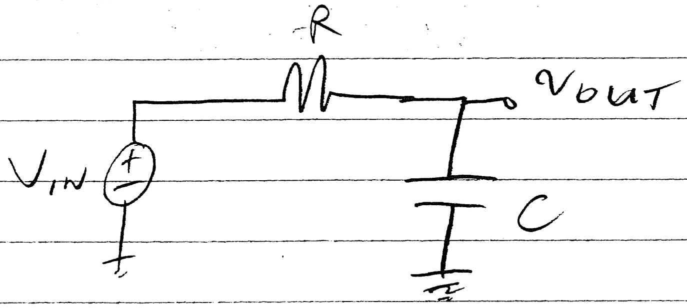
STI

SINGLE TIME CONSTANT CIRCUITS



LOWPASS
SINGLE TIME CONSTANT
(STC)

$$T(s) = \frac{V_{OUT}}{V_{IN}} = \frac{\frac{1}{sC}}{\frac{1}{sC}+R} = \frac{1}{1+sCR}$$

DEFINING $\tau = RC$ & $\omega_0 = \frac{1}{\tau} = \frac{1}{RC}$

$$T(s) = \frac{1}{1+s\tau} = \frac{1}{1+s/\omega_0}$$

IN GENERAL, A LOWPASS STC IS

$$T(s) = \frac{K}{1+s/\omega_0}$$ WHERE $\omega_0 = \frac{1}{\tau} = \frac{1}{RC}$

IF RC IS USED TO REALIZE IT

MAGNITUDE RESPONSE

$$|T(j\omega)| = \left| \frac{K}{1 + j\omega/\omega_0} \right|$$

$$= \frac{K}{\sqrt{1 + (\omega/\omega_0)^2}}$$

$|T(j\omega)| \approx K$ FOR $\omega \ll \omega_0$

$|T(j\omega)| \approx \dfrac{K\omega_0}{\omega}$ FOR $\omega \gg \omega_0$

$|T(j\omega)| = \dfrac{K}{\sqrt{2}}$ FOR $\omega = \omega_0$ 3dB FREQ.

PHASE RESPONSE

$$\angle T(j\omega) = \angle K - \angle\left(1 + j\omega/\omega_0\right)$$

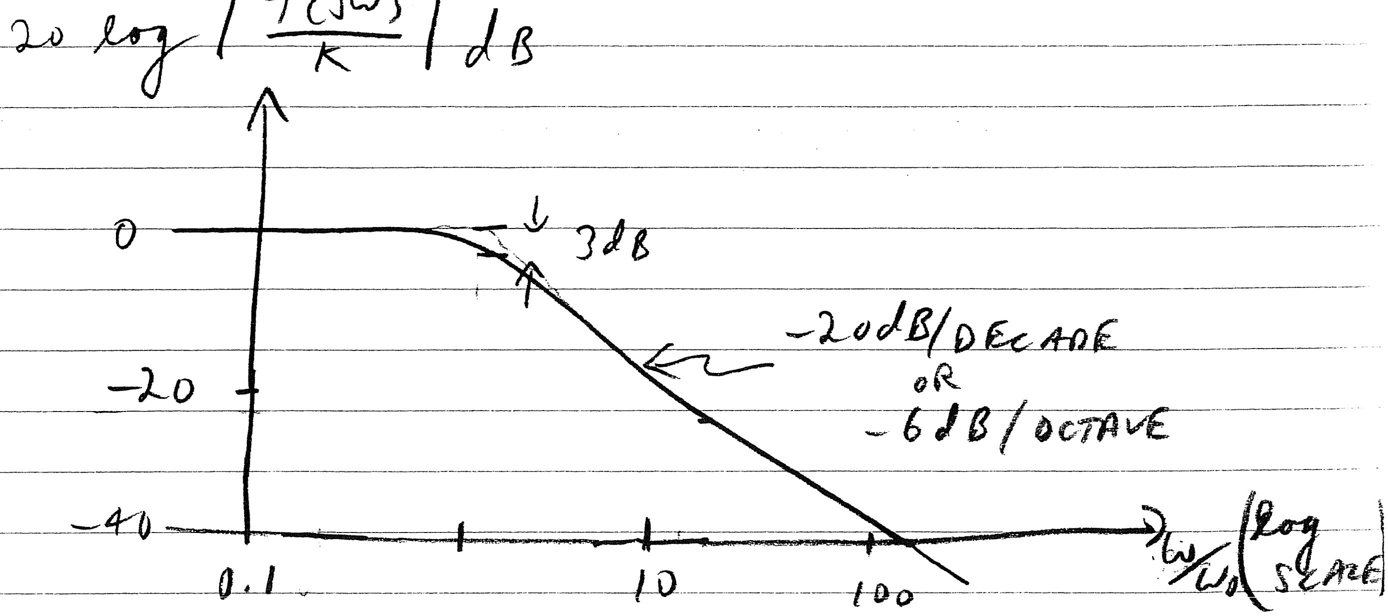
$$= 0 - \tan^{-1}\left(\frac{\omega}{\omega_0}\right)$$

$\angle T(j\omega) \approx 0$ FOR $\omega \ll \omega_0$

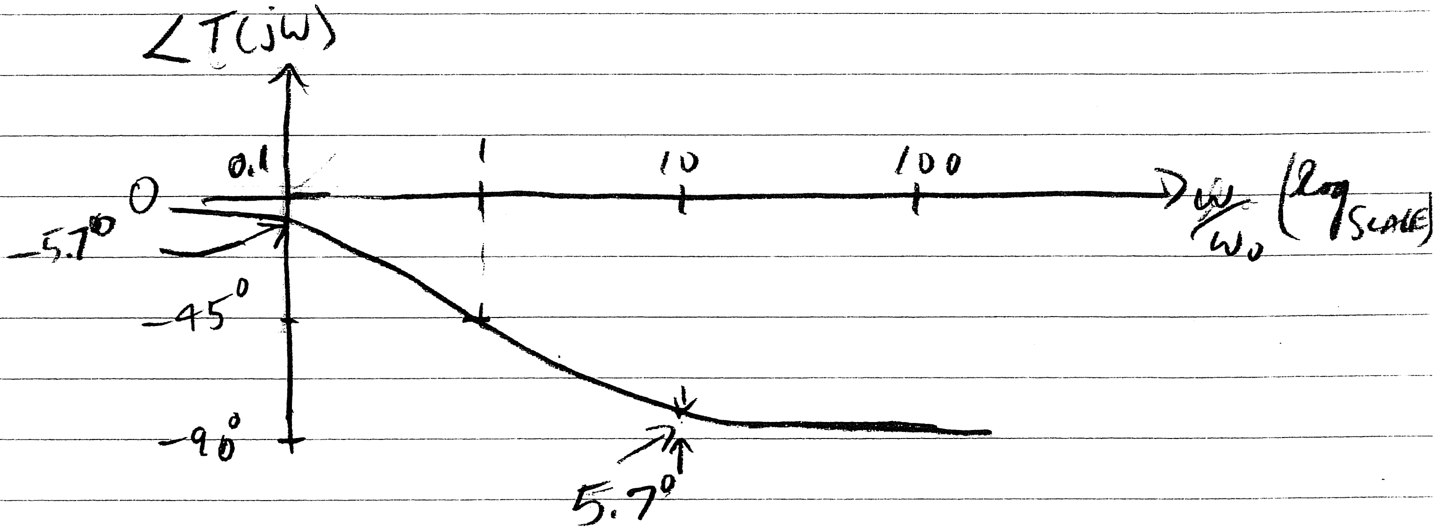
$\angle T(j\omega) \approx -90^\circ$ FOR $\omega \gg \omega_0$

$\angle T(j\omega) = -45^\circ$ FOR $\omega = \omega_0$

ST3

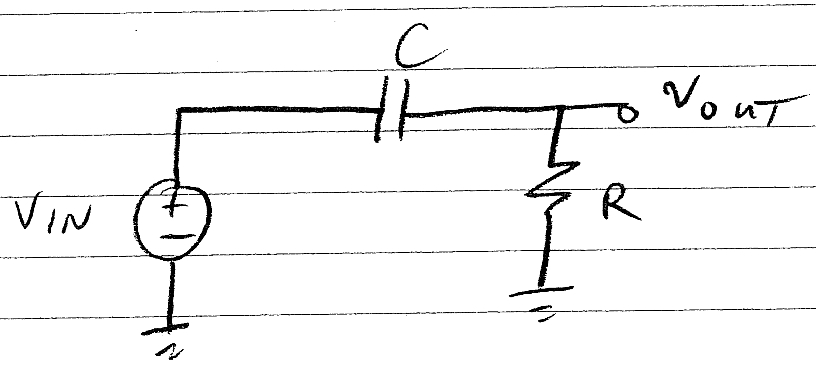


20 log |T(jw)/K| dB
0
3dB
-20dB/DECADE
OR
-6dB/OCTAVE
-20
-40
0.1
10
100
w/w0 (log SCALE)



∠T(jw)
0.1
1
10
100
w/w0 (log SCALE)
0
-5.7°
-45°
-90°
5.7°

HIGHPASS



$$T(s) = \frac{V_{OUT}}{V_{IN}} = \frac{R}{\frac{1}{sC} + R} = \frac{sCR}{sCR+1} = \frac{s}{s + \frac{1}{CR}}$$

DEFINING $\tau = RC$ & $\omega_0 = \frac{1}{\tau} = \frac{1}{RC}$.

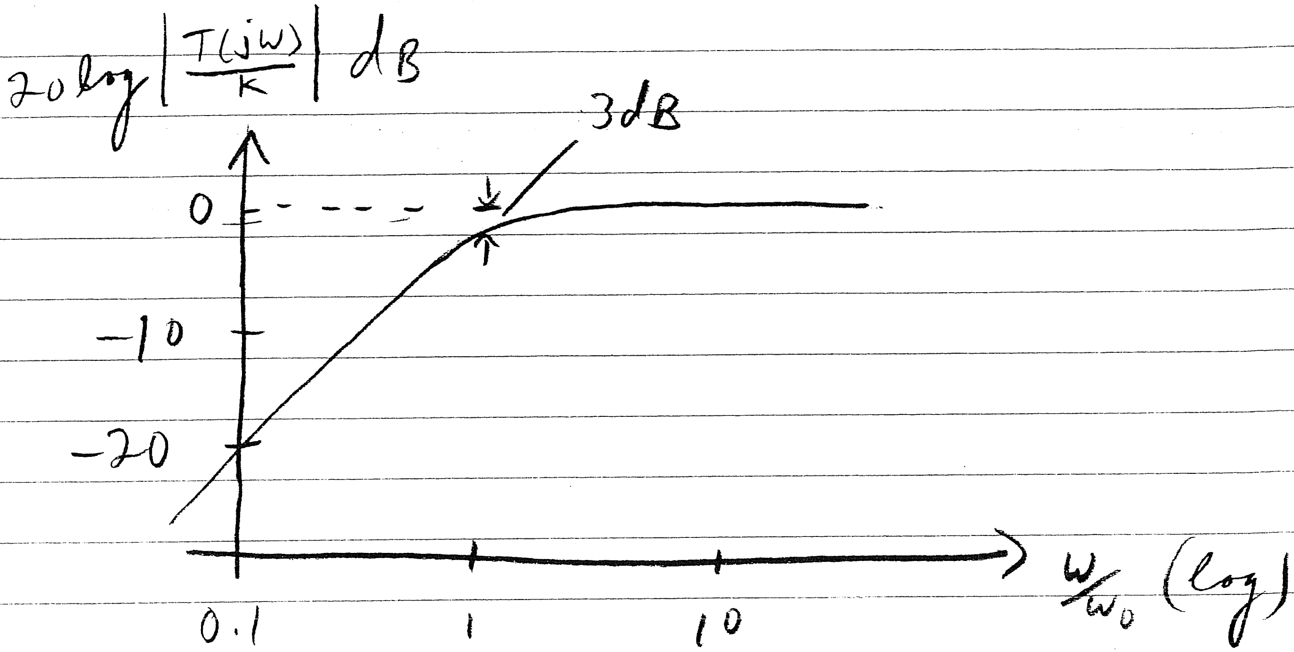
$$T(s) = \frac{s}{s + \frac{1}{\tau}} = \frac{s}{s + \omega_0}$$

IN GENERAL, A HIGHPASS STC IS

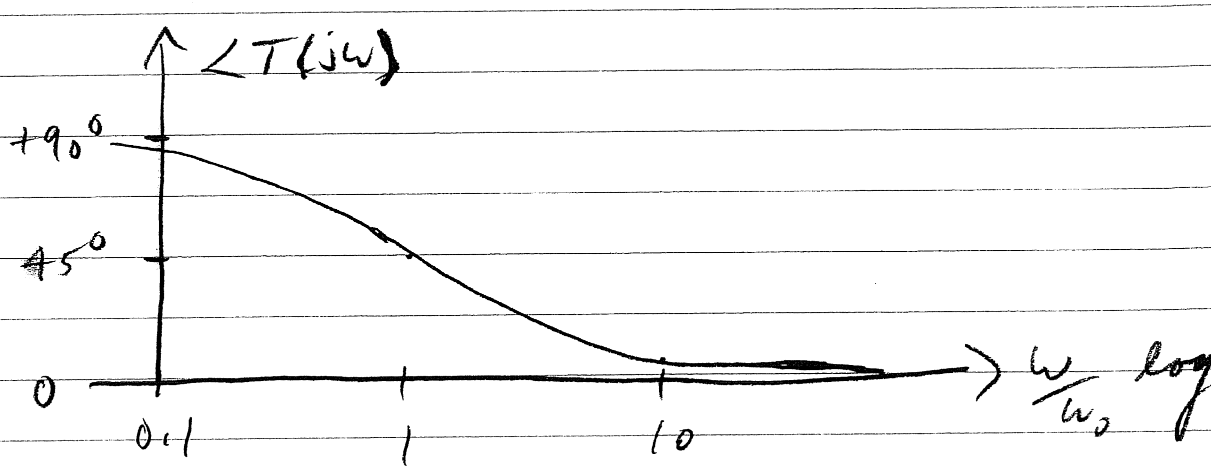
$$T(s) = \frac{ks}{s + \omega_0} \quad \text{WHERE } \omega_0 = \frac{1}{\tau}$$

$$|T(j\omega)| = \frac{k\omega}{\sqrt{\omega^2 + \omega_0^2}} = \frac{k}{\sqrt{1 + \omega_0^2/\omega^2}}$$

$$\angle T(j\omega) = \tan^{-1}\left(\frac{\omega_0}{\omega}\right)$$



20 log |T(jω)/K| dB
3dB
0
−10
−20
0.1
1
10
ω/ω₀ (log)

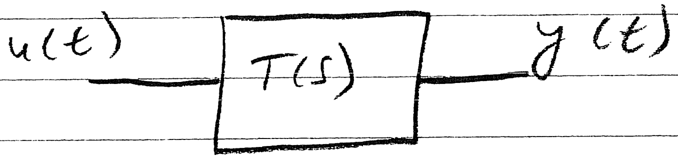


∠T(jω)
+90°
45°
0
0.1
1
10
ω/ω₀ log

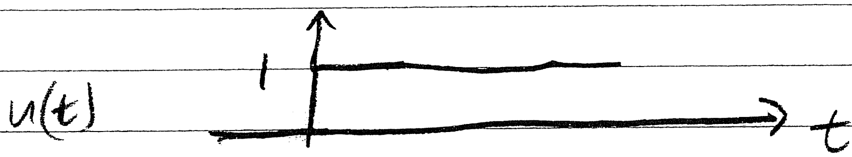
STEP RESPONSE OF STC CIRCUITS

$$y(t) = Y_\infty - (Y_\infty - Y_{0+})\, e^{-t/\tau}$$

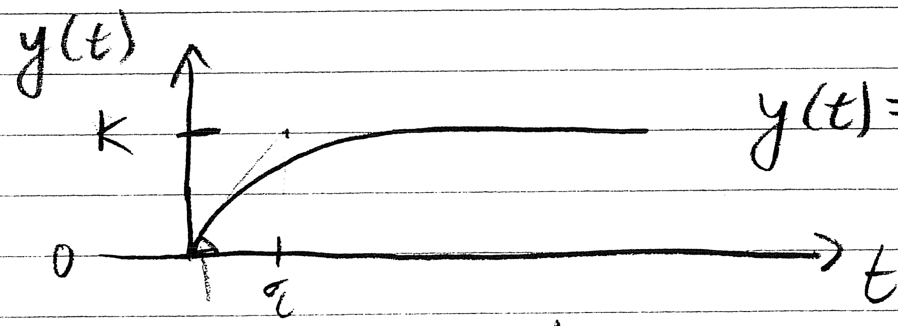
WHERE Y_∞ IS FINAL VALUE
Y_{0+} IS INITIAL "
τ IS TIME CONSTANT



HIGHPASS OR LOWPASS



LOWPASS $\quad T(s) = \dfrac{K}{1 + s/\omega_0} \qquad \omega_0 = \dfrac{1}{\tau}$

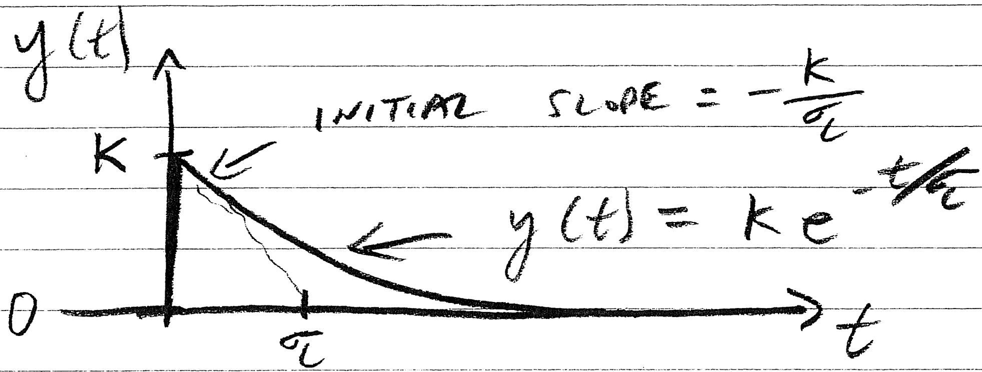


$$y(t) = K\left(1 - e^{-t/\tau}\right)$$

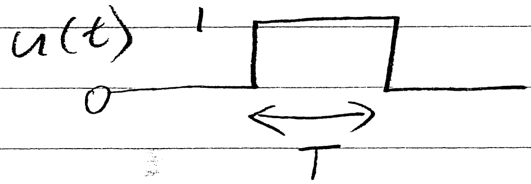
INITIAL SLOPE $= \dfrac{K}{\tau}$

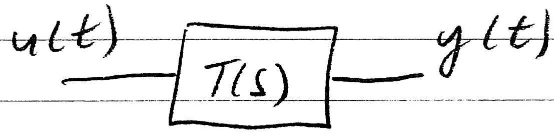
HIGH PASS

$$T(s) = \frac{ks}{s+w_0} \qquad w_0 = \frac{1}{\tau_L}$$

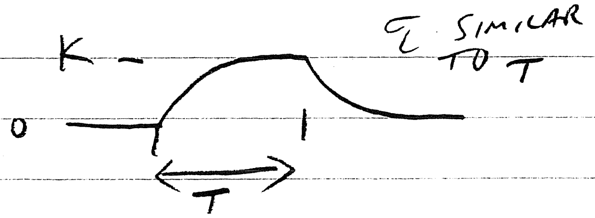


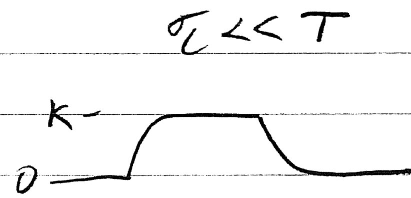
PULSE RESPONSES

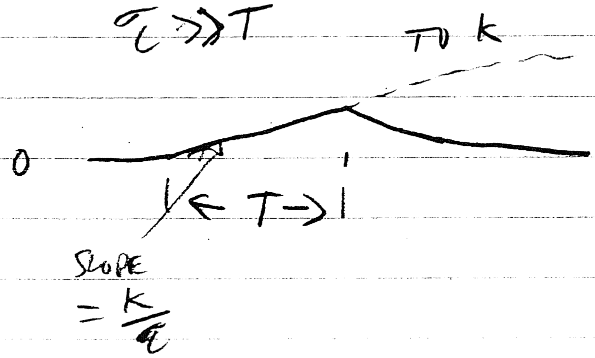




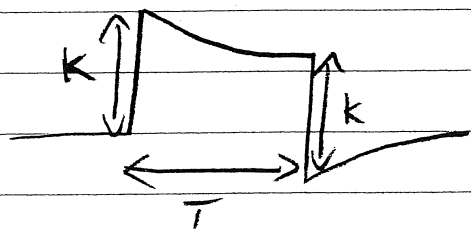
LOWPASS OUTPUT





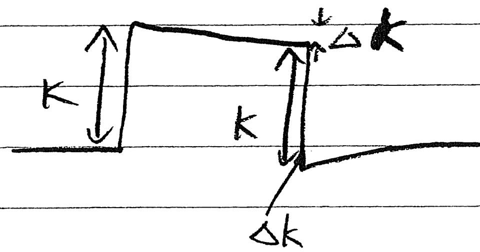


HIGH PASS OUTPUT



τ COMPARABLE TO T

$\tau \gg T$



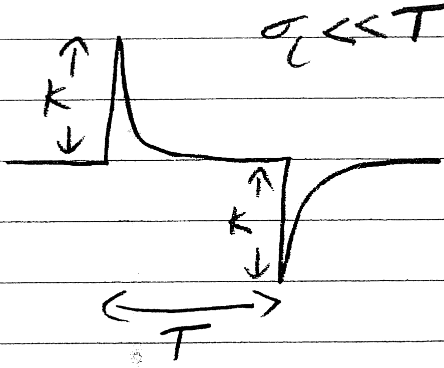
$$\Delta k \approx \frac{K}{\tau} T$$

PERCENTAGE SAG

$$= \frac{\Delta k}{k} \times 100$$

$$\simeq \frac{T}{\tau} \times 100$$

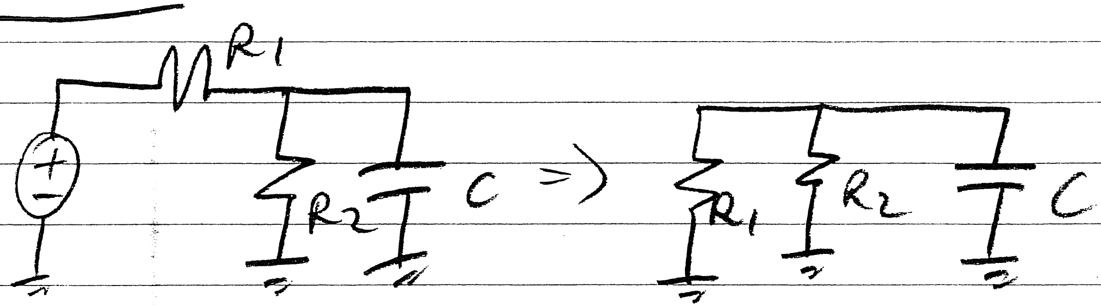
$\tau \ll T$



FINDING τ FROM STC CIRCUITS WITH R's + C's

1) IF SINGLE CAPACITOR + MULTIPLE RESISTORS
   - REDUCE INDEPENDENT SOURCES TO ZERO
   - FIND EQUIVALENT RESISTANCE "SEEN" BY CAPACITOR

EXAMPLE

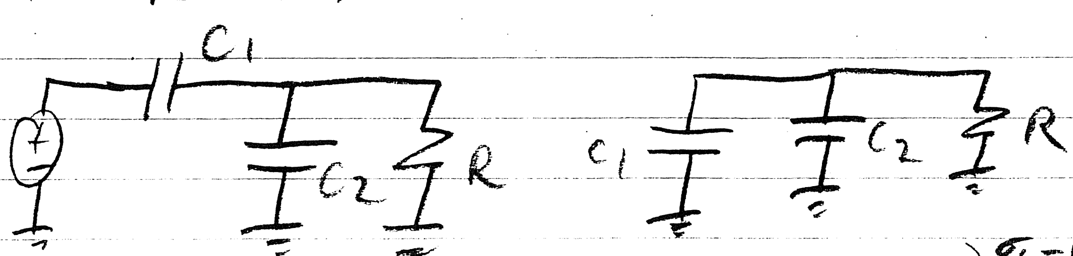


$$R_{EQ} = R_1 \| R_2$$

$$\tau = R_{EQ} C$$

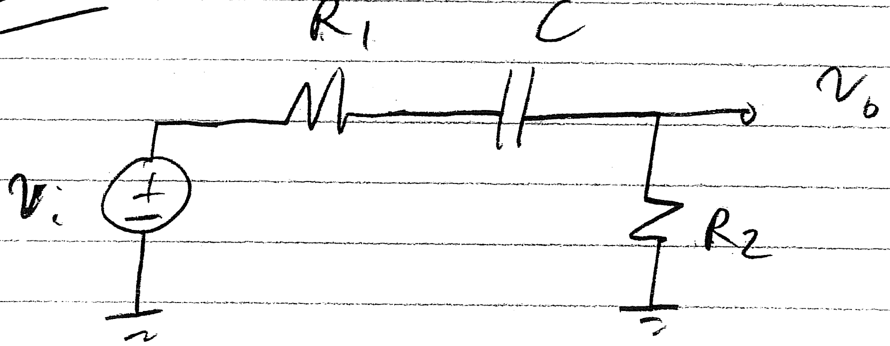
$$= (R_1 \| R_2) C$$

2) IF SINGLE RESISTOR & MULTIPLE CAPACITORS
   - REDUCE INDEPENDENT SOURCES TO ZERO
   - FIND EQUIV CAPACITANCE "SEEN" BY RESISTOR



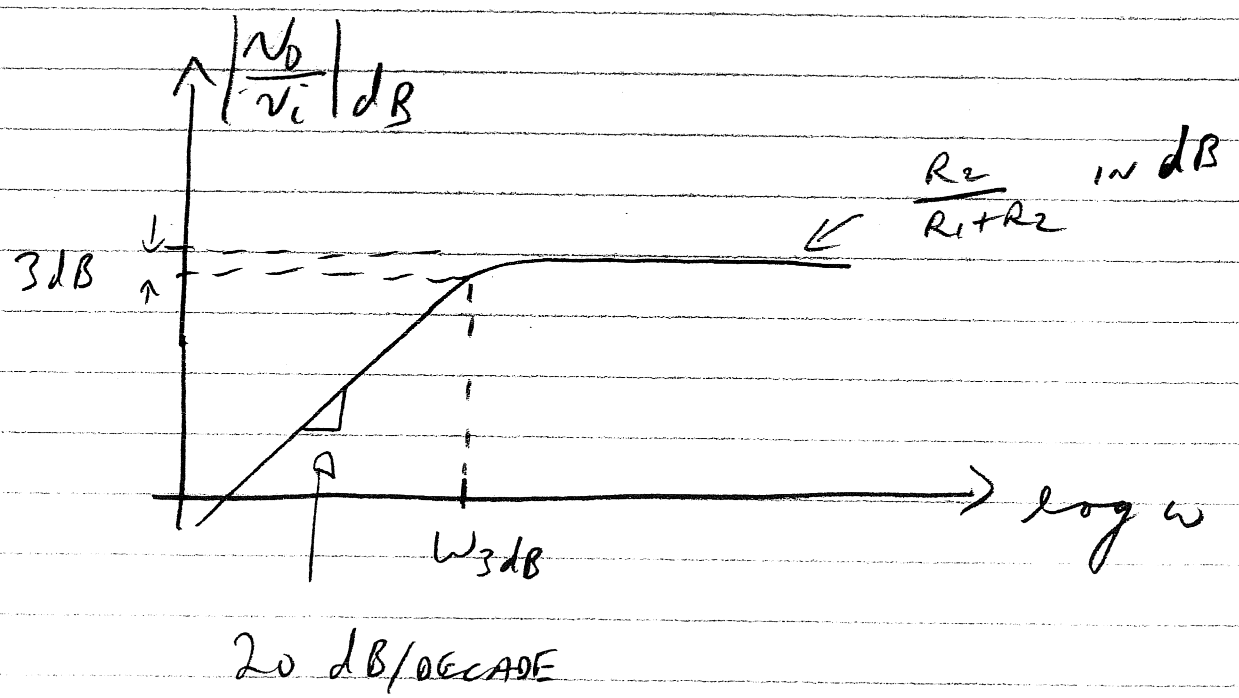
$$C_{EQ} = C_1 + C_2 \Rightarrow \tau = (C_1 + C_2) R$$

Ex

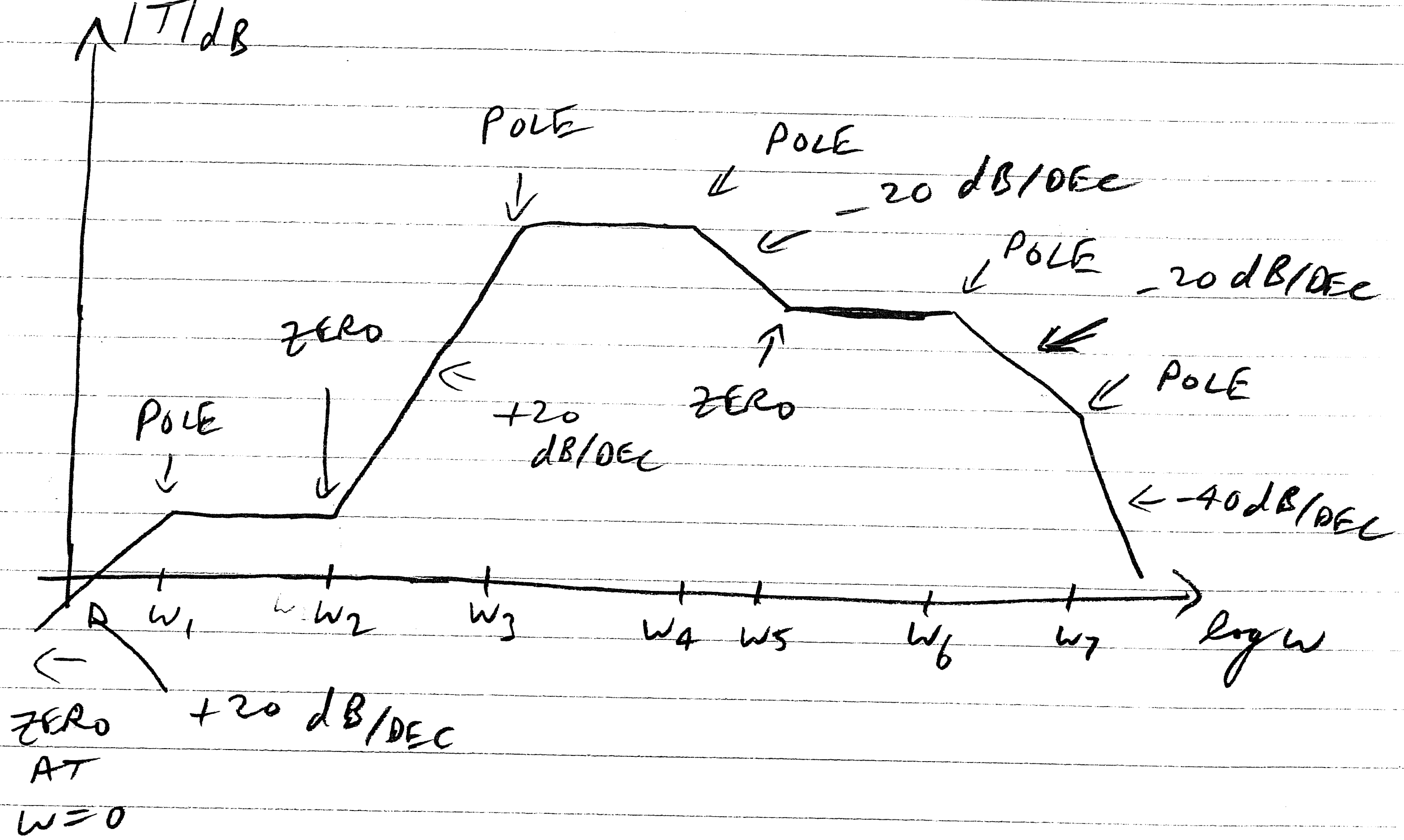


$$\tau = C(R_1 + R_2)$$

$$\omega_{3dB} = \frac{1}{C(R_1+R_2)} \qquad \left|\frac{v_o}{v_i}(j\infty)\right| = \frac{R_2}{R_1+R_2}$$



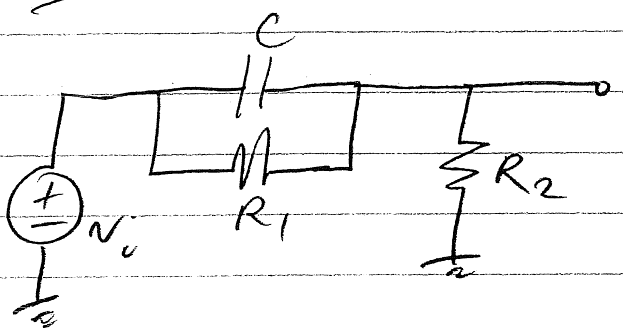
REAL-AXIS POLES & ZEROS BODE PLOT



POLES CAN BE FOUND BY τ AND DO NOT DEPEND ON WHERE INPUT APPLIED OR OUTPUT TAKEN.

ZEROS REQUIRE ANALYSIS (OR EXPERIENCE)

Ex



$T(s) = \frac{v_o}{v_i}$

ASSUME $R_1 \gg R_2$

CAN SOLVE TO FIND $T(s) = \frac{R_2(1 + sCR_1)}{(R_1 + R_2 + sCR_1R_2)}$

$$T(s) = \left(\frac{R_2}{R_1 + R_2}\right)\left(\frac{1 + sCR_1}{1 + sC(R_1 \| R_2)}\right)$$

SO $T(s)$ HAS ZERO AT $\omega_z = \frac{1}{CR_1}$

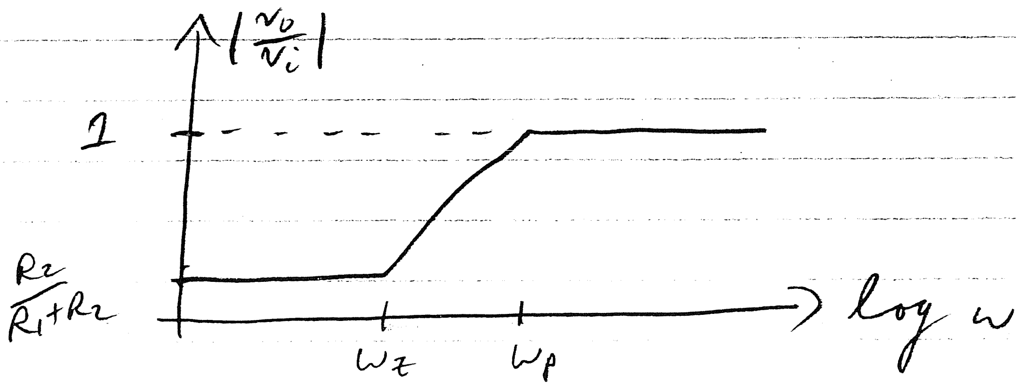
AND POLE AT $\omega_p = \frac{1}{C(R_1 \| R_2)}$

FROM INSPECTION INSTEAD OF ANALYSIS

$$\tau_p = C(R_1 \| R_2) \Rightarrow \omega_p = \frac{1}{C(R_1 \| R_2)}$$

$$\tau_z = CR_1 \Rightarrow \omega_z = \frac{1}{CR_1}$$

SINCE WHEN $\left|\frac{1}{sC} \| R_1\right| = \frac{R_1}{\sqrt{2}}$ $\Leftarrow$ IMPEDANCE GOING TO ZERO

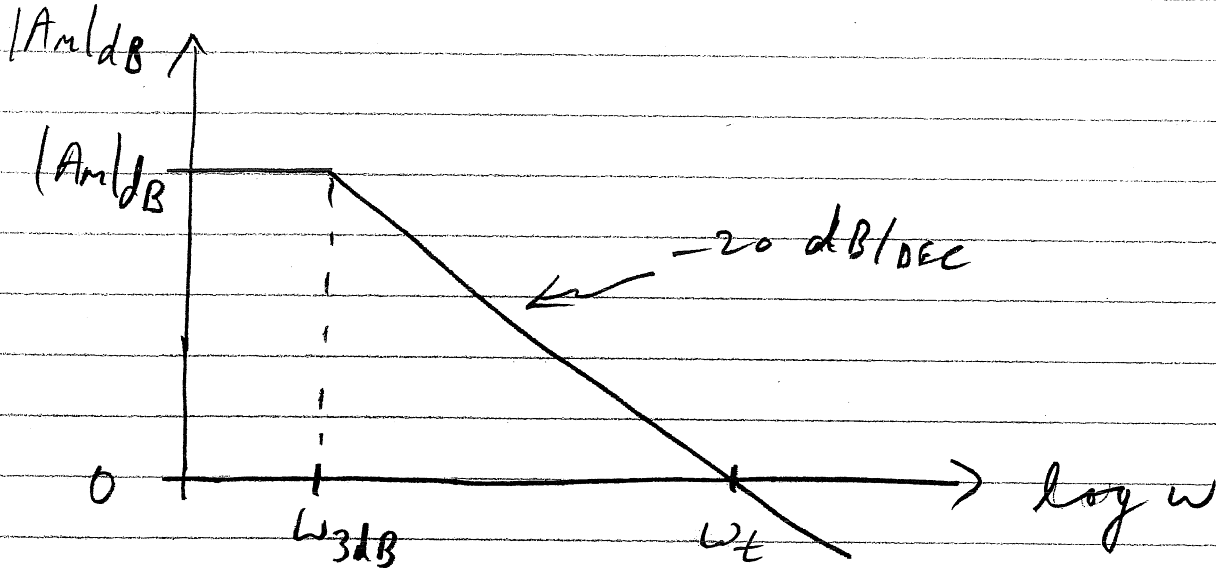


ST13

UNITY GAIN FREQ $f_t = \frac{\omega_t}{2\pi}$
OF A STC WHEN LOW FREQ GAIN $\gg 1$

CONSIDER

$$T(S) = \frac{A_M}{1 + S/\omega_{3dB}} \qquad A_M \gg 1$$



ω_t OCCURS WHEN $|T(j\omega_t)| = 1$

$$\left| \frac{A_M}{1 + \frac{j\omega_t}{\omega_{3dB}}} \right| = 1$$

$\omega_t \gg \omega_{3dB}$ SINCE $A_M \gg 1$

$$\left| \frac{A_M}{j\omega_t/\omega_{3dB}} \right| \approx 1 \Rightarrow \omega_t \approx |A_M|\omega_{3dB}$$

OR $\underline{f_t \approx |A_M| f_{3dB}}$