

REVIEW OF COMPLEX ARITHMETIC

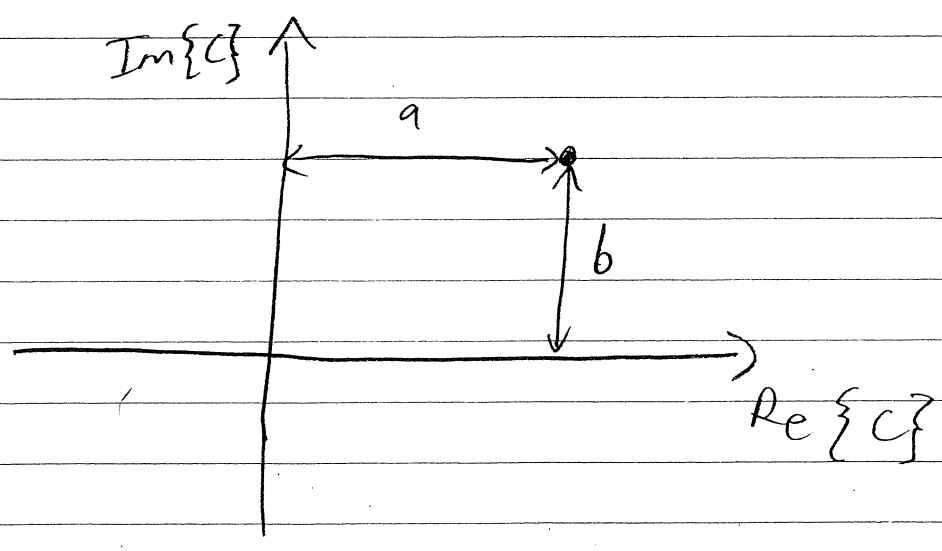
A COMPLEX VALUE "C" HAS A REAL PART AND IMAGINARY PART

$$a = \text{Re}\{C\} \qquad b = \text{Im}\{C\}$$

$$C = a + jb \qquad a, b \text{ BOTH REAL}$$

WHERE $j^2 = -1$

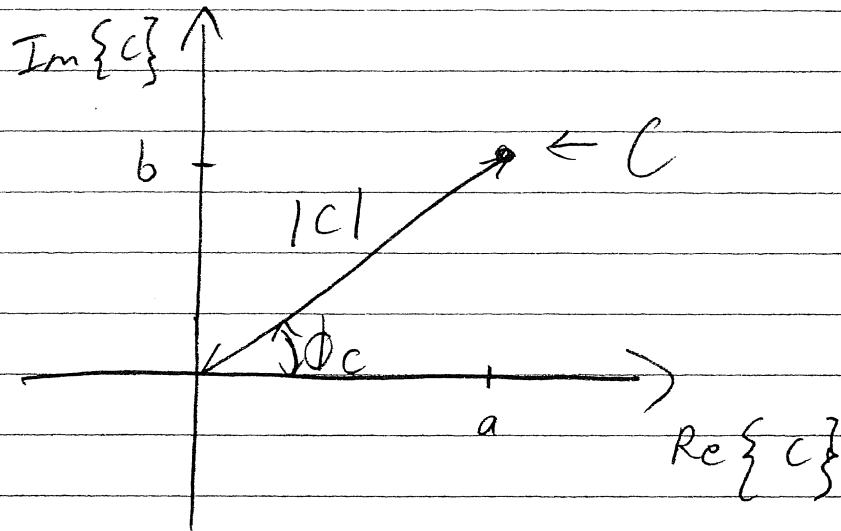
WHILE A REAL NUMBER IS A POINT ON A LINE, A COMPLEX NUMBER CAN BE EXPRESSED AS A POINT ON THE COMPLEX PLANE



(C-2)

VALUE (a, b) ARE CARTESIAN REPRESENTATION
OF C , CAN ALSO USE POLAR

NOTATION



$$|C| = \sqrt{a^2 + b^2}$$

$$\phi_c = \tan^{-1}\left(\frac{b}{a}\right) \quad \text{IF } a > 0$$

$$\phi_c = \tan^{-1}\left(\frac{b}{a}\right) + \pi \quad \text{IF } a < 0$$

(C-3)

IF $|c|$ & ϕ_c GIVEN, CAN
FIND a, b USING

$$a = |c| \cos \phi_c$$

$$b = |c| \sin \phi_c$$

SO

$$C = a + jb$$

$$= |c| \cos \phi_c + j|c| \sin \phi_c$$

$$= |c| (\cos \phi_c + j \sin \phi_c)$$

RECALL EULER'S EQUATION

$$e^{j\phi} = \cos \phi + j \sin \phi$$

SO...

$$C = |c| e^{j\phi_c} = \text{POLAR}$$

$$\downarrow C = a + jb \quad \text{CARTESIAN}$$

(C-4)

CARTESIAN EASIER FOR
ADDING/SUBTRACTING 2 COMPLEX VALUE

$$C_1 = a_1 + jb_1 \quad C_2 = a_2 + jb_2$$

$$C_1 + C_2 = (a_1 + a_2) + j(b_1 + b_2)$$

$$C_1 - C_2 = (a_1 - a_2) + j(b_1 - b_2)$$

POLAR EASIER FOR MULTIPLYING/DIVIDING

$$C_1 = |C_1| e^{j\phi_1} \quad C_2 = |C_2| e^{j\phi_2}$$

$$C_1 C_2 = |C_1| |C_2| e^{j(\phi_1 + \phi_2)}$$

$$C_1 / C_2 = \frac{|C_1|}{|C_2|} e^{j(\phi_1 - \phi_2)}$$

COMPLEX CONJUGATE C^*

$$C = a + jb = |c| e^{j\phi_c}$$

$$\begin{aligned} C^* &= a - jb \\ &= |c| e^{-j\phi_c} \end{aligned}$$

IT IS A REFLECTION IN REAL AXIS

CAN SHOW

$$|c|^2 = C C^*$$

$$C + C^* = 2a$$

$$C - C^* = j2b$$