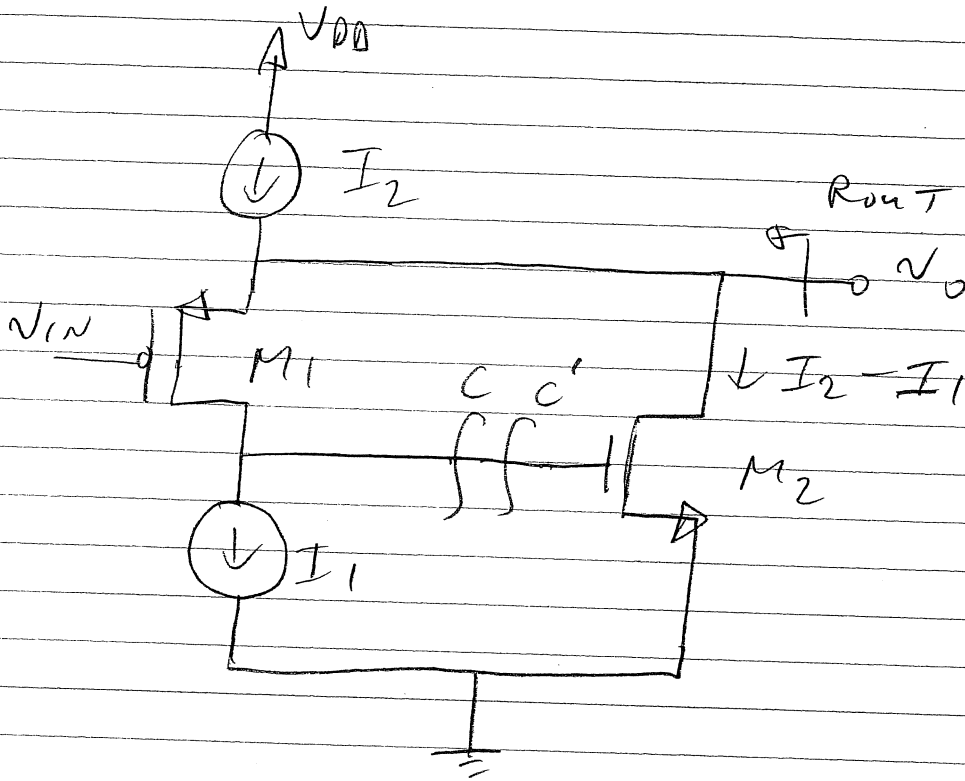
$$R_{out} = R_3 + R_{01} = 1.0385 \quad \checkmark$$

IN GENERAL, TRY TO FIND PORT IMPEDANCES CLOSER TO FEEDBACK LOOP SUCH THAT ONE OF  $L_5$  OR  $L_0$  GOES TO ZERO AND OTHER IS L

EXAMPLE 8

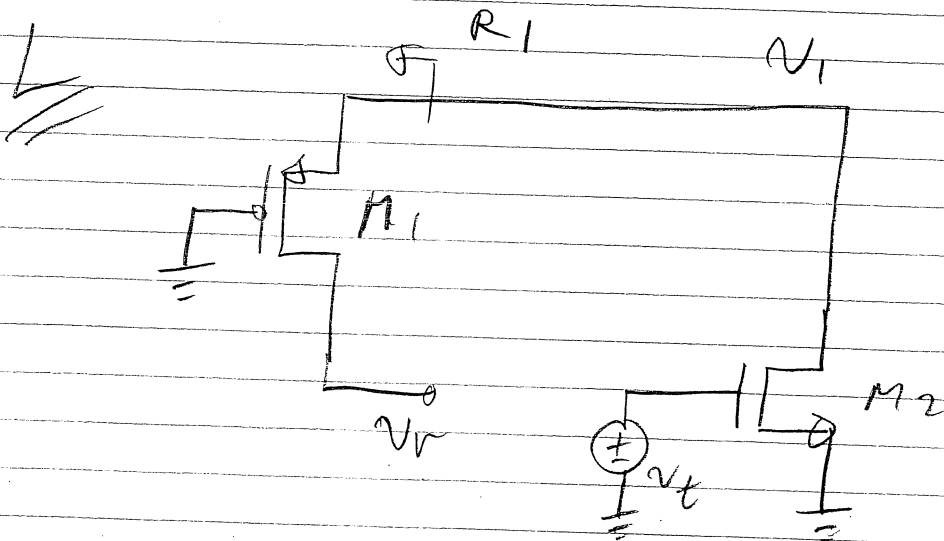
LG26

SUPER SOURCE FOLLOWER



ASSUME  
 $I_1, I_2$  IDEAL

INCLUDE  
 $r_{o1}, r_{o2}$



LG 27

$$R_1 \rightarrow \infty$$

$$v_1 = -g_{m2} r_{o2}$$

$$v_r = i_{sc1} r_o$$

$$i_{sc1} = \frac{g_{m1} r_{o1} + 1}{r_o} v_1$$

$$v_r = (1 + g_{m1} r_{o1}) v_1$$

(SEE MS NOTES)

$$L \equiv -\frac{v_r}{v_t} = g_{m2} r_{o2} (1 + g_{m1} r_{o1})$$

$$L \approx g_{m1} g_{m2} r_{o1} r_{o2}$$

$$R_{out} = R_{10} \left[ \frac{1 + L_5}{1 + L_0} \right]$$

$$L_5 = 0$$

$$L_0 = L$$

$$R_{10} = r_{o2}$$

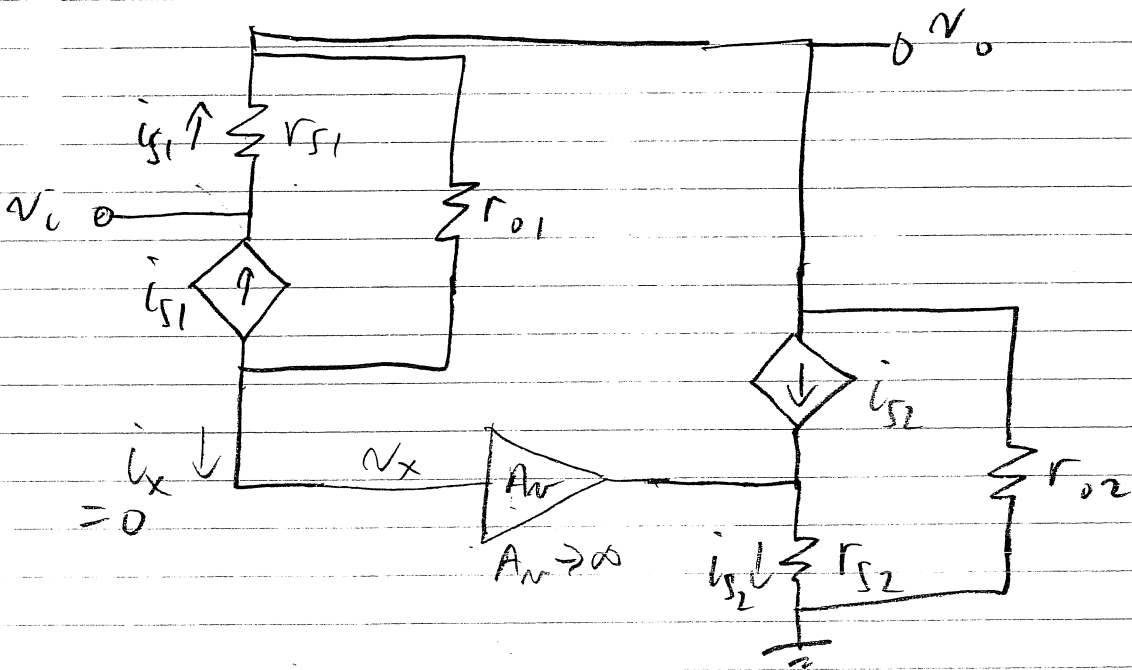
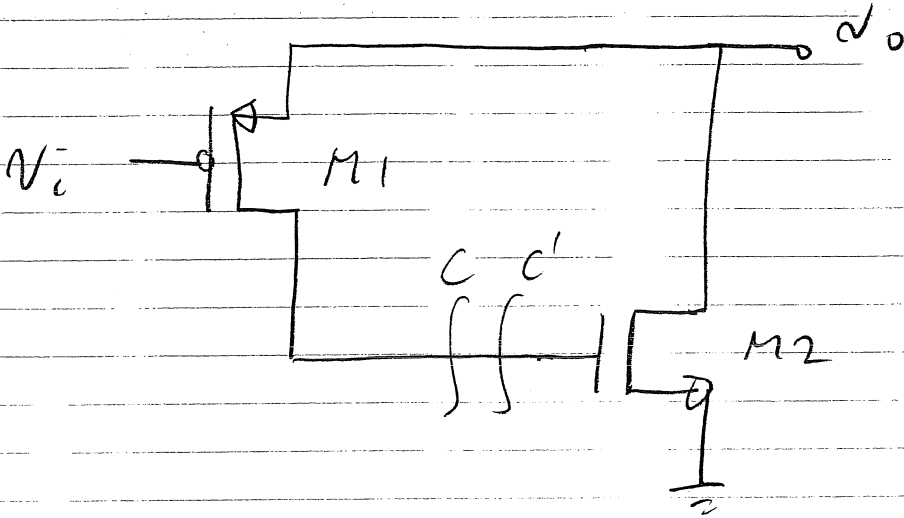
$$R_{out} = \frac{r_{o2}}{1 + g_{m1} g_{m2} r_{o1} r_{o2}} \approx \frac{r_{o2}}{g_{m1} g_{m2} r_{o1} r_{o2}}$$

$$R_{out} \approx \frac{1}{g_{m2} (g_{m1} r_{o1})}$$

REDUCED BY  $g_{m1} r_{o1}$  COMPARED TO SOURCE FOLLOWER



A<sub>∞</sub>



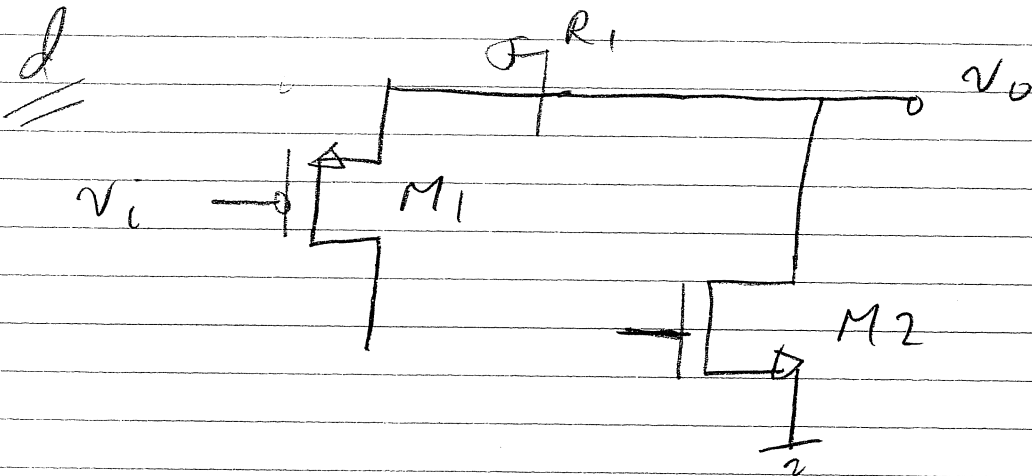
$A_v \rightarrow \infty \Rightarrow v_x \rightarrow 0$

$$\frac{v_o - v_x}{r_{O1}} = i_{S1} = \frac{v_i - v_o}{r_{S1}} + v_x = 0$$

L629

$$\frac{v_o}{r_{o1}} + \frac{v_o}{r_{s1}} = \frac{v_i}{r_{s1}}$$

$$A_{\infty} = \left. \frac{v_o}{v_i} \right|_{L \rightarrow \infty} = \frac{(r_{o1} \parallel r_{s1})}{r_{s1}} \approx 1$$



$$v_o = v_{oc1} \left( \frac{r_{o2}}{r_{o2} + R_1} \right) \quad R_1 \rightarrow \infty$$

$$v_o = 0$$

$$d = 0$$

(L630)

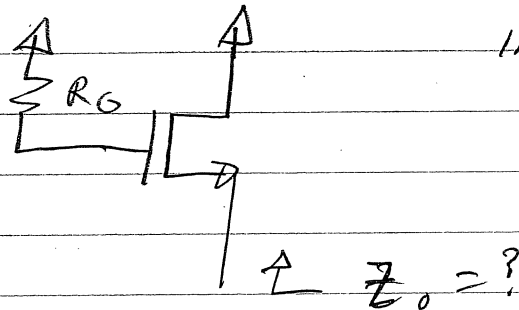
$$A_{CL} = A_{\infty} \left( \frac{L}{1+L} \right) + d \left( \frac{1}{1+L} \right)$$

$$A_{CL} = \left( \frac{r_{o1} \parallel r_{S1}}{r_{S1}} \right) \left( \frac{g_{m2} r_{o2} (1 + g_{m1} r_{o1})}{g_{m2} r_{o2} (1 + g_{m1} r_{o1}) + 1} \right)$$

$$A_{CL} \approx \underline{1}$$

IMPEDANCE EXAMPLE

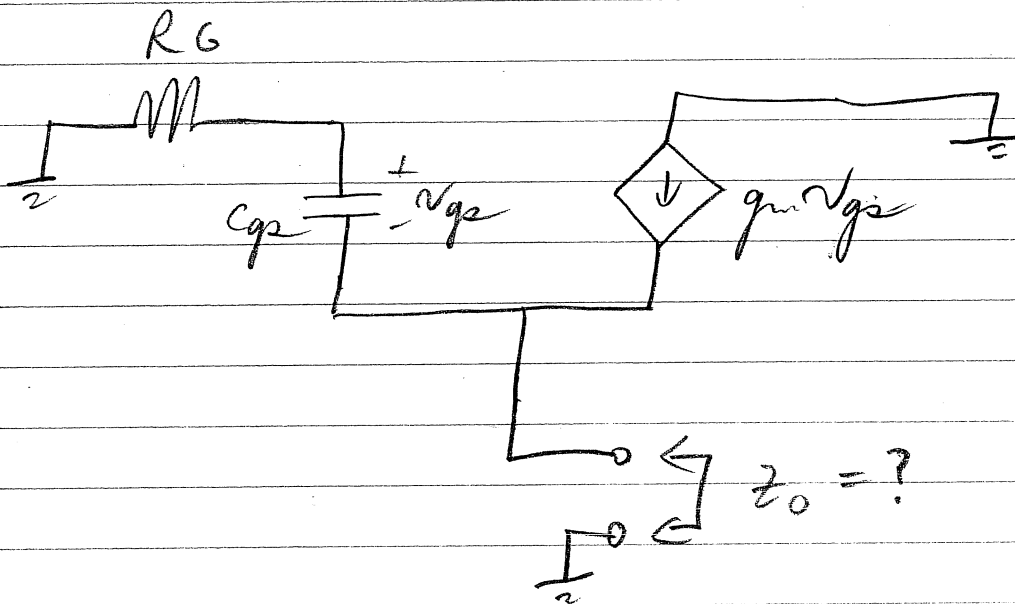
CONSIDER



LET  $r_o \rightarrow \infty$

INCLUDE  $C_{gs}$

SMALL-SIGNAL



$$z_o = z_{p0} \left[ \frac{1 + L_S}{1 + L_0} \right]$$

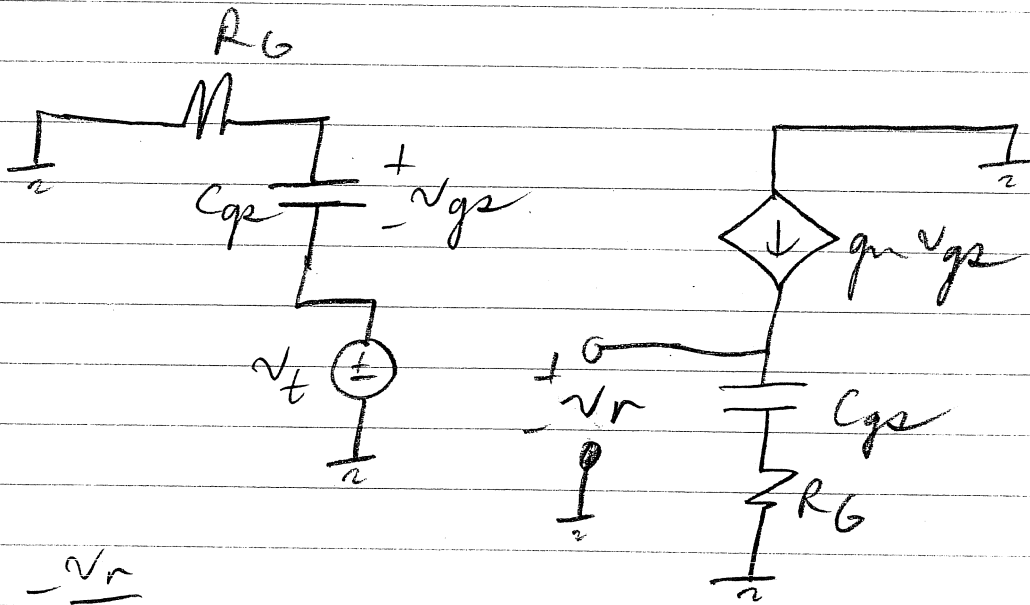
LG32

$$L = 0 \Rightarrow \text{LET } g_m = 0$$

$$Z_{p^0} = R_G + \frac{1}{sC_{gs}} = \frac{1 + sC_{gs}R_G}{sC_{gs}}$$

$$L_s = 0$$

$L_0$



$$L_0 \equiv -\frac{v_r}{v_t}$$

$$L_0 = -\left( \frac{-\frac{1}{sC_{gs}}}{\frac{1}{sC_{gs}} + R_G} \right) g_m \left( \frac{1 + sC_{gs}R_G}{sC_{gs}} \right)$$

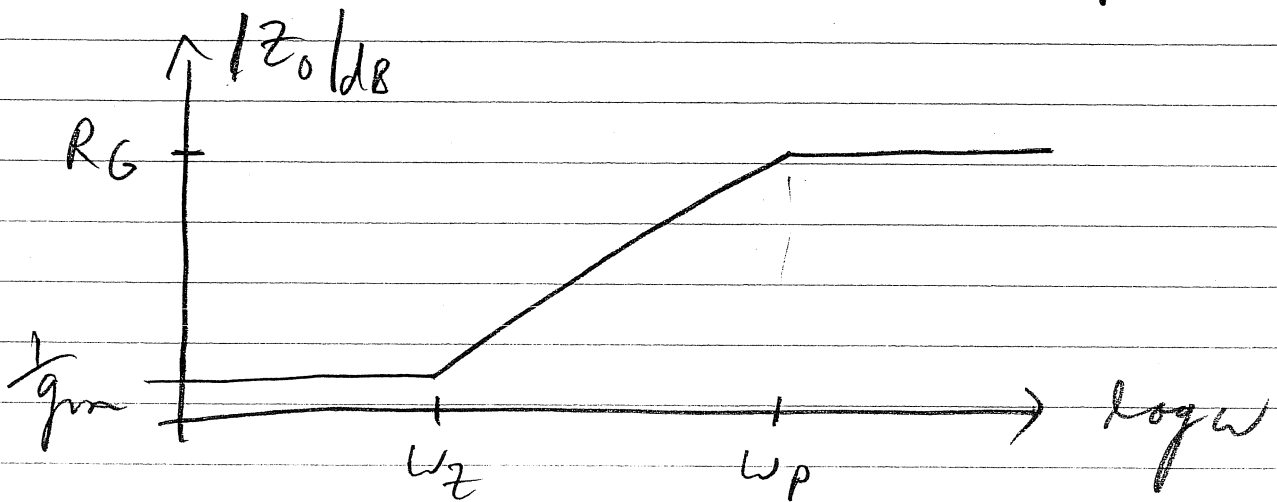
$$L_0 = \frac{g_m}{sC_{gs}}$$

LG33

$$Z_0 = Z_{p0} \left[ \frac{1+s}{1+s0} \right] = \left( \frac{1+s C_{gs} R_G}{s C_{gs}} \right) \left( \frac{1}{1 + \frac{g_m}{C_{gs}}} \right)$$

$$Z_0 = \frac{1 + s C_{gs} R_G}{g_m + s C_{gs}} = \frac{1}{g_m} \left[ \frac{1 + s C_{gs} R_G}{1 + s \frac{C_{gs}}{g_m}} \right]$$

$$Z_0 = \frac{1}{g_m} \left[ \frac{1 + s/\omega_z}{1 + s/\omega_p} \right] \quad \omega_z \equiv \frac{1}{C_{gs} R_G}$$
$$\omega_p \equiv \frac{g_m}{C_{gs}}$$



LG34

SO FOR  $\omega_z \ll \omega \ll \omega_p$

$$Z_0 \approx \frac{1}{g_m} \frac{s}{\omega_z} = s \left( \frac{C_{gs} R_G}{g_m} \right)$$

DEFINE  $L \equiv \frac{C_{gs} R_G}{g_m}$

SO FOR  $\omega_z \ll \omega \ll \omega_p$

$$Z_0 \approx sL \quad \text{WHERE} \quad L = \frac{C_{gs} R_G}{g_m}$$

LOOKS LIKE AN INDUCTOR OF VALUE  $L$

$\Rightarrow$  THIS CIRCUIT IS USED TO EXTEND BANDWIDTH USING INDUCTIVE PEAKING.

$\Rightarrow$  NOTE IF  $C_L$  ADDED  $\Rightarrow$  RINGING MAY OCCUR

