# REVIEW OF COMPLEX ARITHMETIC

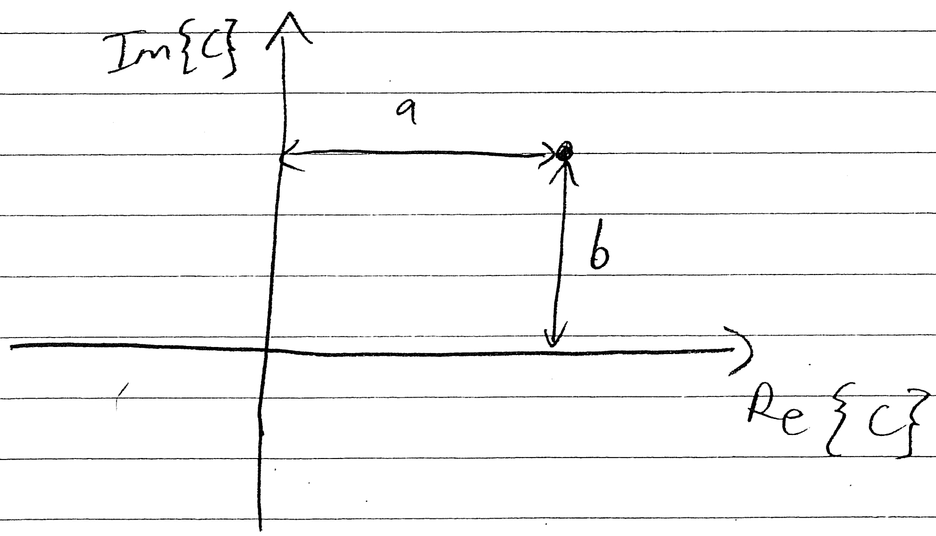
A COMPLEX VALUE "C" HAS A REAL PART AND IMAGINARY PART

$$a = \mathrm{Re}\{C\} \qquad b = \mathrm{Im}\{C\}$$

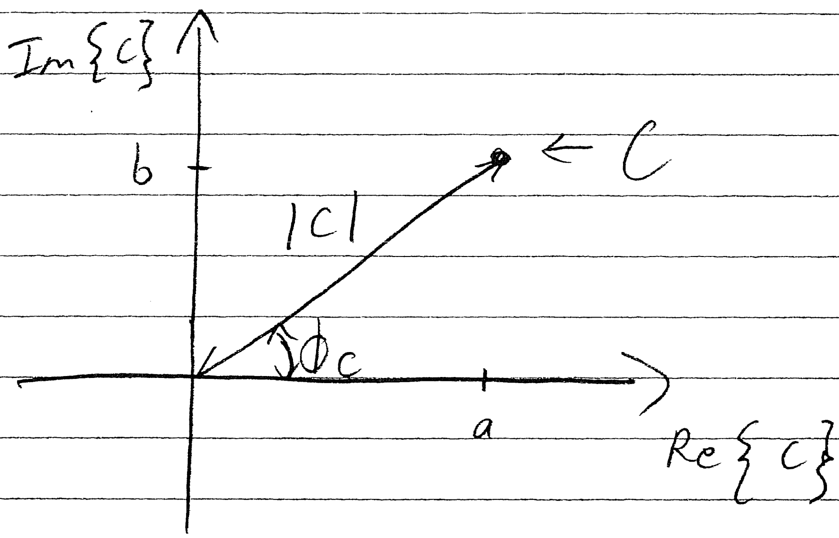
$$C = a + jb \qquad a, b \text{ BOTH REAL}$$

WHERE $j^2 = -1$

WHILE A REAL NUMBER IS A POINT ON A LINE, A COMPLEX NUMBER CAN BE EXPRESSED AS A POINT ON THE COMPLEX PLANE



VALUE $(a, b)$ ARE <u>CARTESIAN</u> REPRESENTATION OF $C$. CAN ALSO USE <u>POLAR</u> NOTATION




$$|C| = \sqrt{a^2 + b^2}$$

$$\phi_C = \tan^{-1}\left(\frac{b}{a}\right) \quad \text{IF} \quad a > 0$$

$$\phi_C = \tan^{-1}\left(\frac{b}{a}\right) + \pi \quad \text{IF} \quad a < 0$$

IF $|c|$ & $\phi_c$ GIVEN, CAN FIND a, b USING

$$a = |c| \cos \phi_c$$

$$b = |c| \sin \phi_c$$

SO

$$C = a + jb$$

$$= |c| \cos \phi_c + j|c| \sin \phi_c$$

$$= |c| (\cos \phi_c + j \sin \phi_c)$$

RECALL EULER'S EQUATION

$$e^{j\phi} = \cos\phi + j \sin \phi$$

SO...

$$C = |c| e^{j\phi_c}$$ POLAR

& $$C = a + jb$$ CARTESIAN

CARTESIAN EASIER FOR ADDING/SUBTRACTING 2 COMPLEX VALUE

$$c_1 = a_1 + jb_1 \qquad c_2 = a_2 + jb_2$$

$$c_1 + c_2 = (a_1 + a_2) + j(b_1 + b_2)$$

$$c_1 - c_2 = (a_1 - a_2) + j(b_1 - b_2)$$

POLAR EASIER FOR MULTIPLYING/DIVIDING

$$c_1 = |c_1| e^{j\phi_1} \qquad c_2 = |c_2| e^{j\phi_2}$$

$$c_1 c_2 = |c_1||c_2| e^{j(\phi_1 + \phi_2)}$$

$$c_1 / c_2 = \frac{|c_1|}{|c_2|} e^{j(\phi_1 - \phi_2)}$$

COMPLEX CONJUGATE $c^*$

$$c = a + jb = |c| e^{j\phi_c}$$

$$c^* = a - jb$$
$$= |c| e^{-j\phi_c}$$

IT IS A REFLECTION IN REAL AXIS

CAN SHOW

$$|c|^2 = c\,c^*$$

$$c + c^* = 2a$$

$$c - c^* = j2b$$