

7.1 For  $I = 10 \mu\text{A}$ :

$$g_m = \frac{I}{V_T} = \frac{10 \mu\text{A}}{25 \text{ mV}} = 0.4 \text{ mA/V}$$

$$r_\pi = \frac{\beta}{g_m} = \frac{100}{0.4 \text{ mA/V}} = 250 \text{ k}\Omega$$

$$r_o = \frac{V_A}{I} = \frac{10 \text{ V}}{10 \mu\text{A}} = 1 \text{ M}\Omega$$

$$A_o = g_m r_o = \frac{V_A}{V_T} = \frac{10 \text{ V}}{0.025 \text{ V}} = 400 \text{ V/V}$$

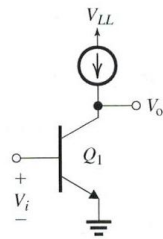
For  $I = 100 \mu\text{A}$ :

$$g_m = \frac{100 \mu\text{A}}{25 \text{ mV}} = 4 \text{ mA/V}$$

$$r_\pi = \frac{100}{4 \text{ mA/V}} = 25 \text{ k}\Omega$$

$$r_o = \frac{10 \text{ V}}{100 \mu\text{A}} = 100 \text{ k}\Omega$$

$$A_o = 4 \text{ mA/V} (100 \text{ k}\Omega) = 400$$



For  $I = 1 \text{ mA}$ :

$$g_m = \frac{1 \text{ mA}}{25 \text{ mV}} = 40 \text{ mA/V}$$

$$r_\pi = \frac{100}{40 \text{ mA/V}} = 2.5 \text{ k}\Omega$$

$$r_o = \frac{10 \text{ V}}{1 \text{ mA}} = 10 \text{ k}\Omega$$

$$A_o = 40 \text{ mA/V} (10 \text{ k}\Omega) = 400$$

$I$	$g_m$	$r_\pi$	$r_o$	$A_o$
10 $\mu\text{A}$	0.4 mA/V	250 k $\Omega$	1 M $\Omega$	400
100 $\mu\text{A}$	4.0 mA/V	25 k $\Omega$	100 k $\Omega$	400
1 mA	40 mA/V	2.5 k $\Omega$	10 k $\Omega$	400

7.3  $g_m = \frac{I_D}{V_{OV}}$ , so

$$I_D = \frac{g_m V_{OV}}{2} = \frac{2 \text{ mA/V} (0.25 \text{ V})}{2} = 0.25 \text{ mA}$$

From chapt. 5,  $k'_n = \mu_n C_{ox}$

since  $g_m = \sqrt{2\mu_n C_{ox} (W/L) \sqrt{I_D}}$ ,

$$2 \text{ mA/V} = \sqrt{2(200 \mu\text{A/V}^2)(W/L)(250 \mu\text{A})}$$

yielding

$$W/L = 40$$

so that

$$W = 40(0.5 \mu\text{m}) = 20 \mu\text{m}$$

7.4 Assuming that the MOSFET is operating above  $V_t$ ,

$$A_o = \frac{V_A' \sqrt{2(\mu_n C_{ox})(W/L)}}{\sqrt{I_D}}$$

If  $I_D$  is decreased to 25  $\mu\text{A}$ ,

$$A_o \text{ is increased by } \frac{1}{\sqrt{1/4}} = 2$$

$$g_m = \sqrt{2(\mu_n C_{ox})(W/L)} \cdot \sqrt{I_D}$$

so,  $g_m$  is decreased by

$$\sqrt{1/4} = 1/2$$

If  $I_D$  is increased to 400  $\mu\text{A}$ ,

$$A_o \text{ is decreased by } \frac{1}{\sqrt{4}} = \frac{1}{2}$$

$$g_m \text{ increases by } \sqrt{4} = 2$$

7.5 In this problem, we have two relevant equations, but three unknowns. so, one parameter can be chosen.

If we multiply the equations for  $A_o$  and  $g_m$ , we can eliminate  $I_D$  and  $L$ :

$$A_o \cdot g_m = \frac{V_A' \sqrt{2(\mu_n C_{ox})(WL)}}{\sqrt{I_D}}$$

$$\sqrt{2(\mu_n C_{ox})(WL)} \cdot \sqrt{I_D} \text{ so that,}$$

$$A_o \cdot g_m = V_A' (2\mu_n C_{ox})W, \text{ and}$$

$$W = \frac{A_o \cdot g_m}{V_A' (2\mu_n C_{ox})} = \frac{25(1 \text{ mA/V})}{5 \text{ V}/\mu\text{m}(2)(387 \mu\text{A}/\text{V}^2)}$$

$$= 6.46 \mu\text{m}$$

Since  $L$  must be  $\geq 0.18 \mu\text{m}$ , arbitrarily choose  $L = 0.3 \mu\text{m}$

$$\text{Then, } \frac{W}{L} = \frac{6.46 \mu\text{m}}{0.3 \mu\text{m}} = 21.5$$

To find  $I_D$ :

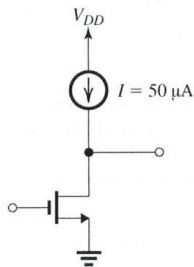
$$g_m^2 = 2 \mu_n C_{ox} (W/L) I_D$$

so,

$$I_D = \frac{g_m^2}{2 \mu_n C_{ox} (W/L)} = \frac{(1 \text{ mA/V})^2}{2 (387 \mu\text{A}/\text{V}^2)(21.5)} = 60 \mu\text{A}$$

Note that many answers are possible.

7.9



Since  $A_o = \frac{2V_A' L}{V_{OV}}$ , and the current source is ideal,

$$L = \frac{A_o V_{OV}}{2V_A'} = \frac{100(0.2 \text{ V})}{2(20 \text{ V}/\mu\text{m})} = 0.5 \mu\text{m}$$

$$\text{Since } I_D = \frac{1}{2}(\mu_n C_{ox})\left(\frac{W}{L}\right)V_{OV}^2,$$

$$\frac{W}{L} = \frac{2I_D}{(\mu_n C_{ox})V_{OV}^2} = \frac{2(50 \mu\text{A})}{(200 \mu\text{A}/\text{V}^2)(0.2 \text{ V})^2} = 12.5$$

7.12 If both MOSFETs have the same current-source values, and  $|V_{An}| = |V_{Ap}|$ , all  $r_{O}$  values are equal.

The small-signal model shows all  $r_{OS}$ :

$$V_O = -g_{m2} V_{gs2} (r_{O2} \parallel r_{o2})$$

$$= -\frac{1}{2} g_{m2} V_{gs2} r_O$$

$$V_{gs2} = -g_{m1} V_{gs1} (r_{O1} \parallel r_{o1})$$

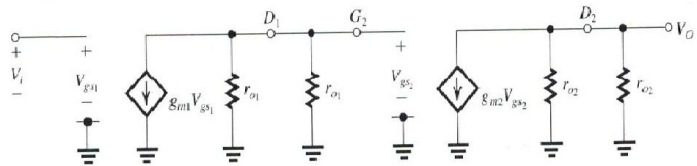
$$= -g_{m2} V_{gs1} r_O$$

$$\text{since } V_{gs1} = V_i,$$

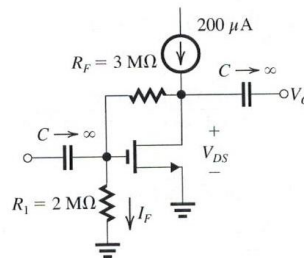
$$A_V = \frac{V_O}{V_i} = \left(-\frac{1}{2} g_{m2} r_O\right) \left(-\frac{1}{2} g_{m1} r_O\right)$$

$$A_V = \frac{1}{4} g_{m1} g_{m2} r_O^2$$

This figure belongs with 7.12



7.13



(a) If we neglect the current through  $R_F$ ,

$$I_D = 200 \mu\text{A} = \frac{1}{2} k_n' (W/L) V_{OV}^2$$

$$V_{OV} = \sqrt{\frac{2I_D}{k_n' (W/L)}} = \sqrt{\frac{2(200 \mu\text{A})}{2 \text{ mA}/\text{V}^2}} = 0.45 \text{ V}$$

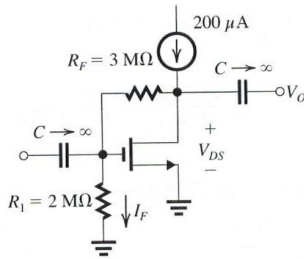
$$V_{GS} = V_i + V_{OV} = 0.5 + 0.45 = 0.95 \text{ V}$$

The current through the feedback network is

$$I_F = \frac{V_G}{R_1} = \frac{0.95 \text{ V}}{2 \text{ M}\Omega} = 0.475 \mu\text{A}$$

This is  $\ll 200 \mu\text{A}$ , so this assumption is justified.

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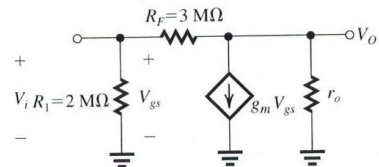
$V_{GS} = V_i + V_{OV} = 0.5 + 0.45 = 0.95 \text{ V}$   
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$$V_{DS} \approx I_F (R_F + R_1) = 0.475 \mu\text{A} (3 \text{ M}\Omega + 2 \text{ M}\Omega) = 2.38 \text{ V} \approx 2.4 \text{ V}$$

(b) small-signal model:



KCL at the output node yields

$$\frac{V_o}{r_o} + g_m V_{gs} + \frac{V_o - V_i}{R_F} = 0$$

since  $V_{gs} = V_i$

$$\frac{V_o}{r_o} + g_m V_i + \frac{V_o}{R_F} - \frac{V_i}{R_F} = 0 \text{ or}$$

$$\frac{V_o}{V_i} = \frac{\left(\frac{1}{R_F} - g_m\right)}{\left(\frac{1}{r_o} + \frac{1}{R_F}\right)}$$

$$g_m = \frac{2I_D}{V_{OV}} = \frac{2(200 \mu\text{A})}{0.45 \text{ V}} = 0.89 \text{ mA/V}$$

$$r_o = \frac{V_A}{I_D} = \frac{20 \text{ V}}{200 \mu\text{A}} = 100 \text{ k}\Omega$$

so,

$$\frac{V_o}{V_i} = \frac{\frac{1}{3000 \text{ K}} - 0.89 \text{ mA/V}}{\frac{1}{100 \text{ K}} + \frac{1}{3000 \text{ K}}} = -86.1 \text{ V/V}$$

To find the peak of the maximum sinewave output possible, we note that the current source is assumed to be ideal. Therefore, sinewave amplitude will be limited by the negative excursion. Since this happens when

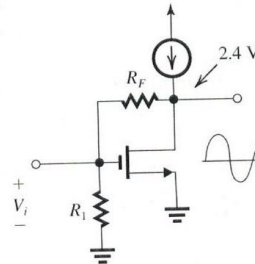
$$V_{DS} = V_{OV} = 0.45 \text{ V,}$$

the maximum peak amplitude will be

$$2.4 - 0.45 = 1.95 \text{ V}$$

(That is, the output will vary between 0.45V and

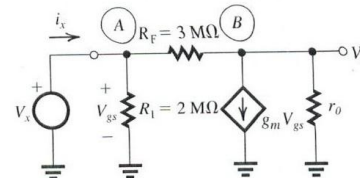
$$2.4 + 1.95 = 4.35 \text{ V.})$$



The corresponding input voltage is

$$V_{i\text{peak}} = \frac{V_{o\text{peak}}}{|A_v|} = \frac{1.95 \text{ V}}{86.1 \text{ V/V}} = 23 \text{ mV}_{\text{peak}}$$

(c) To find  $R_{in}$ , we apply a test voltage  $V_x$  to the input



KCL at node A:

$$i_x = \frac{V_x}{R_1} + \frac{V_x - V_o}{R_F}$$

KCL at node B:

$$\frac{V_x - V_o}{R_F} = \frac{V_o}{r_o} + g_m V_x$$

$$\Rightarrow V_o = \frac{V_x \left(\frac{1}{r_o} - g_m\right)}{\frac{1}{r_o} + \frac{1}{R_F}}$$

Substituting into the first equation, we get

$$i_x = \frac{V_x}{R_1} + \frac{V_x}{R_F} - \frac{V_x}{R_F} \frac{\left(\frac{1}{r_o} - g_m\right)}{\left(\frac{1}{r_o} + \frac{1}{R_F}\right)}$$

so that

$$R_{in} = \frac{V_x}{i_x} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_F} - \frac{1}{R_F} \frac{\frac{1}{r_o} - g_m}{\frac{1}{r_o} + \frac{1}{R_F}}}$$

$$R_{in} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_F} - \frac{1}{(R_F)^2} + \frac{g_m}{R_F} + \frac{1}{r_o}}$$

$$R_{in} = \frac{1}{\frac{1}{2 \text{ m}\Omega} + \frac{1}{3 \text{ m}\Omega} - \frac{1}{(3 \text{ m}\Omega)^2} + \frac{0.89 \text{ mA/V}}{3 \text{ m}\Omega} + \frac{1}{0.1 \text{ m}\Omega}}$$

$$R_{in} = 33.9 \text{ k}\Omega$$