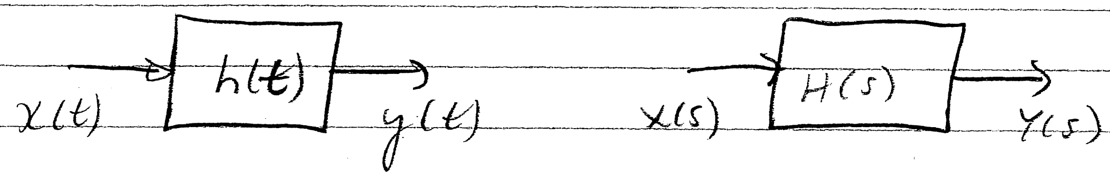
12.1 ANALOG FILTER TYPES AND SPECS



TIME DOMAIN FREQ DOMAIN

SYSTEM SHOULD BE LINEAR TIME-INVARIANT SO THAT FREQ ANALYSIS CAN BE USED.

- $h(t)$ IMPULSE RESPONSE OF FILTER.
- $H(s)$ LAPLACE TRANSFORM OF $h(t)$

$$y(t) = h(t) \circledast x(t) \qquad Y(s) = H(s)X(s)$$

$$H(s) = \frac{Y(s)}{X(s)}$$

"s" IS COMPLEX VARIABLE $\sigma + jw$
"w" IS PHYSICAL FREQ IN RAD/S

$H(s)$ IS FILTER'S "TRANSFER-FUNCTION"

FOR PHYSICAL FREQUENCIES CAN FIND FILTER'S RESPONSE BY:

$$H(s)\Big|_{s=jw} = H(jw) = |H(jw)|\, e^{j\phi(w)}$$

$|H(jw)|$ IS MAGNITUDE RESPONSE

$\phi(w)$ IS PHASE RESPONSE

GAIN IN dB

OFTEN MAGNITUDE RESPONSE EXPRESSED IN dB

$$|H(\omega)|_{dB} = 20 \log |H(j\omega)| \quad \text{in dB}$$

$|H(j\omega)|_{dB}$ IS FILTER GAIN IN dB

EXAMPLE

$\|H(j\omega)\|$	$\|H(j\omega)\|_{dB}$
1	0
$0.7071 = 1/\sqrt{2}$	−3.01
0.5	−6.02
0.1	−20
0.01	−40
0.001	−60

GROUP DELAY

$\phi(\omega)$ IS PHASE LEAD THROUGH FILTER.

$$\text{GROUP DELAY}, \quad T_d(\omega) \equiv -\frac{d\,\phi(\omega)}{\partial \omega}$$

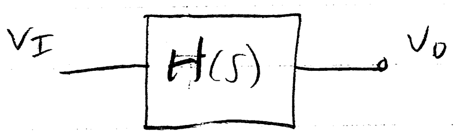
INDICATION OF DELAY THROUGH A FILTER.

— A LINEAR PHASE HAS A CONSTANT GROUP DELAY

WHEN DESIGNING FILTERS

OFTEN ONLY MAGNITUDE IS OF CONCERN (CAN TOLERATE VARYING GROUP DELAY) IN APPLICATIONS SUCH AS ANTI-ALIASING FILTER (DSP CAN FIX UP DELAY BUT NOT FOLD-OVER NOISE) OR SPEECH SIGNAL PROCESSING (THE EAR IS NOT SENSITIVE TO PHASE DISTORTION)

WHY "LINEAR-PHASE" FILTERS?

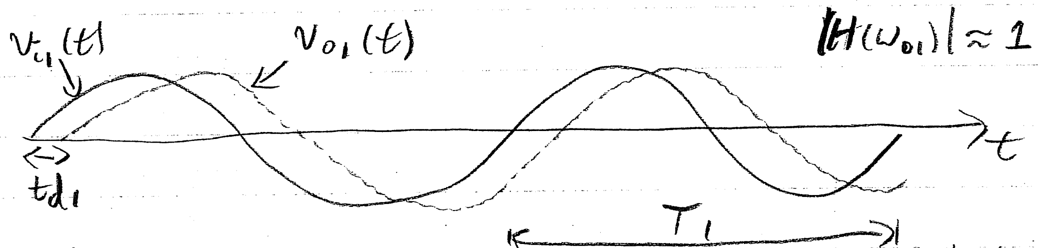


$$v_{i1}(t) = \cos(\omega_{01} t) \Rightarrow v_{01}(t) = |H(\omega_{01})| \cos(\omega_{01} t - \phi(\omega_{01}))$$

$$v_{i2}(t) = \cos(\omega_{02} t) \Rightarrow v_{02}(t) = |H(\omega_{02})| \cos(\omega_{02} t - \phi(\omega_{02}))$$

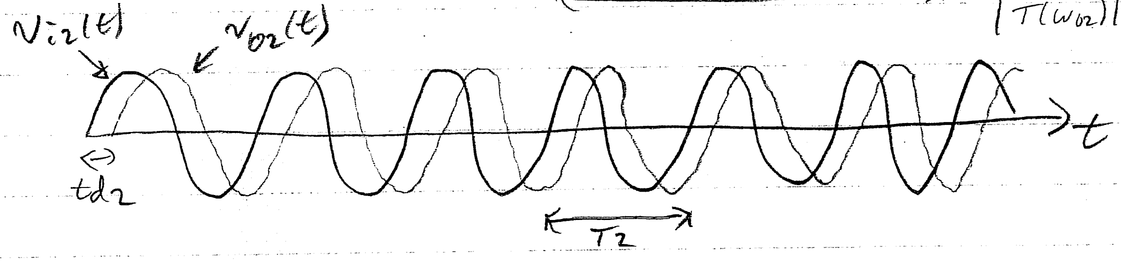
EX

$|H(\omega_{01})| \approx 1$



$$T_1 = \frac{1}{f_{01}} = \frac{2\pi}{\omega_{01}}$$

$|T(\omega_{02})| \approx 1$



$$T_2 = \frac{1}{f_{02}} = \frac{2\pi}{\omega_{02}}$$

WANT $td_1 = td_2$ SO THAT ALL FREQUENCIES IN PASSBAND EXPERIENCE THE SAME DELAY THROUGH FILTER – IMPORTANT FOR DATA COMMUNICATIONS & AUDIO (NOT FOR SPEECH USUALLY)

$$td_1 = \frac{\phi(\omega_{01})}{2\pi} \times T_1 = \frac{\phi(\omega_{01})}{2\pi} \times \frac{2\pi}{\omega_{01}} = \frac{\phi(\omega_{01})}{\omega_{01}}$$

SIMILARILY $td_2 = \dfrac{\phi(\omega_{02})}{\omega_{02}}$

$\therefore$ IF $td = td_1 = td_2 \Rightarrow \left.\begin{array}{l}\phi(\omega_{01}) = \omega_{01}\, td \\ \& \ \phi(\omega_{02}) = \omega_{02}\, td\end{array}\right\}$ $\boxed{\phi(\omega) = \omega\, td}$

FOR ALL FREQ TO HAVE SAME td

$\Rightarrow$ LINEAR-PHASE

ALTERNATE APPROACH

$$v_0(t) = v_I(t - t_d)$$

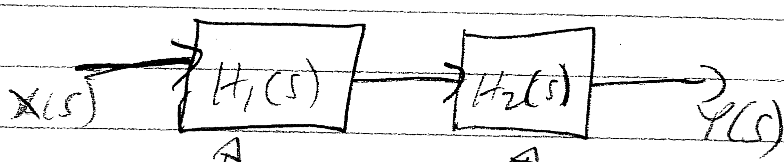
$$\therefore \quad V_0(s) = e^{-st_d} V_I(s)$$

$$\therefore \quad H(s) = e^{-st_d} \Rightarrow H(j\omega) = e^{-j\omega t_d}$$

$$\therefore \quad \phi(\omega) = -\omega t_d \quad \& \quad |H(\omega)| = 1$$

SO LINEAR PHASE GIVES A STRAIGHT DELAY.

IF MAGNITUDE & PHASE (OR GROUP DELAY)
IMPORTANT OFTEN USE

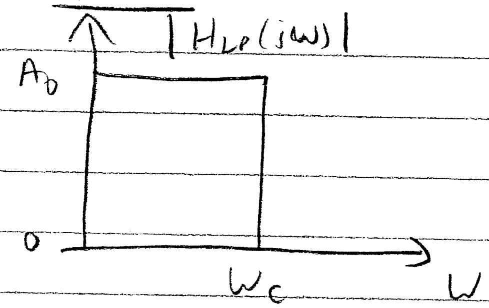


MAGNITUDE

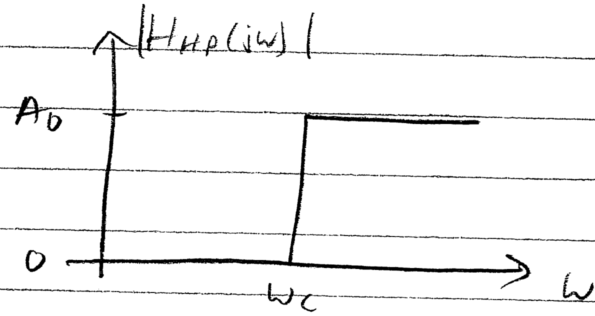
DELAY (OR PHASE)
EQUALIZER

FILTER CHARACTERISTICS

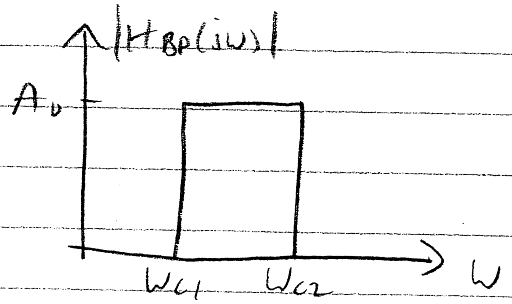
IDEAL



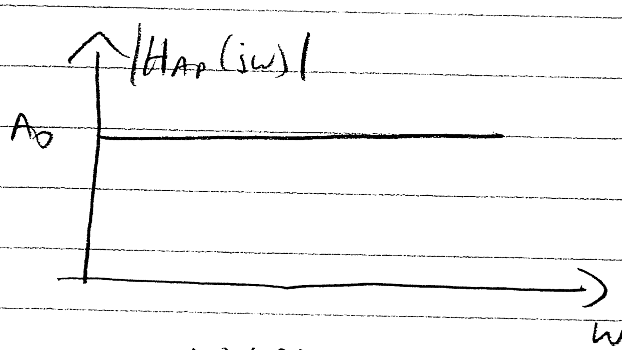
LOWPASS



HIGHPASS



BANDPASS

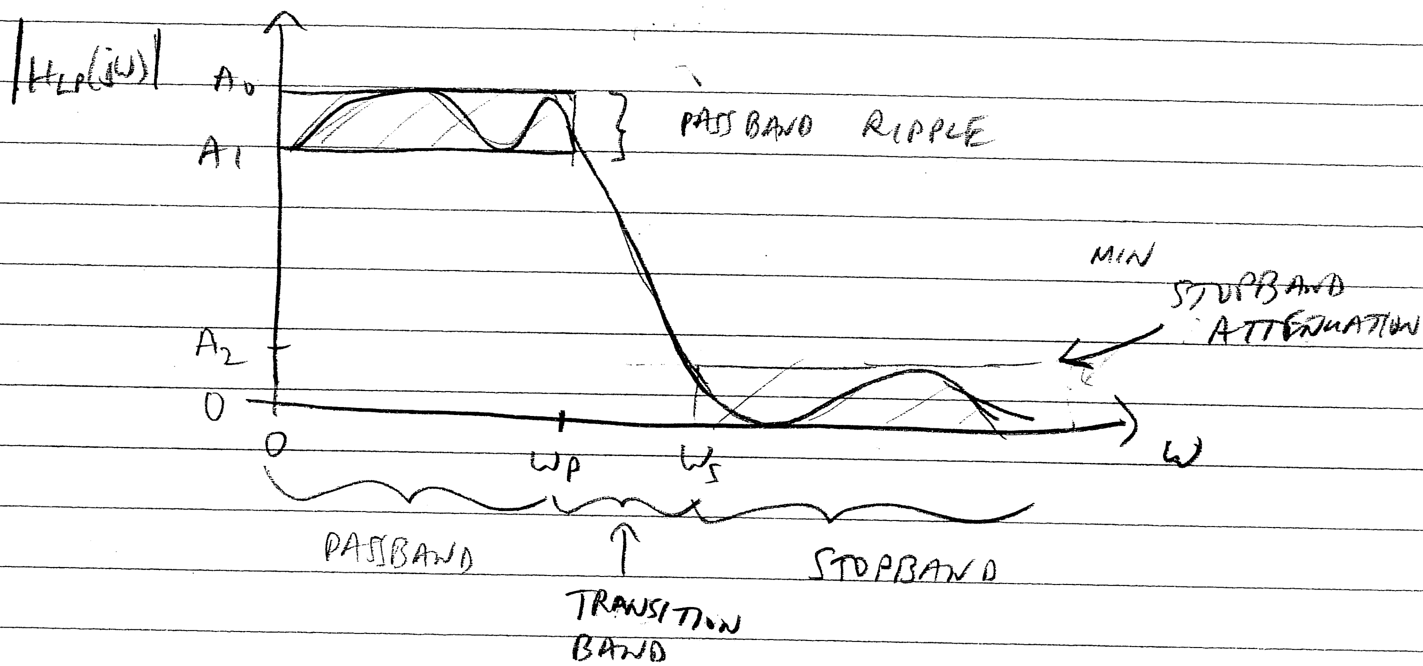


ALLPASS
(USED IN DELAY EQUALIZER)

- CANNOT REALIZE THESE "BRICK WALL" RESPONSES

NEED PASSBAND & STOPBAND VARIATIONS AND TRANSITION BAND

LOWPASS



- PASSBAND RIPPLE $0 < \omega < \omega_p$ $A_{MAX} \triangleq 20\log\left(\frac{A_0}{A_1}\right)$ (IN dB)

$20\log(A_0) - 20\log(A_1)$ (SMALLER THE BETTER)

STOPBAND ATTENUATION $\omega_s < \omega < \infty$ $A_{MIN} \triangleq 20\log\left(\frac{A_0}{A_2}\right)$ (LARGER THE BETTER)

TRANSITION BAND $\omega_p < \omega < \omega_s$ (SMALLER THE BETTER) : RESPONSE SHOULD BE BOUNDED BELOW A_0 (USUALLY MONOTONICALLY FALLING)

$$\text{ATTENUATION} = \frac{1}{\text{GAIN}}$$

$$\text{ATTEN (dB)} = -\ \text{GAIN (dB)}$$

EXAMPLE

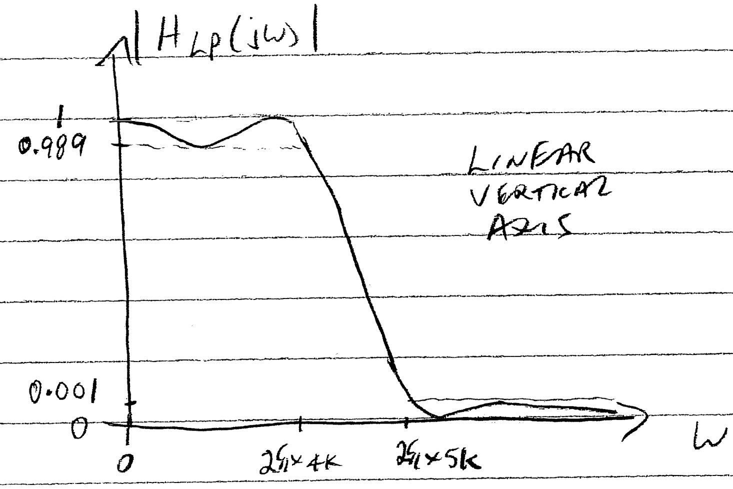
6 dB ATTENUATION = −6 dB GAIN

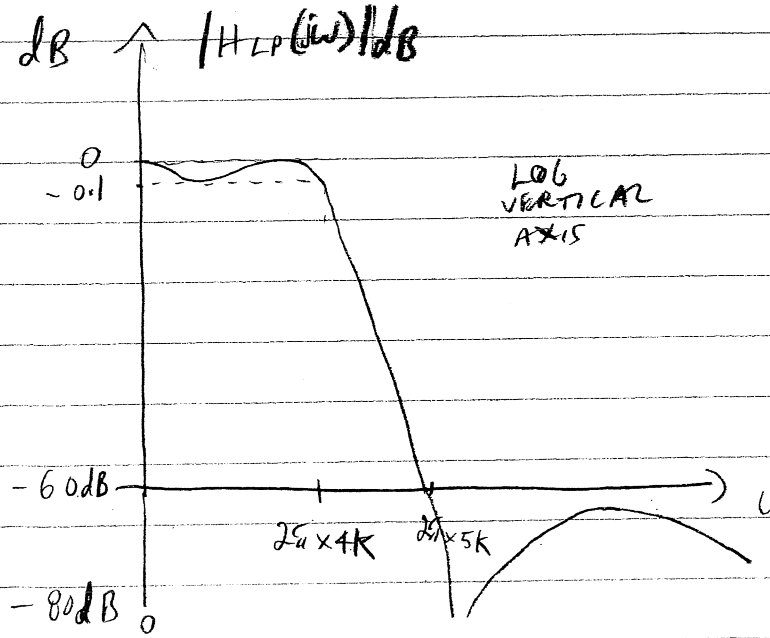
SIMILAR DEFINITIONS FOR HIGHPASS, BANDPASS, etc.

TYPICAL SPECIFICATIONS

LOWPASS

VOICE	PASSBAND	0 → 4KHZ	RIPPLE	0.1 dB ← AMAX
CODEC	STOPBAND	5KHZ → ∞	ATTEN	60 dB ← AMIN





EXAMPLE

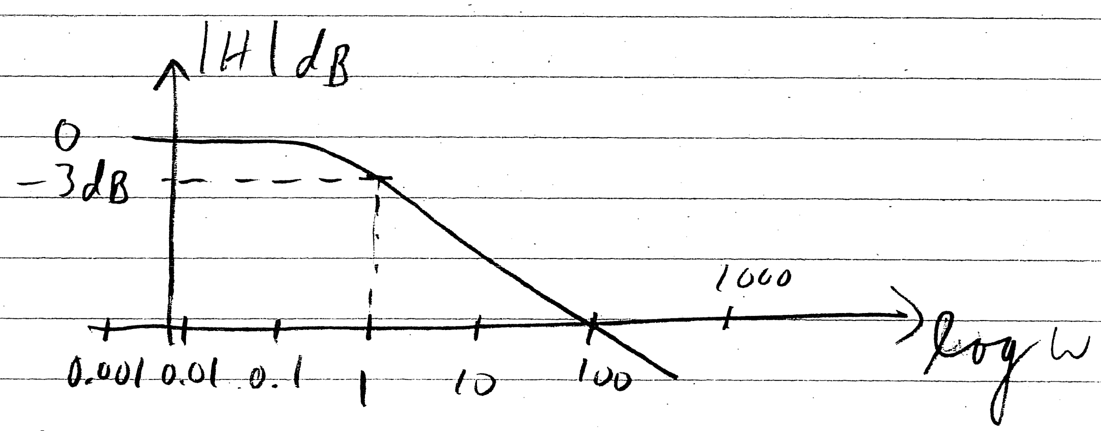
$$H(s) = \frac{1}{s+1}$$

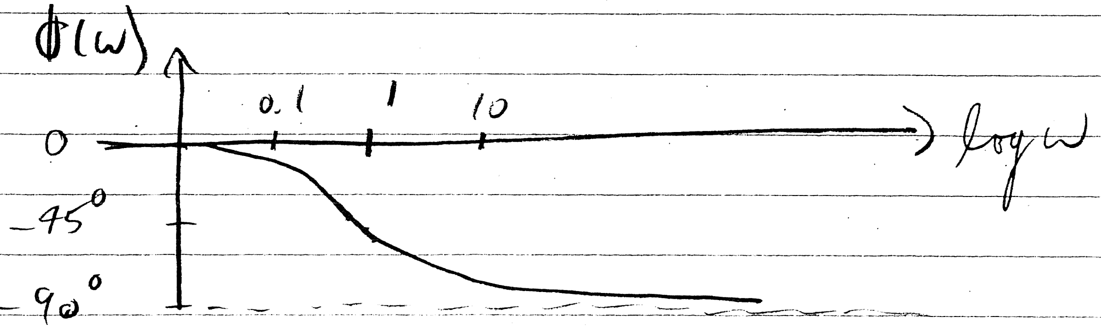
$$|H(j\omega)| = \left|\frac{1}{j\omega+1}\right| = \sqrt{\frac{1}{\omega^2+1}}$$

$$\phi(\omega) = \angle(1) - \angle(j\omega+1)$$

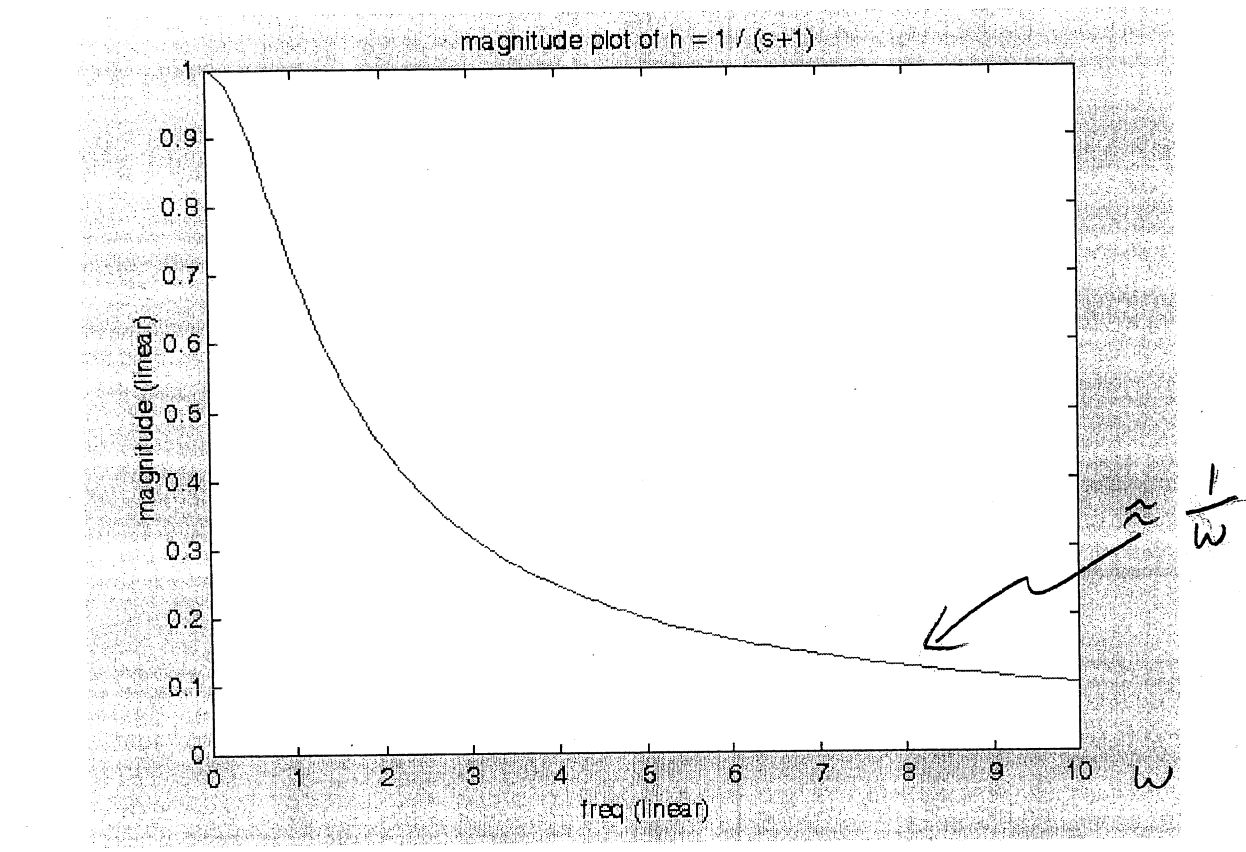
$$= 0^\circ - \tan^{-1}(\omega)$$

$$\phi(\omega) = -\tan^{-1}(\omega)$$

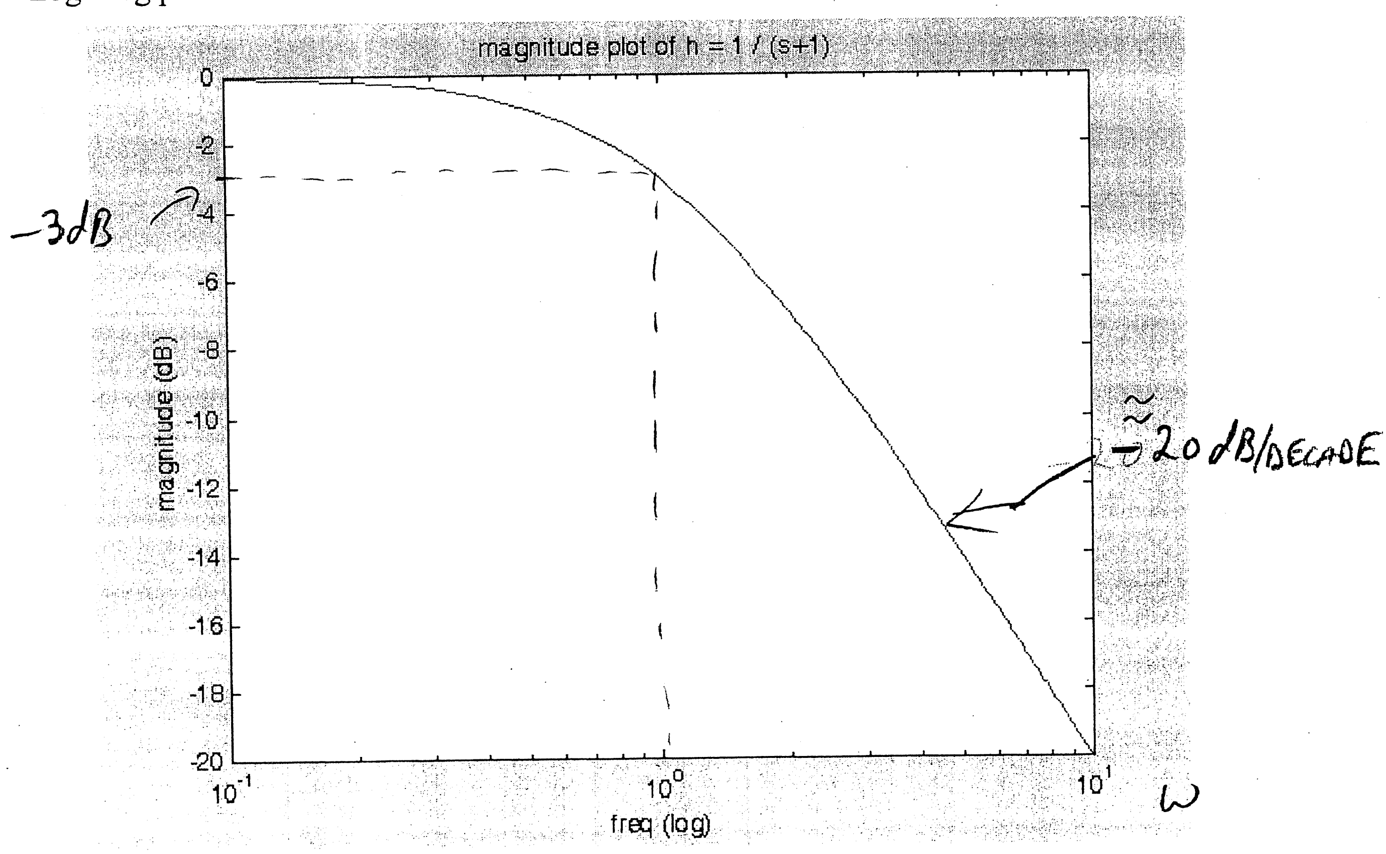




Linear - linear plot



Log - log plot

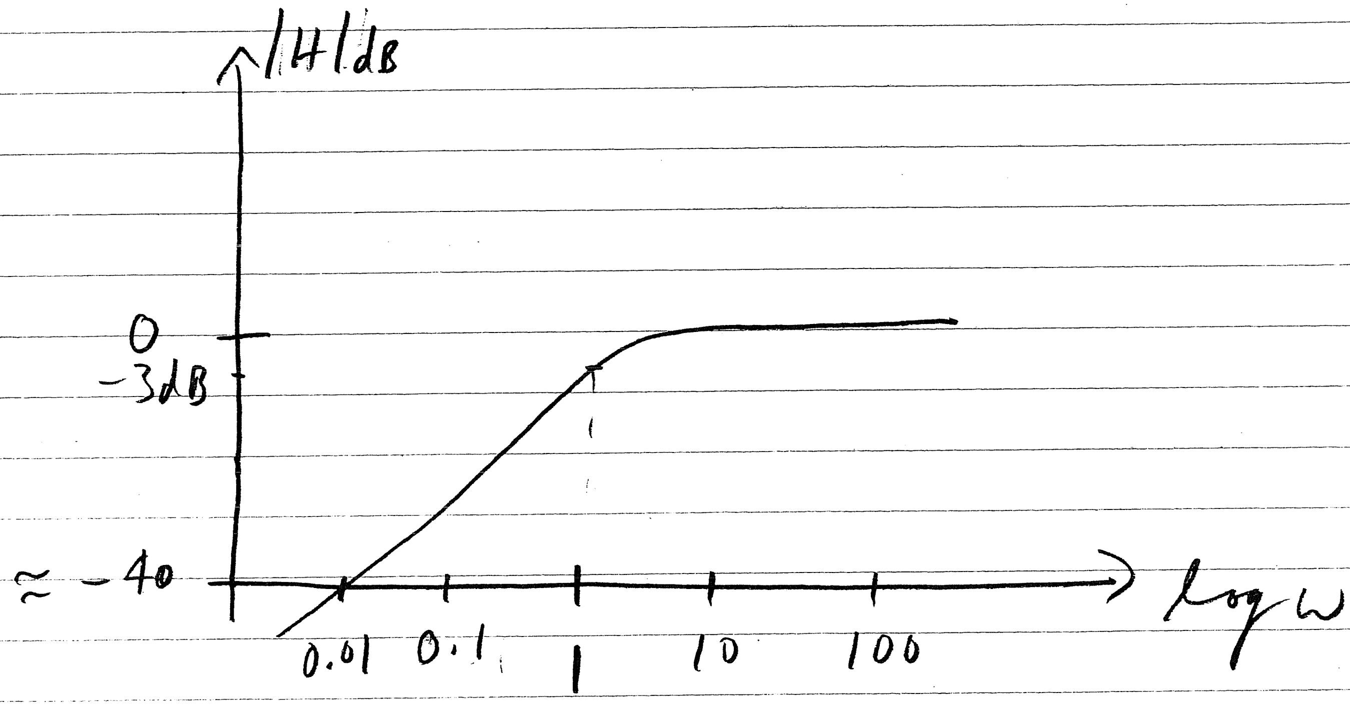


EXAMPLE HIGH PASS

$$H(s) = \frac{s}{s+1}$$

$$|H(j\omega)| = \frac{|j\omega|}{|j\omega+1|} = \frac{\omega}{\sqrt{\omega^2+1}}$$

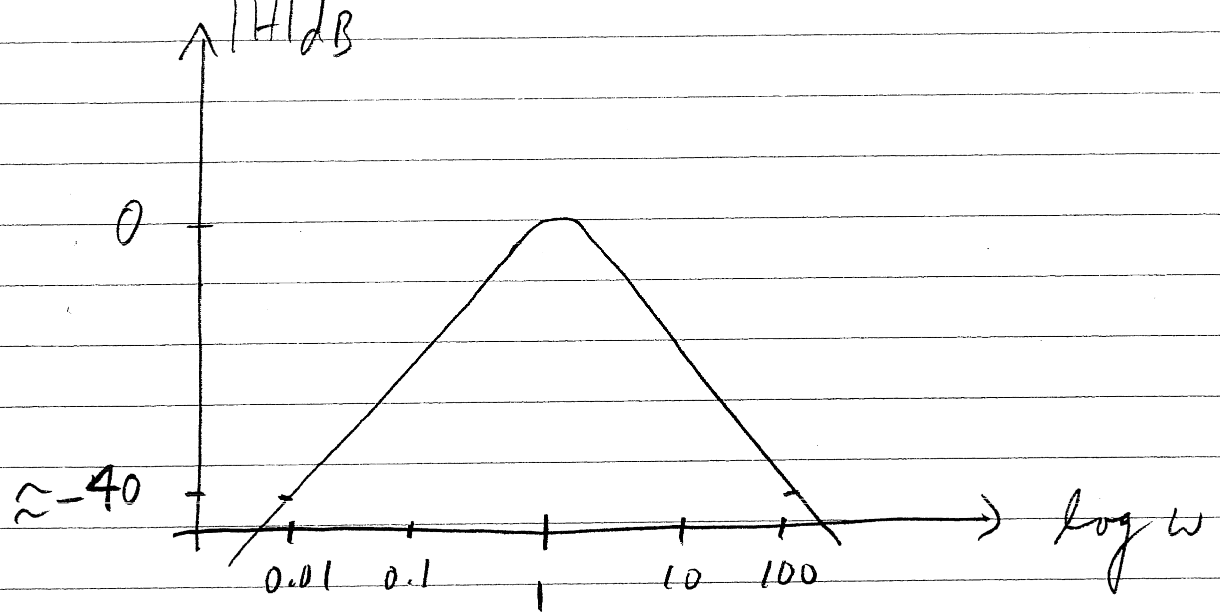
$$= \sqrt{\frac{\omega^2}{\omega^2+1}}$$



EXAMPLE

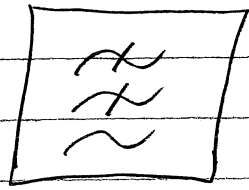
BAND PASS

$$H(s) = \frac{s}{s^2 + s + 1}$$

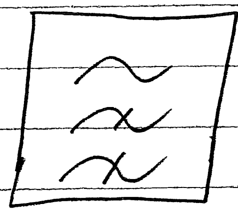


FILTER NOTATION SYMBOL

LOWPASS



HIGH PASS



BANDPASS