ST1

STABILITY OF A FEEDBACK SYSTEM

- POLES OF A SYSTEM DETERMINE STABILITY OF A SYSTEM & SETTLING BEHAVIOUR

- POLES OF A SYSTEM <u>DO NOT</u> DEPEND ON WHERE INPUT IS APPLIED OR OUTPUT TAKEN

- LOOP GAIN, $L(s)$, CAN BE USED TO FIND POLES OF A SYSTEM + HENCE STABILITY [LOOP GAIN IS FUNCTION OF FREQ]

- IF ONLY MAGNITUDE + PHASE OF $L(s)$ KNOWN $\Rightarrow$ CAN USE INFORMATION TO DETERMINE <u>SETTLING</u> BEHAVIOUR OF A FEEDBACK SYSTEM

- RECALL

$$A_f = A_\infty \left(\frac{L}{1+L}\right) + \frac{d}{1+L}$$

ST2

IF $L \Rightarrow L(s)$ A FUNCTION OF FREQ

$$A_f(s) = A_\infty \frac{L(s)}{1+L(s)} + \frac{d}{1+L(s)}$$

SO POLES OF $A_f(s)$ ARE WHERE

$1 + L(s) = 0$ [i.e. WHERE $A_f(s) \to \infty$]

FOR PHYSICAL FREQ $s = j\omega$

& LOOP GAIN $L(j\omega) = |L(j\omega)|\, e^{j\phi_L(\omega)}$

$|L(j\omega)|$ MAGNITUDE RESPONSE OF $L(s)$

$\phi_L(\omega)$ PHASE RESPONSE OF $L(s)$

$1 + L(j\omega) = 0$ WHERE $L(j\omega) = -1$

ie $|L(j\omega)| = 1$ & $\phi_L(\omega) = 180^\circ$

- IN FACT, THIS IS HOW SOME OSCILLATORS ARE CREATED.

[NOTE ASSUME $L(s)$ HAS REAL AXIS NEGATIVE LEFT HALF PLANE POLES FROM HERE ON]

ST3

FROM NYQUIST PLOT

IF $|L(j\omega_1)| = 1$ & $\phi_L(j\omega_1) < (-180^\circ)$

THEN OSCILLATIONS GROW & SYSTEM IS UNSTABLE (POSITIVE FEEDBACK WITH LOOP GAIN GREATER THAN 1)

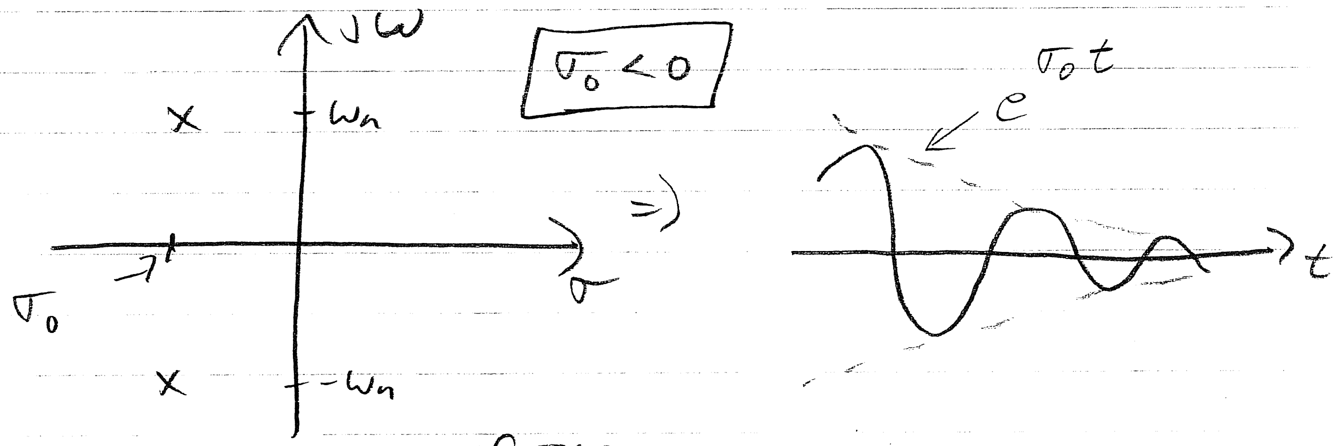
STABILITY & POLE LOCATION

CONSIDER A SYSTEM WITH A POLE PAIR AT $s = \sigma_0 \pm j\omega_n$ (COMPLEX POLE PAIR)

IF THERE IS A DISTURBANCE OF SOME SORT (POWER SUPPLY SWITCH, SIGNAL STEP, ETC) THE TRANSIENT WILL HAVE TERMS

$$v(t) = e^{\sigma_0 t}\left[e^{+j\omega_n t} + e^{-j\omega_n t}\right] = 2e^{\sigma_0 t}\cos(\omega_n t)$$

$\boxed{\sigma_0 < 0}$

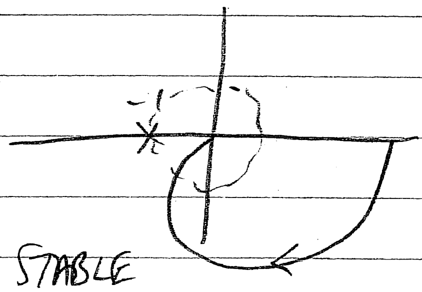


STABLE

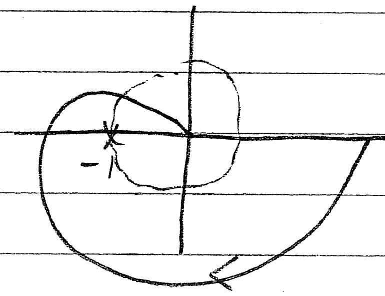
ST3A

EXAMPLE

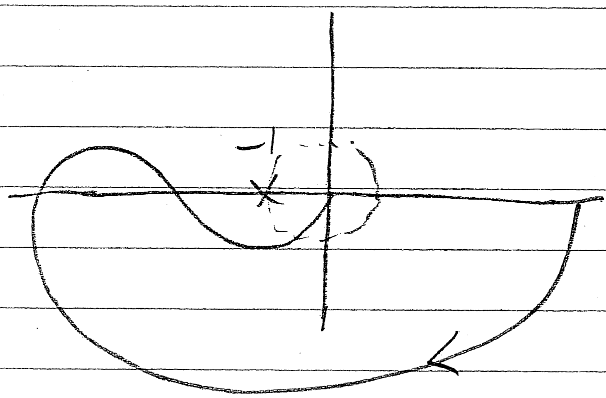
NYQUIST PLOTS FOR $L(s)$ HAVING REAL-AXIS LHP POLES & REAL-AXIS ZEROS



STABLE



UNSTABLE



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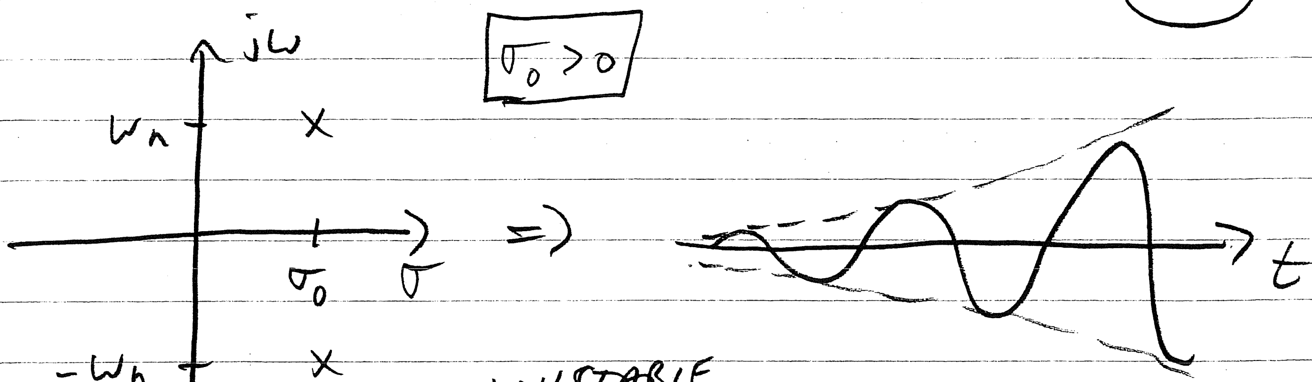
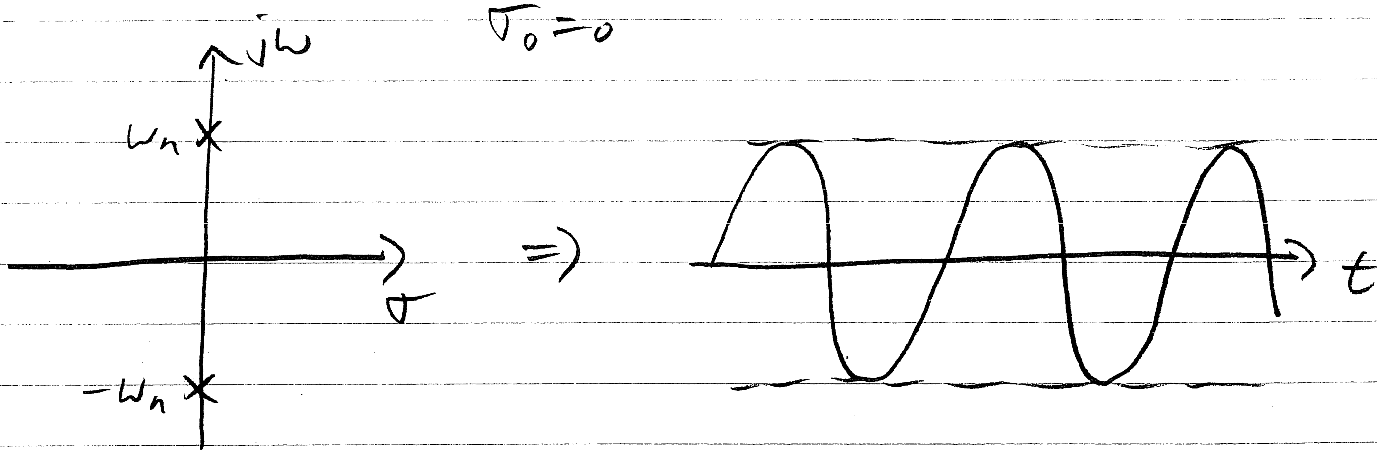


UNSTABLE



OSCILLATING

IT CAN BE SHOWN THAT ANY RIGHT HALF PLANE POLES RESULTS IN UNSTABLE SYSTEM.

ST5

$L(s) = A(s)\,\beta(s)$

FOR SIMPLICITY WE WILL ASSUME

β IS INDEPENDENT OF FREQ $\Rightarrow$ $\beta \leq 1$
& $\beta > 0$

& $A(s)$ HAS ONLY REAL-AXIS POLES
& POLES ARE IN LEFT HALF PLANE
(ALL ZEROS AT INFINITY)

$$A(s) = \frac{A_0}{(1+s/\omega_{p1})(1+s/\omega_{p2})\ldots}$$

A_0 IS dc GAIN $\quad$ ω_{pi} ARE REAL AXIS POLES

& RECALL FOR IDEAL CASE

$$A_f(s) = \frac{A(s)}{1+A(s)\beta}$$

CONSIDER 3 CASES

$A(s)$ SINGLE POLE
$A(s)$ 2 POLES
$A(s)$ 3 OR MORE POLES

ST6

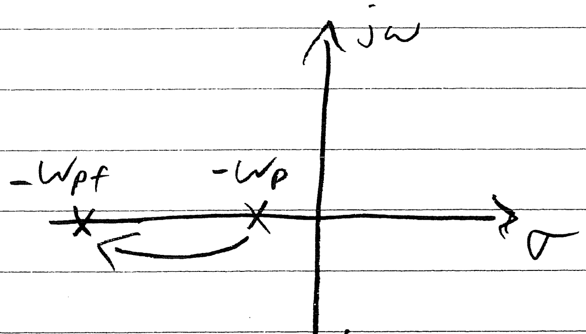
$A(s)$ SINGLE POLE

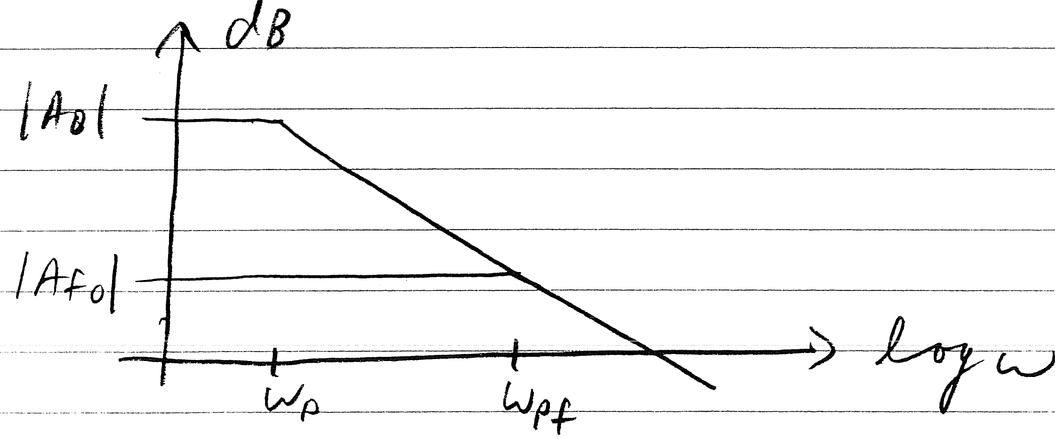
$$A(s) = \frac{A_0}{1 + s/\omega_p}$$

CAN SHOW A_f POLE

IS AT $\omega_{pf} = \omega_p(1 + A_0\beta)$

& $A_{f0} = A_0/(1+A_0\beta)$





SO CLOSED-LOOP $A_f(s)$ IS

ALWAYS STABLE FOR ANY β.

$0 < \beta \leq 1$

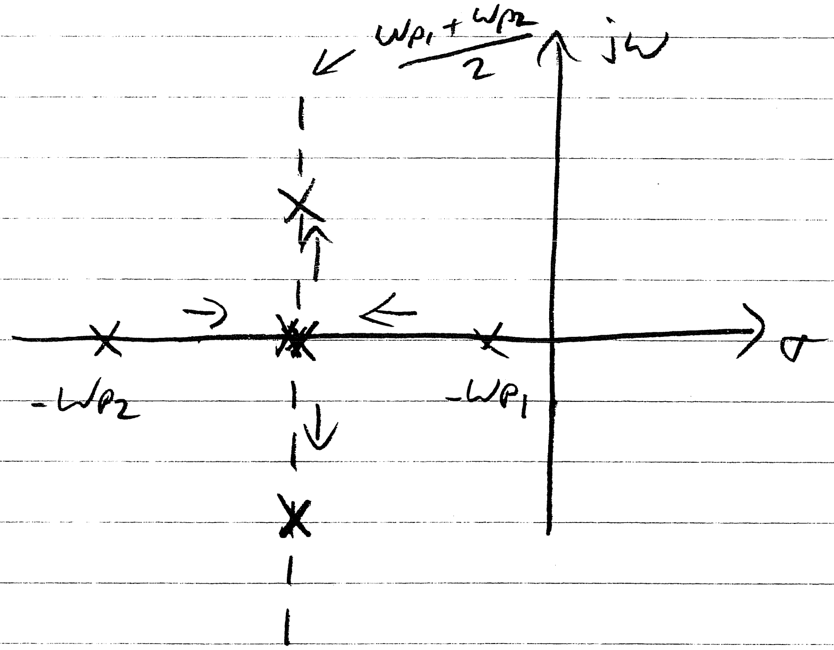
ST7

A(S) 2 POLES

$$A(s) = \frac{A_0}{(1 + s/\omega_{p1})(1 + s/\omega_{p2})}$$

$\omega_{p1} \neq \omega_{p2}$

CAN SHOW POLES OF $A_f(s)$ FOLLOW

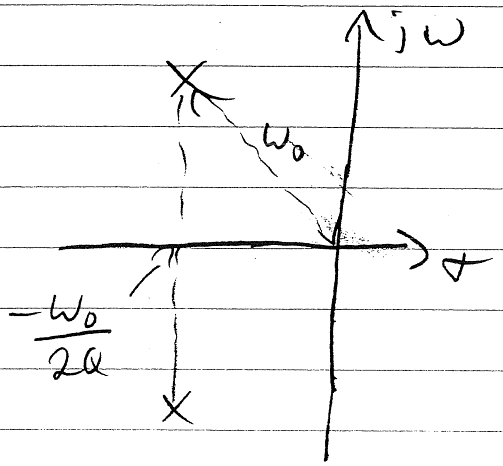


AS $A_0 \beta$ IS INCREASED (dc LOOP GAIN INCREASED)

POLES COME TOGETHER & THEN SPLIT INTO COMPLEX CONJUGATE PAIR OF POLES

ST8

PAIR OF COMPLEX CONJUGATE POLES



POLE PAIR CAN BE WRITTEN AS

$$s^2 + \frac{\omega_0}{Q}s + \omega_0^2 = 0 \quad \left(\text{ROOTS ARE POLES}\right)$$

ω_0 IS DISTANCE TO POLES FROM ORIGIN

Q GIVES INDICATION OF HOW CLOSE POLES ARE TO $j\omega$ AXIS
(IF POLES ON $j\omega$ AXIS $\Rightarrow Q \to \infty$)

THE HIGHER THE $Q \Rightarrow$ CLOSER TO INSTABILITY
AND LONGER TRANSIENT RINGING

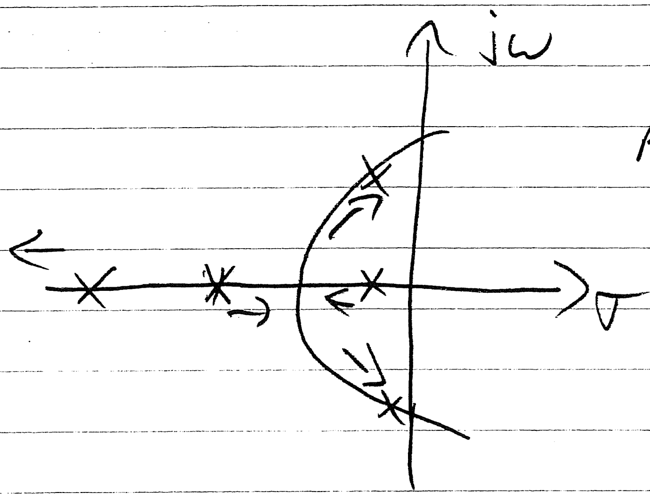
$Q \leq 0.5 \Rightarrow$ POLES ON REAL AXIS

$Q = 0.707 = \frac{1}{\sqrt{2}} \Rightarrow$ POLES AT 45°
(MAXIMALLY FLAT RESPONSE)

$Q > \frac{1}{\sqrt{2}}$ PEAKING IN FREQ RESPONSE

ST9

A(S) 3 OR MORE POLES



AS $A_0\beta$ INCREASES POLES GO INTO RIGHT HALF PLANE
$\Rightarrow$ UNSTABLE.

COMMON APPROACH FOR STABILITY STUDY IS TO USE

PHASE MARGIN
& GAIN MARGIN

USEFULL FOR LOOP GAINS WITH REAL AXIS POLES & ZEROS.

STABILITY STUDY USING PHASE MARGIN & GAIN MARGIN.

$$L(s) = A(s)\,\beta$$

POLES OF CLOSED-LOOP SYSTEM OCCUR AT

$$1 + L(s) = 0$$

SO WHEN $L(j\omega) = -1$

OSCILLATIONS OCCUR (POLES ON $j\omega$ AXIS)

& IF $|L(j\omega)| > 1$
WHERE $\angle L(j\omega) = 180^\circ$ THEN
SYSTEM UNSTABLE (POLES IN RIGHT HALF PLANE)

SO PLOT $L(j\omega)$ IN MAGNITUDE & PHASE & DEFINE ω_1 TO BE WHERE

$$|L(j\omega_1)| = 1$$

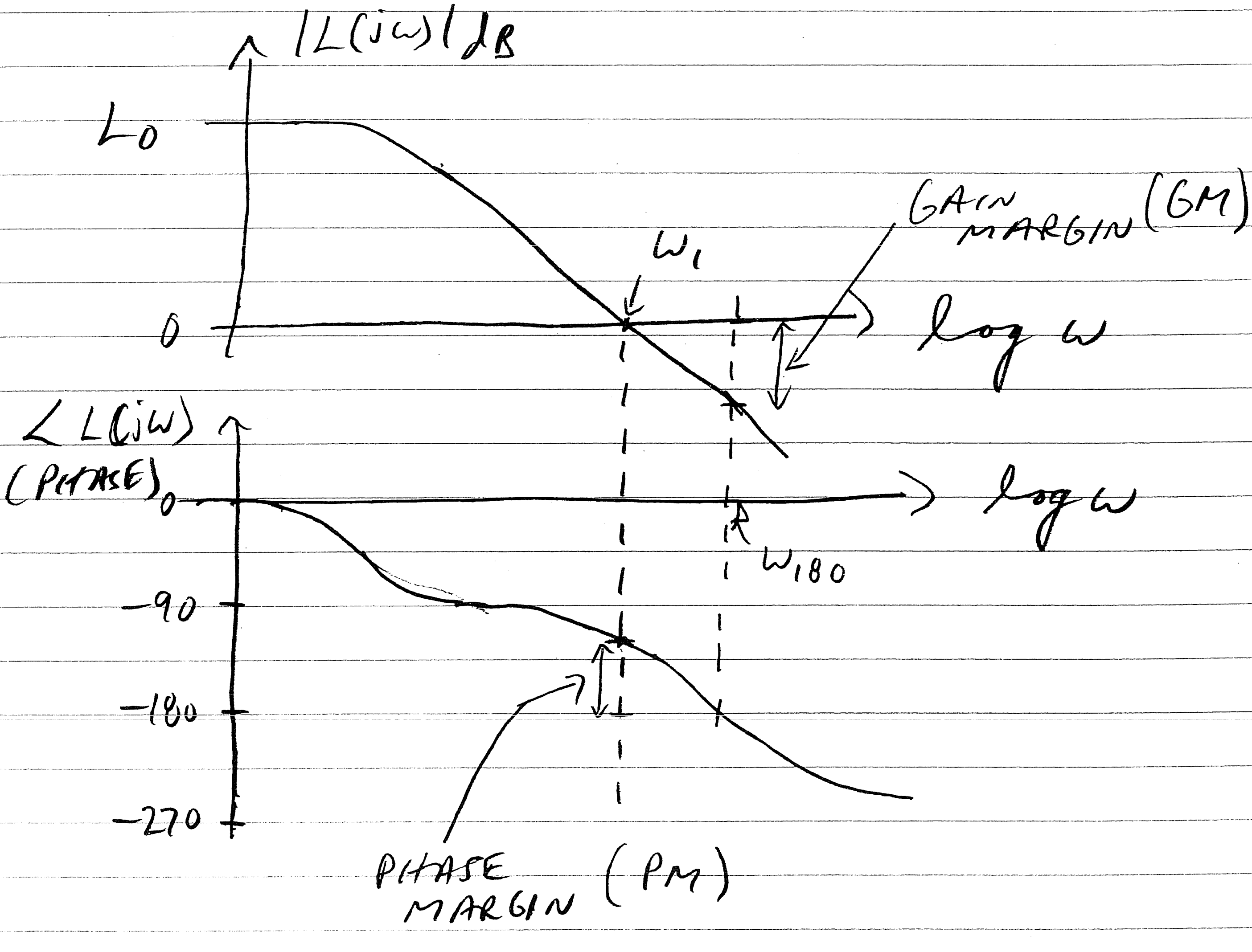
WHERE LOOP GAIN MAGNITUDE $= 1$

ST11

& ω_{180} WHERE

$$\angle L(j\omega_{180}) = -180^0$$

WHERE PHASE OF $L(j\omega)$ EQUALS -180^0



$$PM \equiv \angle L(j\omega_1) - (-180^0)$$

$$= \angle L(j\omega_1) + 180^0$$

$$GM \equiv 0 - |L(j\omega_{180})|_{dB}$$
$$= -|L(j\omega_{180})|_{dB}$$

GAIN MARGIN GIVES INDICATION ON HOW ROBUST DESIGN IS TO VARIATIONS IN PHASE, GAIN VARIATIONS.

PHASE MARGIN GIVES INFORMATION ON SETTLING BEHAVIOR OF CLOSED LOOP SYSTEM.

TYPICAL PHASE MARGIN $\approx 60^\circ$ OR HIGHER
(A FIRST-ORDER $L(s)$ HAS A $PM = 90^\circ$)

A SYSTEM WITH $PM = 45^\circ$ WILL HAVE 23% OVERSHOOT IN A STEP RESPONSE + 2.28 dB FREQ PEAK IN FREQ RESPONSE

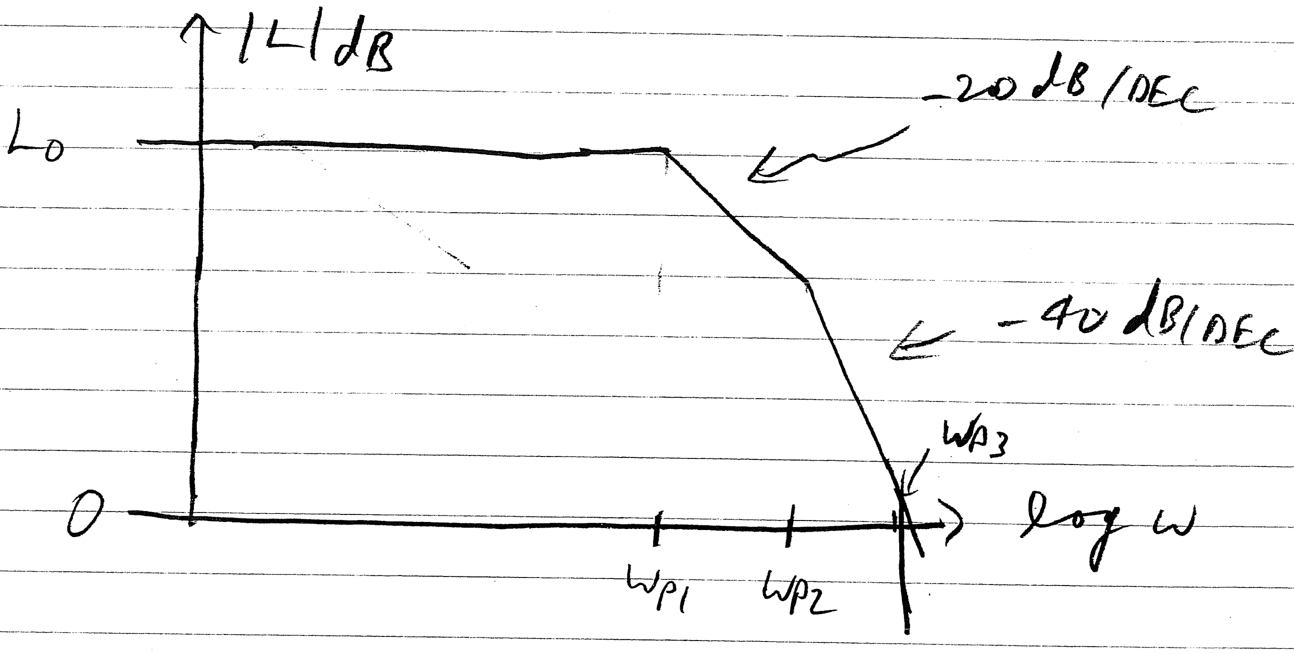
ST13

PM	STEP RESPONSE PERCENTAGE OVERSHOOT
45	23 %
55	13.3 %
65	4.7 %
75	0.008 %

ST14

FREQUENCY COMPENSATION

ASSUME IT IS DESIRED TO HAVE 45° PM (FOR SIMPLICITY) & THAT POLES ARE WIDELY SPACED APART



SYSTEM IS UNSTABLE SINCE

ω_{p1} ADDS -90°

ω_{p2} ADDS ANOTHER -90°

ω_{p3} ADDS -45° AT ω_{p3}

$$-90^\circ - 90^\circ - 45^\circ = -135^\circ$$

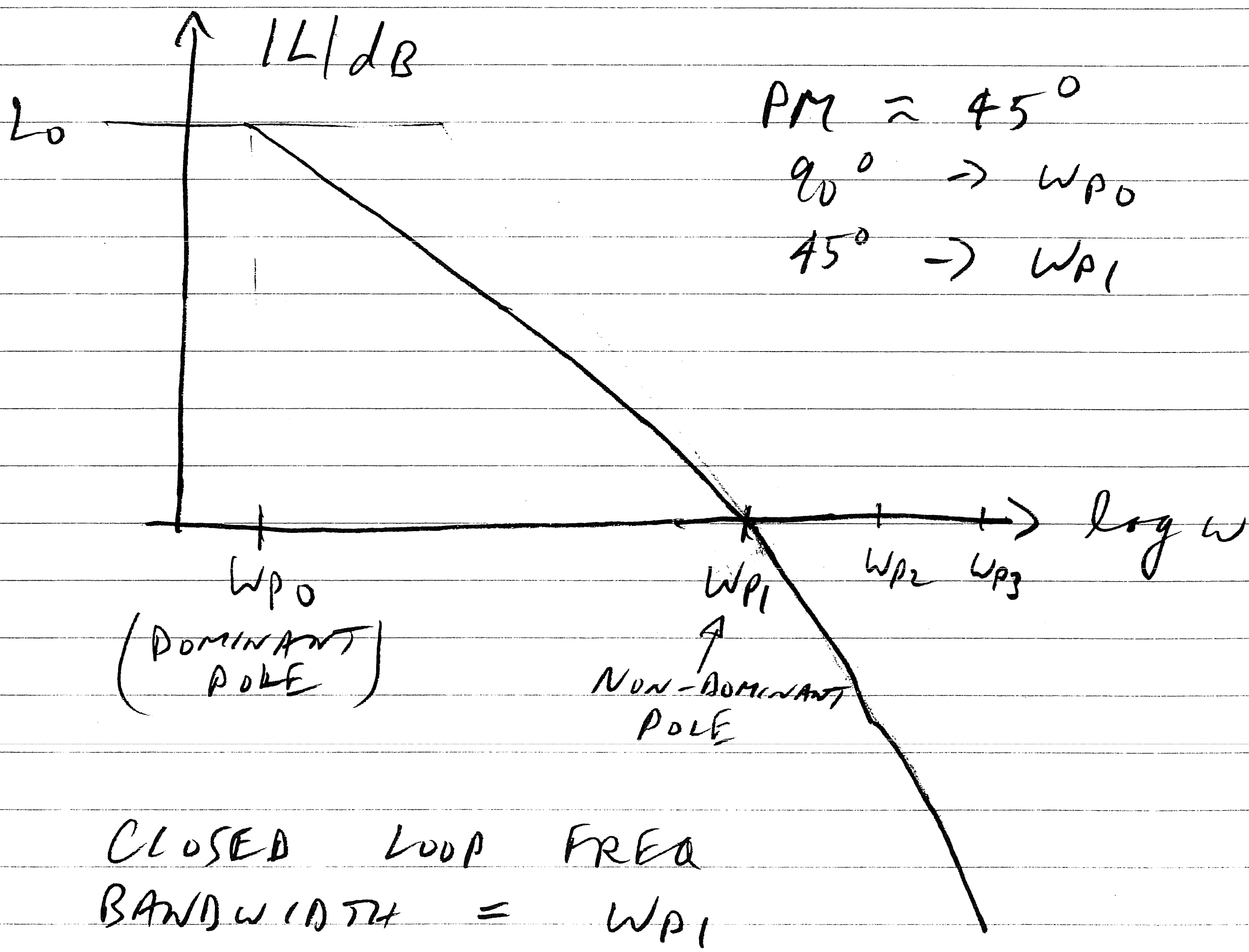
& $|L(j\omega_{p3})|_{dB} > 0\,dB$

ST15

APPROACH #1

INTRODUCE W_{P0} SUCH THAT

$$|L(jW_{P1})|_{dB} = 0$$



PM ≈ 45°

90° → W_{P0}

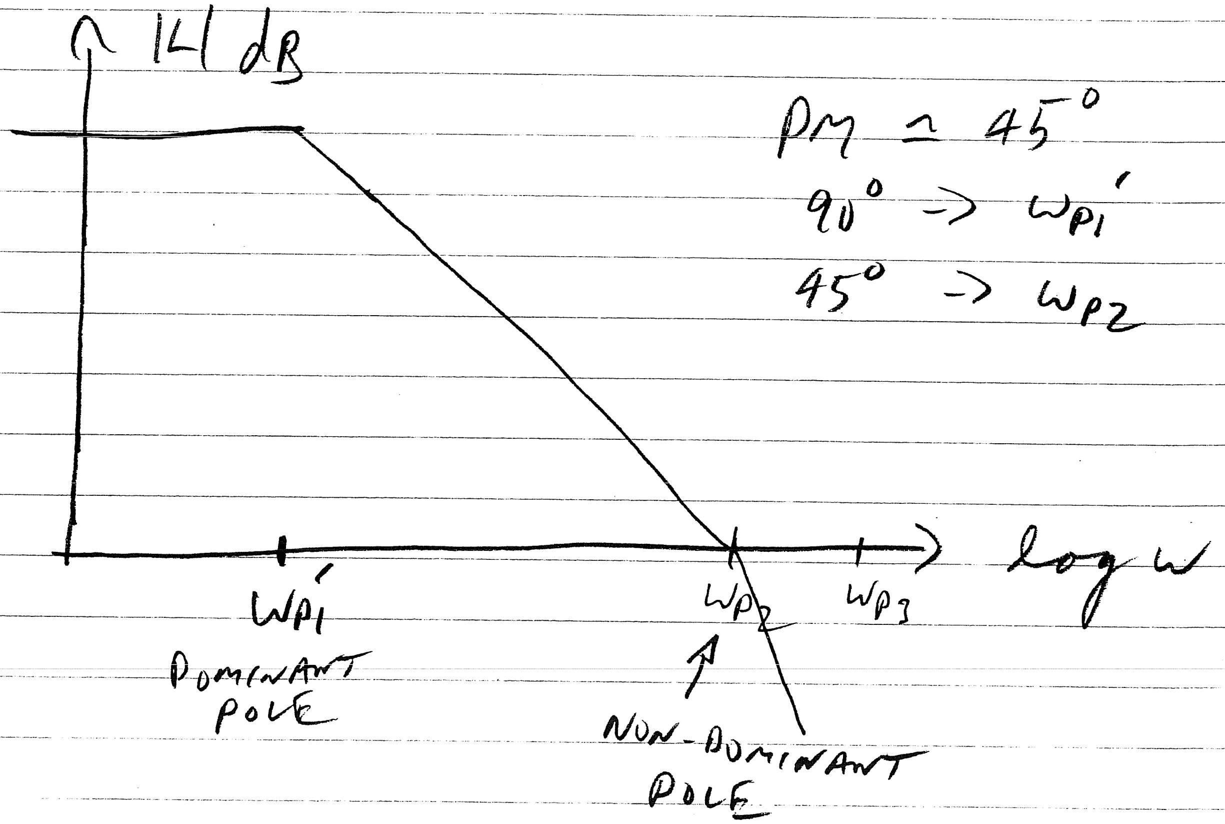
45° → W_{P1}

CLOSED LOOP FREQ BANDWIDTH = W_{P1}

ST16

APPROACH #2 (PREFERRED)

FIND NODE CAUSING ω_{p1}
AND ADD CAPACITANCE TO
MOVE ω_{p1} TO ω_{p1}' (DOMINANT POLE)
SO NON-DOMINANT POLE IS ω_{p2}



PM ≃ 45°
90° → ω_{p1}'
45° → ω_{p2}

CLOSED-LOOP FREQ
BANDWIDTH = ω_{p2}

EXAMPLE

GIVEN $L(s) = \dfrac{L_0}{\left(1+ s/\omega_{p1}\right)\left(1+ s/\omega_{p2}\right)\left(1+ s/\omega_{p3}\right)}$

& $L_0 \gg 1$ $\quad \omega_{p1} < \omega_{p2} \ll \omega_{p3}$

FIND ω_{p1}' SUCH THAT $PM \approx 60^0$
IN TERMS OF L_0 & ω_{p2}

ω_{p1}' REPLACES ω_{p1} SUCH THAT $\omega_{p1}' \ll \omega_{p2}$

SINCE $L_0 \gg 1$

FOR $\omega_{p1}' \ll \omega_{p2} \ll \omega_{p3}$

$$L(s) \approx \frac{L_0}{\left(s/\omega_{p1}'\right)\left(1 + s/\omega_{p2}\right)}$$

$$\angle L(j\omega_1) \approx -90^0 - \tan^{-1}\left(\frac{\omega_1}{\omega_{p2}}\right)$$

$$PM = \angle L(j\omega_1) - (-180^0) = 60^0 \text{ (GIVEN)}$$

$$\angle L(j\omega_1) = -120^0$$

ST16B

$$\tan^{-1}\left(\frac{\omega_1}{\omega_{p2}}\right) = 30^0$$

$$\omega_1 = \left(\sqrt{\tfrac{1}{3}}\right)\omega_{p2}$$

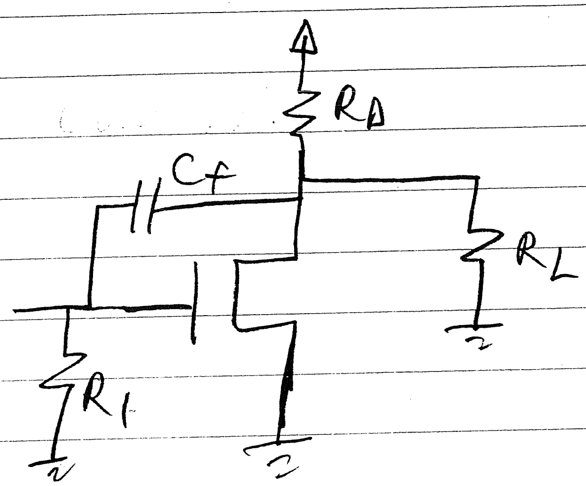
$$|L(j\omega_1)| = 1 = \frac{L_0}{\left(\frac{\omega_1}{\omega_{p1}'}\right)\left(1+\left(\frac{\omega_1}{\omega_{p2}}\right)^2\right)^{1/2}}$$

$$1 = \frac{L_0\,\omega_{p1}'}{\left(\sqrt{\tfrac{1}{3}}\,\omega_{p2}\right)\left(\tfrac{4}{3}\right)^{1/2}}$$

$$\boxed{\omega_{p1}' = \frac{\left(\frac{2}{3}\right)\omega_{p2}}{L_0}}$$

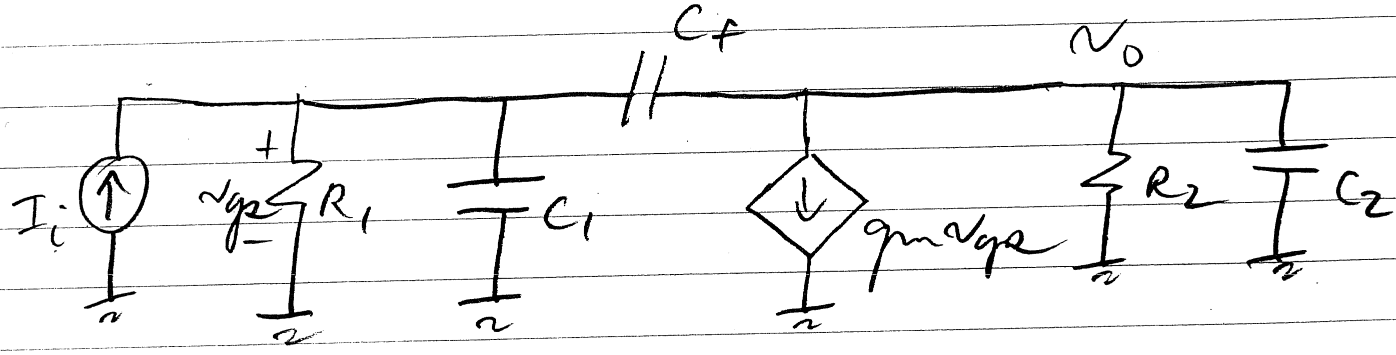
ST17

MILLER COMPENSATION & POLE SPLITTING



LET $R_2 = R_L \| R_D$

⇓ MODEL



IF $C_f = 0$ (NOT PRESENT)

$$\omega_{P1} = \frac{1}{R_1 C_1} \qquad \omega_{P2} = \frac{1}{R_2 C_2}$$

ST18

IF C_f ADDED

CAN SHOW

$$\omega_{p_1}' \cong \frac{1}{g_m R_2 C_f R_1}$$

$$\omega_{p_2} \cong \frac{g_m C_f}{C_1 C_2 + C_f (C_1 + C_2)}$$

SO AS C_f INCREASED

$\omega_{p_1} \downarrow$ & $\omega_{p_2} \uparrow$

POLE SPLITTING. SO BETTER THAN JUST ADDING C_C IN PARALLEL WITH C_1

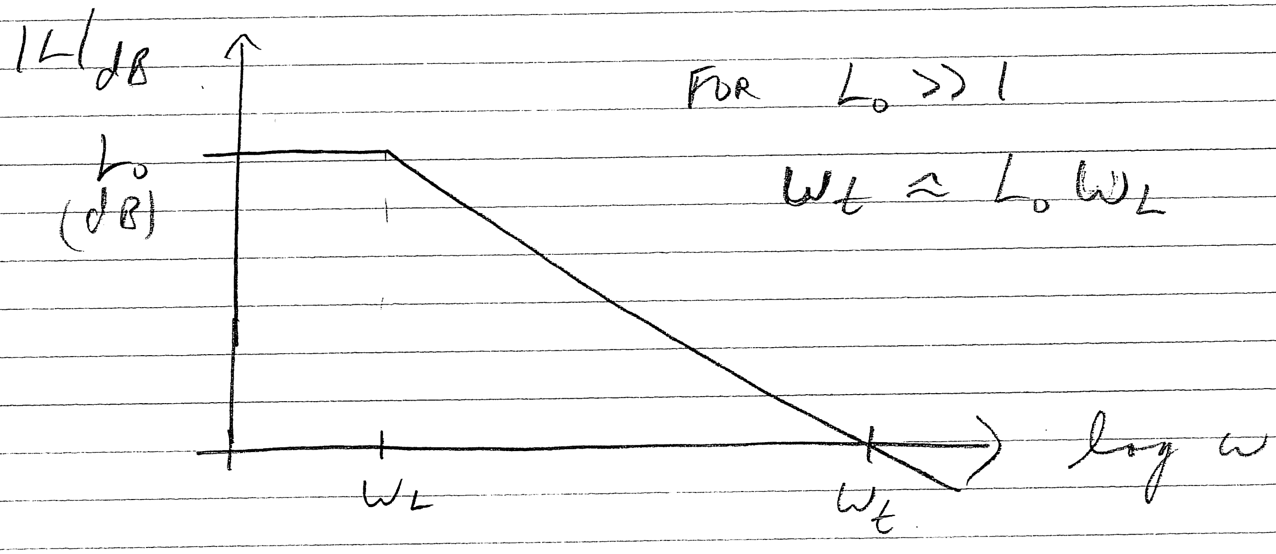
ESTIMATING SETTLING TIME FOR A FEEDBACK SYSTEM WITH PHASE-MARGIN > 60°

(PM)

SINCE PM > 60°, CAN APPROXIMATE $L(s)$ AS 1ST ORDER SYSTEM

$$L(s) \simeq \frac{L_o}{1 + s/\omega_L}$$

$L(s) \Rightarrow$ LOOP GAIN



FOR $L_o \gg 1$

$$\omega_t \simeq L_o \omega_L$$

WILL FIND FOR CLOSED-LOOP SYSTEM

$$\omega_{CL} \simeq \omega_t$$

$$\therefore \quad \tau_{CL} = \frac{1}{\omega_{CL}} \simeq \frac{1}{\omega_t}$$

ST20

TO SEE THIS RESULT

RECALL POLES OF CLOSED-LOOP SYSTEM FOUND FROM

$$1 + L(s) = 0$$

$$1 + \frac{L_0}{1 + s/\omega_L} = 0 \Rightarrow \text{POLE AT}$$

$$s = -(1 + L_0)\,\omega_L$$

ASSUMING $L_0 \gg 1 \Rightarrow$ POLE AT

$$s \approx -L_0\omega_L \cong -\omega_t$$

SO CLOSED-LOOP POLE AT $s \approx -\omega_t$

SO $\omega_{CL} \approx \omega_t$

& $\tau_{CL} = \dfrac{1}{\omega_t}$