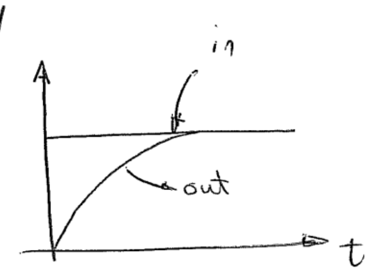
Problem 5

D. 8

$$V_{out}(t) = V_{final} + (V_{initial} - V_{final})e^{-t/\tau}$$



$$= 10 + (0 - 10)e^{-t/\tau}$$

$$= 10 - 10e^{-t/\tau}$$

$$\Rightarrow e^{-t/\tau} = \frac{10 - V_{out}(t)}{10} \Rightarrow \frac{t}{\tau} = \ln\left(\frac{10}{10 - V_{out}(t)}\right)$$

$$\Rightarrow t = \tau \cdot \ln\left(\frac{10}{10 - V_{out}(t)}\right)$$

$V_{out}(t)$	5 (V)	9 (V)	9.9 (V)	9.99 (V)
t	$\tau \ln(2)$	$\tau \ln(10)$	4.6τ	6.9τ

$$f_{3db} = 100\,MHz = f_0$$

$$\Rightarrow \omega_0 = 2\pi f_0 = \frac{1}{\tau} = \frac{1}{RC} \Rightarrow \tau = \frac{1}{2\pi \times 100 \times 10^6} = 1.59 \text{ nS}$$

$$V_{final} + (V_{initial} - V_{final}) e^{-t/\tau} = V_{out}(t)$$

assume $V_{initial} = 0$,

$V_{final} = V_f$

$$\Rightarrow V_{out}(t) = V_f - V_f e^{-t/\tau}$$

$$\Rightarrow t = \tau \ln\left(\frac{V_f}{V_f - V_{out}(t)}\right)$$

$V_{out}(t_1) = 0.1\, V_f$

$$\Rightarrow t_1 = \tau \ln\left(\frac{1}{0.9}\right) = 0.105\,\tau$$

$V_{out}(t_2) = 0.9\, V_f$

$$\Rightarrow t_2 = \tau \ln\left(\frac{1}{0.1}\right) = 2.303\,\tau$$

$$\Delta t = t_{rise} = t_2 - t_1 = 2.2\,\tau \cong 3.5 \text{ nS}$$

$$R \parallel \frac{1}{sC_2} = \left(\frac{1}{R} + sC_2\right)^{-1} = \frac{R}{1+sRC_2}$$

$$\frac{V_o(s)}{V_i(s)} = \frac{\dfrac{R}{1+sRC_2}}{\dfrac{1}{sC_1} + \dfrac{R}{1+sRC_2}} = \frac{R}{\dfrac{1+sRC_2+sRC_1}{sC_1}}$$

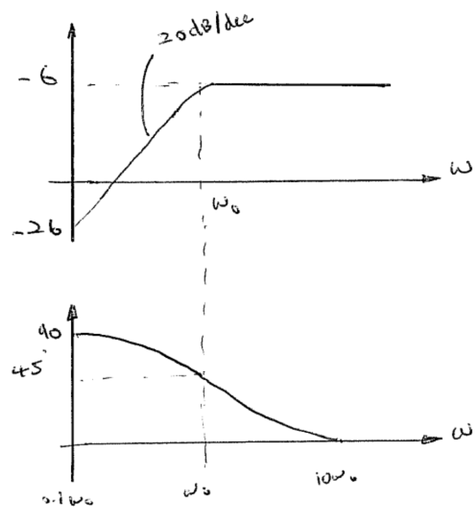
$$= \frac{sRC_1}{1+s(RC_1+RC_2)} = \frac{RC_1}{RC_1+RC_2} \quad \frac{s}{\dfrac{1}{R(C_1+C_2)}+s}$$

$$= \frac{C_1}{C_1+C_2} \cdot \frac{s}{\dfrac{1}{R(C_1+C_2)}+s}$$

It is a HPF. $\omega_0 = \frac{1}{\tau} = (R(C_1+C_2))^{-1} = 10\,Hz \rightarrow$ pole

$$k = \frac{C_1}{C_1+C_2} = \frac{1}{2}$$

We have a zero at 0.



The bode plot not drawn to scale.

E.2 a)

$$T(s) = \frac{R_L}{R_L + R_s + \frac{1}{sC}} = \frac{R_L sC}{1 + sC(R_L + R_s)}$$

$$= \frac{R_L C}{C(R_L+R_s)} \cdot \frac{s}{\frac{1}{C(R_L+R_s)} + s} = \frac{R_L}{R_L + R_s} \cdot \frac{s}{\frac{1}{C(R_L+R_s)} + s}$$

b) at high frequencies:

$$\frac{V_o(s)}{V_i(s)} = \frac{R_L}{R_L + R_s} = \frac{R_L}{R_L + 10^k} > 0.7 \quad --- \text{I}$$

$$|T(j\omega)| = \frac{R_L}{R_L + R_s} \cdot \sqrt{\frac{\omega^2}{\omega^2 + \frac{1}{C^2(R_L+R_s)^2}}}$$

at 10Hz, $\omega = 2\pi \times 10 = 62.8$ rad/s

$$\Rightarrow \frac{R_L}{R_L + 10^k} \cdot \sqrt{\frac{1}{3947 + \frac{1}{C^2(R_L+R_s)^2}}} \cdot 62.8 =$$

$$\frac{62.8 \cdot R_L \cdot C}{\sqrt{1 + 3947 C^2 (R_L+R_s)^2}} > 0.1 \quad --- \text{II}$$

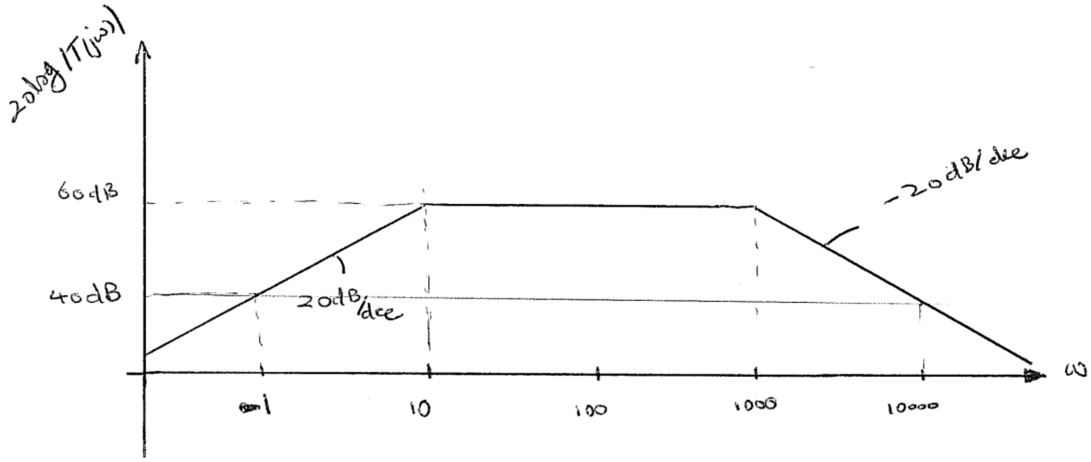
Smallest value of R_L to satisfy I is: $R_L = 23.3^k$

Substituting R_L into II:

$$C = 6.89 \times 10^{-8} \text{ F} = 68.9 \text{ nF}$$

(5)

$$T(s) = \frac{10^6 \cdot s}{(s+10)(s+10^3)} = \frac{10^2 \cdot s}{\left(1+\frac{s}{10}\right)\left(1+\frac{s}{10^3}\right)}$$



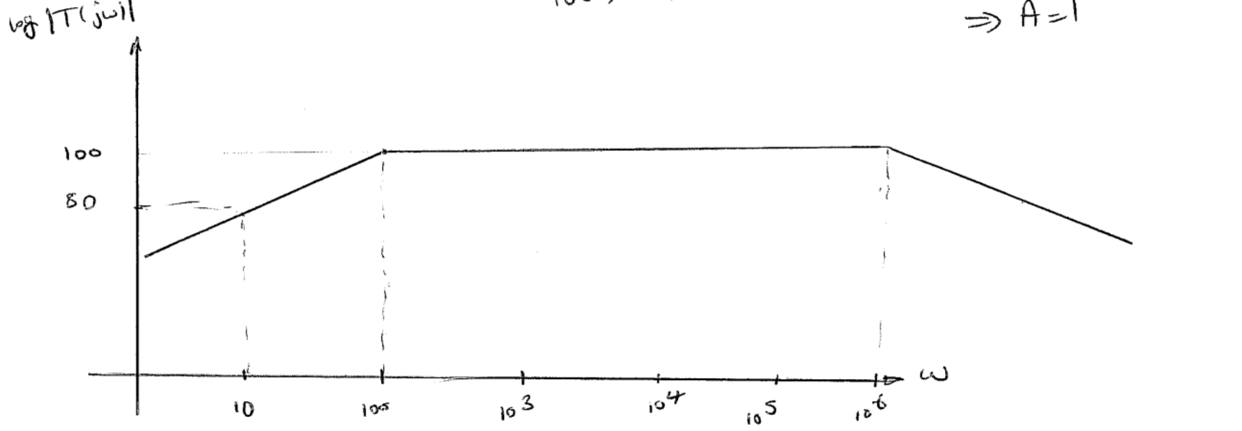
freq.	1	10	10^2	10^3	10^4	10^5
magnitude	20	40	40	40	20	0

b

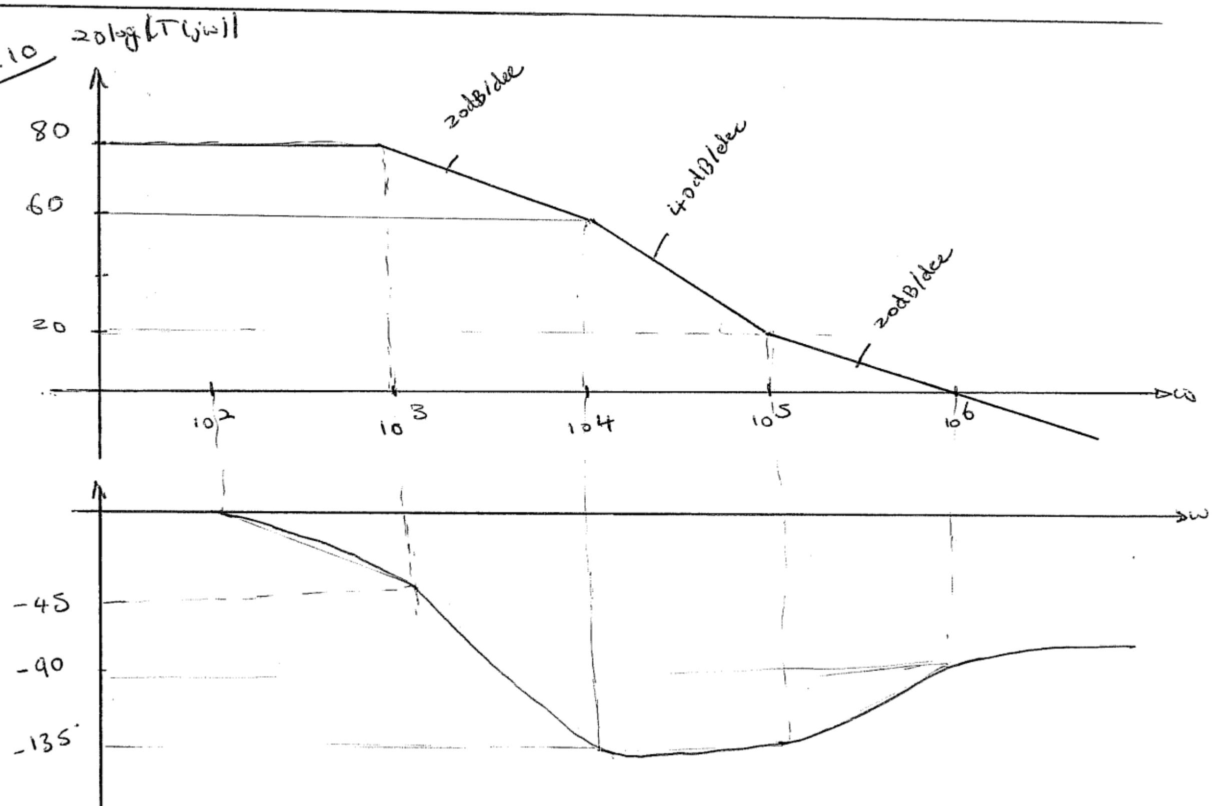
$$T(s) = \frac{s \times A}{\left(1+\frac{s}{100}\right)\left(1+\frac{s}{10^6}\right)}$$

, to find A: $\frac{A\omega}{\frac{\omega}{100}} \approx |T(j\omega)|_{\omega=10^4}$

$\Rightarrow A=1$



E.10



at $\omega = 10^6$, mag $\approx$ 0dB

phase $\approx -90°$

9.1 Using eq. $A_{VO} = \dfrac{-g_m R_D}{1 + g_m R_S}$

$$A_M = \frac{-g_m \cdot R_D}{1 + g_m \cdot R_S} = \frac{-1\text{ mA/V} \cdot 10\text{ k}\Omega}{1 + 1\text{ mA/V} \cdot 6\text{ k}\Omega}$$

$$= \frac{-10}{1+6}$$

$$A_M = 1.43\text{ V / V}$$

$$f_L = \frac{1}{2\pi\left(\frac{1}{g_m} \parallel R_S\right) \cdot C_S} = 20\text{ Hz}$$

$$C_S = \frac{1}{2\pi \cdot 20(1\text{ K} \parallel 6\text{ K})} = 9.3\ \mu\text{F}$$

9.2 $f_{C_{C2}} = \dfrac{1}{2\pi \cdot C_{C2}(R_L + R_D \parallel r_O)}$

≤ 10 Hz

$$\Rightarrow C_{C2} \ge \frac{1}{10 \cdot 2\pi\ (10\text{ K} + 10\text{ K} \parallel 100\text{ K})}$$

$C_{C2} \ge 0.83\ \mu$F

If I_D is doubled with both r_O and R_D halved

$$f_{C_{C2}} = \frac{1}{2\pi .\, 0.83\ \mu\ (10\text{ K} + 5\text{ K} \parallel 50\text{ K})}$$

$= 13.18$ Hz

For higher power designs, where I_D is incremented and r_o and R_D are reduced, eventually $r_O \parallel R_D << R_L$ then R_L becomes dominant in determinig the corner frequency:

$$f_{C_{C2}} = \frac{1}{2\pi \cdot C_{C2} \cdot R_L} = 19.17\text{ Hz}$$

9.3 Refer to Fig. P. 9.3: $g_m = 5$ mA/V

$$A_M = \frac{-R_G}{R_G + R_{sig}} \cdot g_m\,(R_D \parallel R_L)\ \text{where}$$

$$R_G = 47\text{ M} \parallel 10\text{ M} = 8.25\text{ M}\Omega$$

$$A_M = \frac{-8.25}{8.25 + 0.1} \cdot 5 \times 10^{-3} \cdot (4.7 \parallel 10)\ .\ 10^3$$

$$= -15.79\text{ V/V}$$

$$f_{P1} = \frac{1}{2\pi\ C_{C1}(R_G + R_{sig})}\quad \text{(Eq. 9.2)}$$

$$f_{P1} = \frac{1}{2\pi \cdot 0.01 \times 10^{-6} \cdot (8.25 + 0.1) \times 10^6}$$

$= 1.9$ Hz

$$f_{P2} = \frac{1}{2\pi \cdot C_S \cdot \left(R_S \parallel \frac{1}{g_m}\right)}$$

$$= \frac{1}{2\pi \cdot 10 \times 10^{-6}\left(2\text{ K} \parallel \frac{1}{5}\text{ K}\right)} = 87.5\text{ Hz}$$

$$f_{P3} = \frac{1}{2\pi \cdot C_{C2} \cdot (R_D + R_L)}$$

$$= \frac{1}{2\pi \cdot 0.1 \times 10^{-6}(4.7 + 10) \times 10^3} = 108.3\text{ Hz}$$

$f_L \simeq 108.3$ Hz

9.4 $A_M = -\dfrac{R_G}{R_G + R_{sig}} g_m\,(r_O \parallel R_D \parallel R_L)$

$$= -\frac{2 \times 3}{2 + 0.5}(20\text{ K} \parallel 10\text{ K}),\ (r_O = \infty)$$

$$A_M = -16\text{ V / V}$$

$$f_{P1} = \frac{1}{2\pi C_{C1}(R_G + R_{sig})} \Rightarrow 3\text{ Hz}$$

$$= \frac{1}{2\pi\ C_{C1}(2 + 0.5) \times 10^6}$$

$\Rightarrow C_{C1} = 21.2$ nF

$$f_{P2} = \frac{g_m}{2\pi C_S} \Rightarrow 50\text{ Hz} = \frac{3 \times 10^{-3}}{2\pi \times C_S}$$

$\Rightarrow C_S = 9.6\ \mu$F

$$f_{P3} = \frac{1}{2\pi C_{C2}(R_D + R_L)}$$

$$\Rightarrow 10\text{ Hz} = \frac{1}{2\pi C_{C2}(10\text{ K} + 20\text{ K})}$$

$\Rightarrow C_{C2} = 0.5\ \mu$F

$f_L = 50$ Hz

9.5 Refer to Fig. 9.1

$g_m = 5$ mA/V, $R_{sig} = 200$ kΩ, $R_G = 10$ MΩ, $R_D = 3$ kΩ, $R_L = 5$ kΩ

If $C_{C1} = C_{C2} = C_S = 1\ \mu$F then from Eq 9.2, 9.4, 9.6

$$f_{P1} = \frac{1}{2\pi \cdot C_{C1}(R_G + R_{sig})}$$

$$= \frac{1}{2\pi \cdot 1 \times 10^{-6}(10 + 0.2) \times 10^6} = 15.6\text{ mHz}$$

$$f_{P2} = \frac{g_m}{2\pi \cdot C_S} = \frac{5 \times 10^{-3}}{2\pi \cdot 10^{-6}} = 795.8\text{ Hz}$$

$$f_{P3} = \frac{1}{2\pi \cdot C_{C2}(R_D + R_L)}$$

$$= \frac{1}{2\pi \cdot 10^{-6}(3 + 5) \times 10^3} = 19.9\text{ Hz}$$

From highest to lowest :

$f_{P2} = 785.8$ Hz, $f_{P3} = 19.9$ Hz, $f_{P1} = 15.6$ mHz

$$\frac{f_{P2}}{f_{P3}} = 39.5\quad \frac{f_{P3}}{f_{P1}} = 1275.6$$

If $f_{P2} = 10$ Hz

$$\Rightarrow C_S = \frac{5 \times 10^{-3}}{2\pi \cdot 10} = 79.6\ \mu\text{F}$$