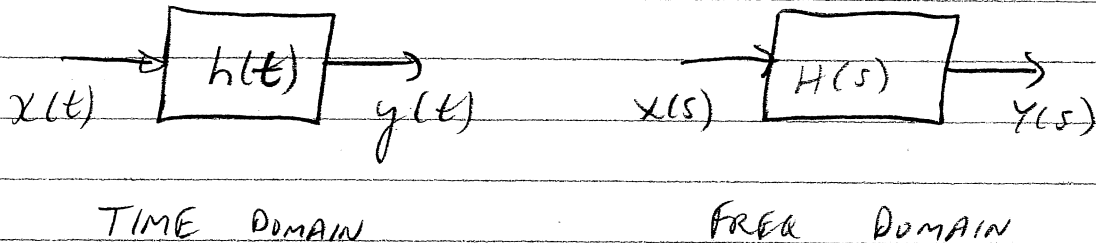


# 12.1 ANALOG FILTER TYPES AND SPECS



SYSTEM SHOULD BE LINEAR TIME-INVARIANT SO THAT FREQ ANALYSIS CAN BE USED.

- $h(t)$  IMPULSE RESPONSE OF FILTER.
- $H(s)$  LAPLACE TRANSFORM OF  $h(t)$

$$y(t) = h(t) \otimes x(t) \qquad Y(s) = H(s)X(s)$$

$$H(s) = \frac{Y(s)}{X(s)}$$

"s" IS COMPLEX VARIABLE  $\sigma + j\omega$   
 "w" IS PHYSICAL FREQ IN RAD/S

$H(s)$  IS FILTER'S "TRANSFER-FUNCTION"

FOR PHYSICAL FREQUENCIES CAN FIND FILTER'S RESPONSE BY:

$$H(s) \Big|_{s=j\omega} = H(j\omega) = |H(j\omega)| e^{j\phi(\omega)}$$

$|H(j\omega)|$  IS MAGNITUDE RESPONSE

$\phi(\omega)$  IS PHASE RESPONSE

GAIN IN dB

OFTEN MAGNITUDE RESPONSE EXPRESSED IN dB

$$|H(\omega)|_{dB} = 20 \log |H(j\omega)| \quad \text{in dB}$$

$|H(j\omega)|_{dB}$  IS FILTER GAIN IN dB

EXAMPLE

$ H(j\omega) $	$ H(j\omega) _{dB}$
1	0
$0.7071 = \frac{1}{\sqrt{2}}$	-3.01
0.5	-6.02
0.1	-20
0.01	-40
0.001	-60

GROUP DELAY

$\phi(\omega)$  IS PHASE LEAD THROUGH FILTER.

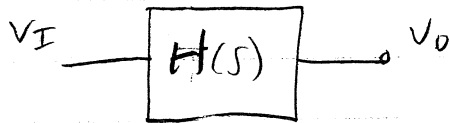
$$\text{GROUP DELAY, } T_d(\omega) \equiv - \frac{d\phi(\omega)}{d\omega}$$

INDICATION OF DELAY THROUGH A FILTER.

- A LINEAR PHASE HAS A CONSTANT GROUP DELAY  
WHEN DESIGNING FILTERS

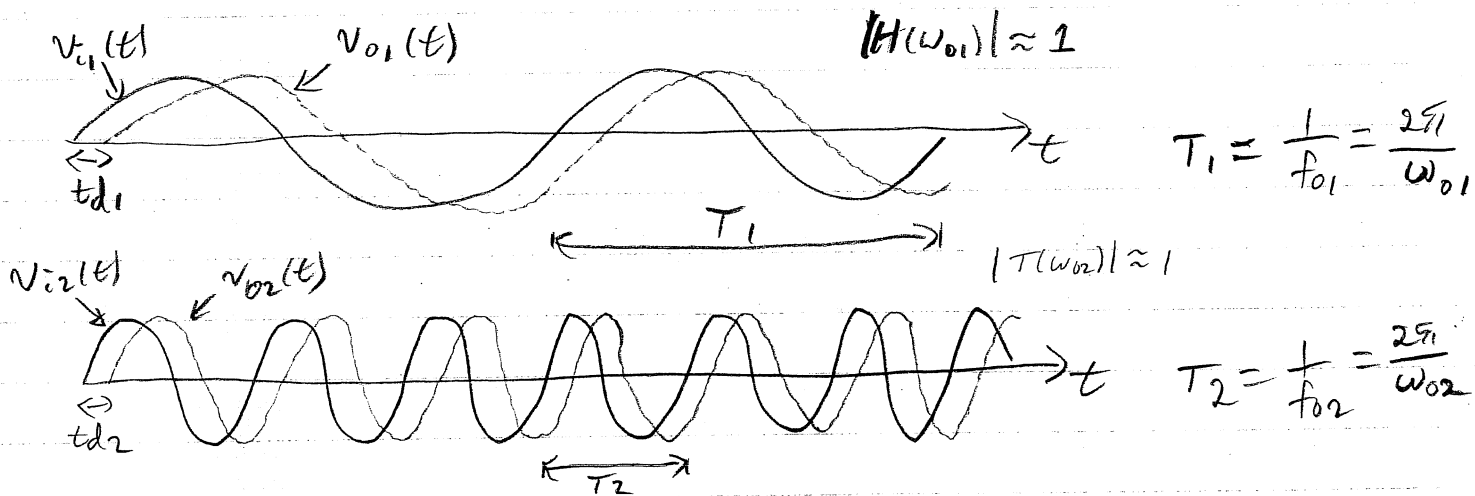
OFTEN ONLY MAGNITUDE IS OF CONCERN  
 (CAN TOLERATE VARYING GROUP DELAY) IN APPLICATIONS  
 SUCH AS ANTI-ALIASING FILTER (DSP CAN  
 FIX UP DELAY BUT NOT FOLD-OVER NOISE)  
 OR SPEECH SIGNAL PROCESSING (THE EAR IS NOT  
 SENSITIVE TO PHASE DISTORTION)

# WHY "LINEAR-PHASE" FILTERS?



$$V_{i1}(t) = \cos(\omega_{01}t) \Rightarrow V_{o1}(t) = |H(\omega_{01})| \cos(\omega_{01}t - \phi(\omega_{01}))$$

$$V_{i2}(t) = \cos(\omega_{02}t) \Rightarrow V_{o2}(t) = |H(\omega_{02})| \cos(\omega_{02}t - \phi(\omega_{02}))$$



WANT  $t_{d1} = t_{d2}$  SO THAT ALL FREQUENCIES IN PASSBAND EXPERIENCE THE SAME DELAY THROUGH FILTER - IMPORTANT FOR DATA COMMUNICATIONS & AUDIO (NOT FOR SPEECH USUALLY)

$$t_{d1} = \frac{\phi(\omega_{01})}{2\pi} \times T_1 = \frac{\phi(\omega_{01})}{2\pi} \times \frac{2\pi}{\omega_{01}} = \frac{\phi(\omega_{01})}{\omega_{01}}$$

SIMILARLY  $t_{d2} = \frac{\phi(\omega_{02})}{\omega_{02}}$

o.o IF  $t_d = t_{d1} = t_{d2} \Rightarrow \left. \begin{aligned} \phi(\omega_{01}) &= \omega_{01} t_d \\ \phi(\omega_{02}) &= \omega_{02} t_d \end{aligned} \right\} \boxed{\phi(\omega) = \omega t_d}$   
 FOR ALL FREQ TO HAVE SAME  $t_d$

$\Rightarrow$  LINEAR-PHASE

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## ALTERNATE APPROACH

$$v_o(t) = v_i(t - t_d)$$

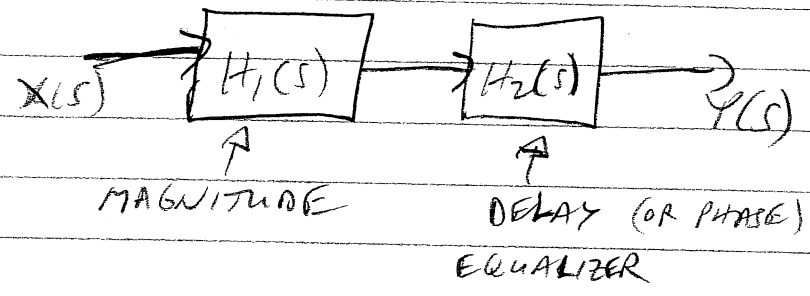
$$V_o(s) = e^{-s t_d} V_i(s)$$

$$H(s) = e^{-s t_d} \Rightarrow H(j\omega) = e^{-j\omega t_d}$$

$$\therefore \phi(\omega) = -\omega t_d \quad \& \quad |H(\omega)| = 1$$

SO LINEAR PHASE GIVES A STRAIGHT DELAY.

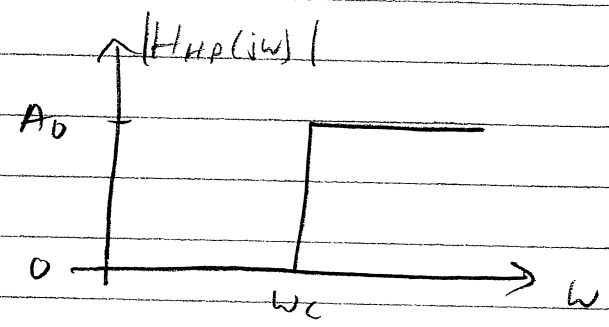
IF MAGNITUDE & PHASE (OR GROUP DELAY)  
IMPORTANT OFTEN USE



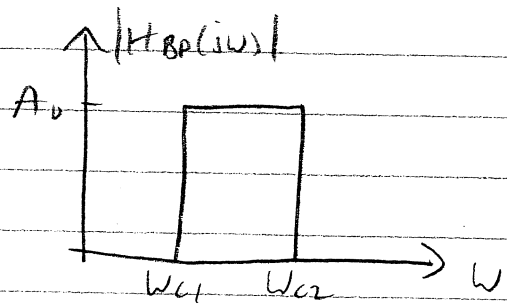
FILTER CHARACTERISTICS



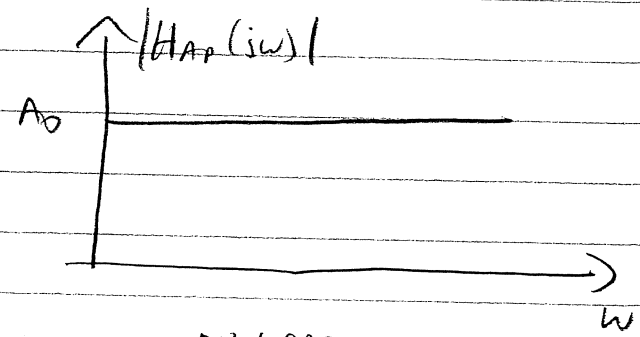
LOWPASS



HIGHPASS



BANDPASS



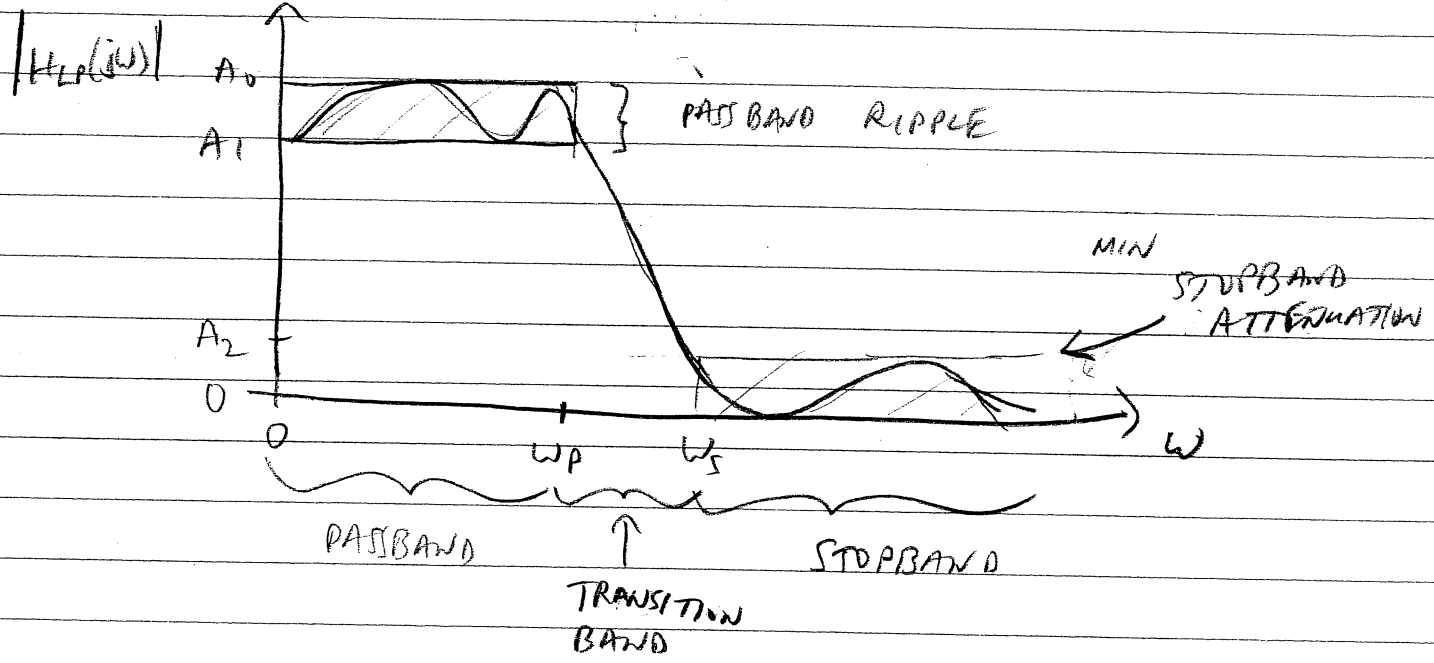
ALLPASS

(USED IN DELAY EQUALIZER)

- CANNOT REALIZE THESE "BRICK WALL" RESPONSES

NEED PASSBAND + STOPBAND VARIATIONS AND TRANSITION BAND

### LOW PASS



- PASSBAND RIPPLE  $A_{max} \triangleq 20 \log \left( \frac{A_0}{A_1} \right)$  (IN DB)  
 $0 < w < w_p$   
 $20 \log(A_0) - 20 \log(A_1)$  (SMALLER THE BETTER)

STOPBAND ATTENUATION  
 $w_s < w < \infty$   $A_{min} \triangleq 20 \log \left( \frac{A_0}{A_2} \right)$  (LARGER THE BETTER)

TRANSITION BAND : RESPONSE SHOULD BE BOUNDED BELOW  $A_0$  (USUALLY MONOTONICALLY FALLING)  
 $w_p < w < w_s$   
 (SMALLER THE BETTER)

7A

$$\text{ATTENUATION} = \frac{1}{\text{GAIN}}$$

$$\text{ATTEN (dB)} = - \text{GAIN (dB)}$$

EXAMPLE

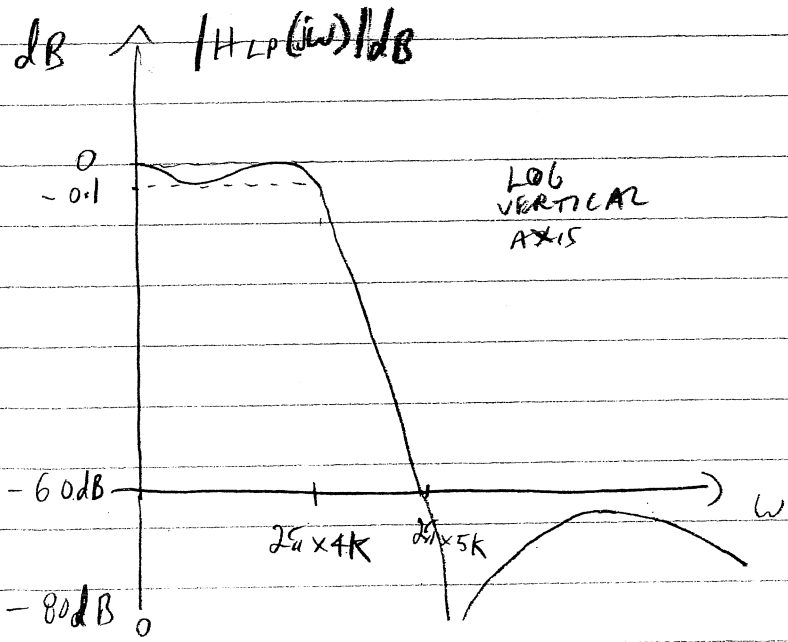
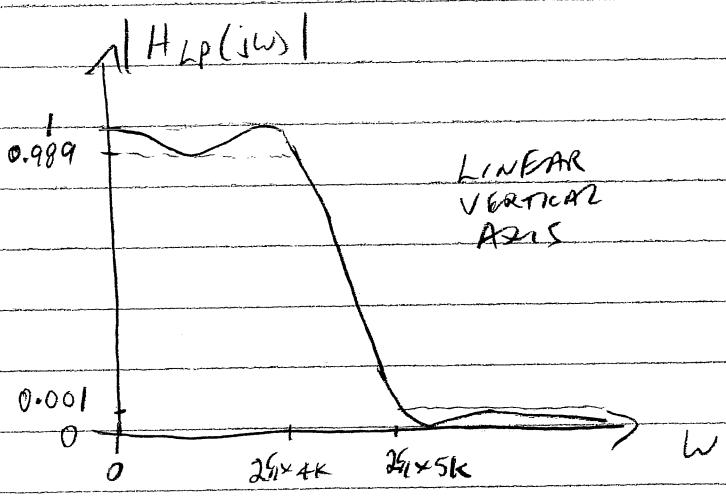
$$6 \text{ dB ATTENUATION} = -6 \text{ dB GAIN}$$

SIMILAR DEFINITIONS FOR HIGHPASS, BANDPASS, etc.

TYPICAL SPECIFICATIONS

LOWPASS

VOICE	PASSBAND	$0 \rightarrow 4\text{KHz}$	RIPPLE	$0.1\text{ dB} \leftarrow A_{MAX}$
CODEC	STOPBAND	$5\text{KHz} \rightarrow \infty$	ATTEN	$60\text{ dB} \leftarrow A_{MIN}$



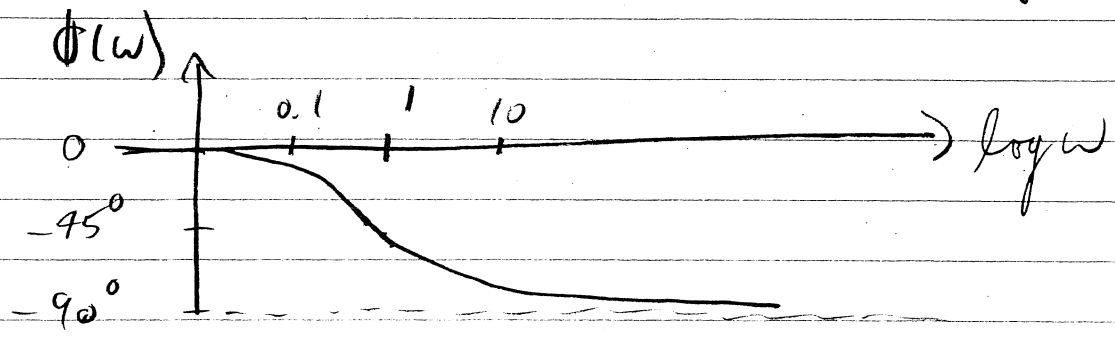
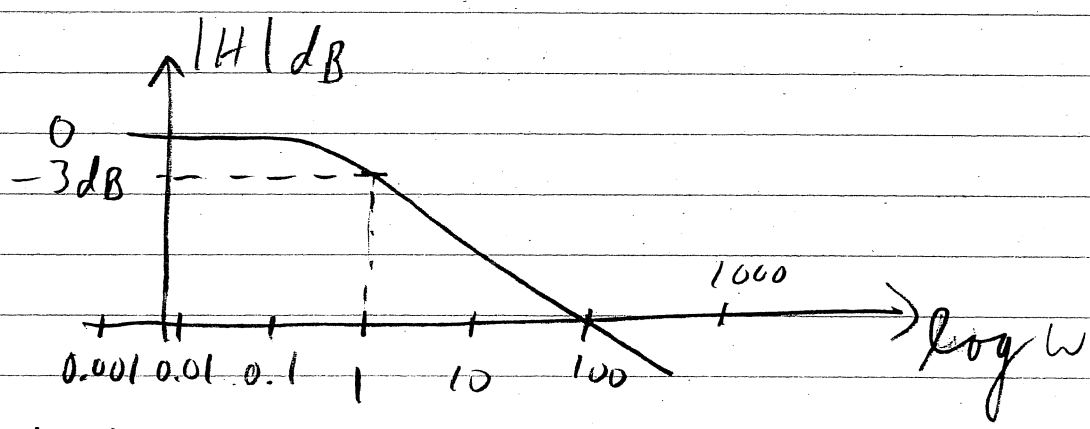
EXAMPLE

$$H(s) = \frac{1}{s+1}$$

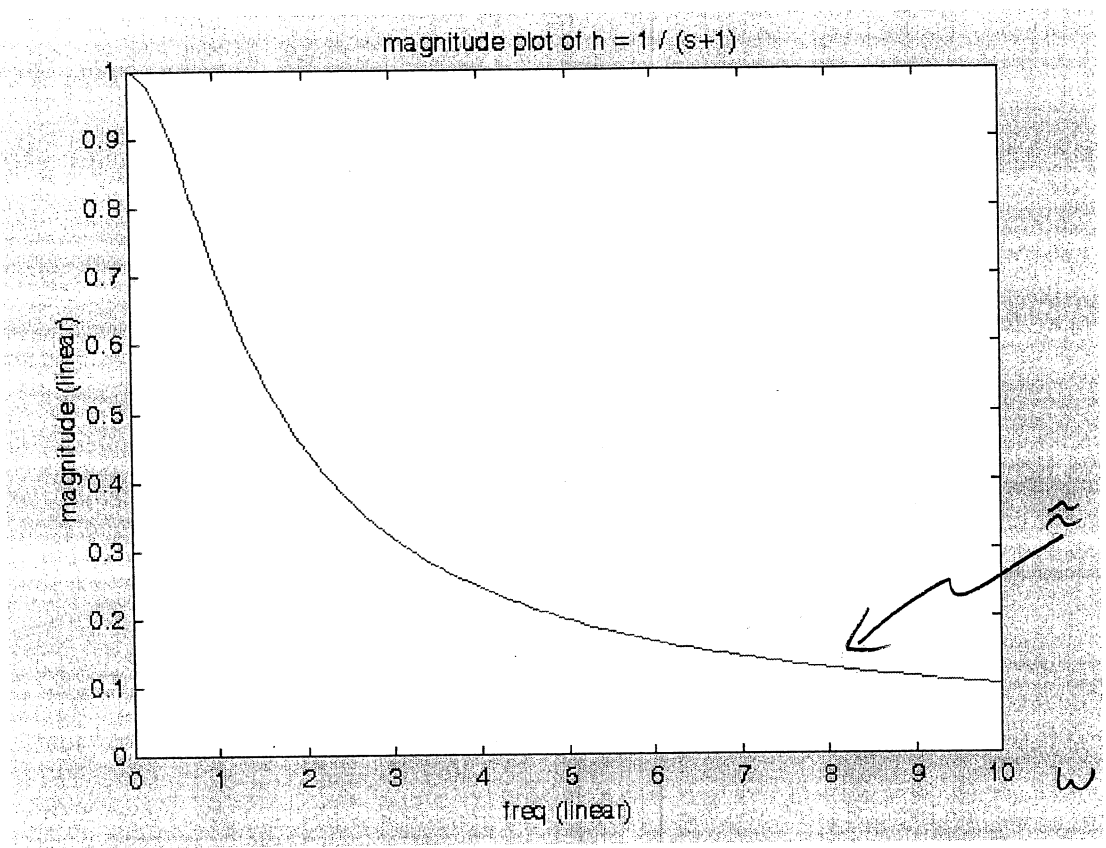
$$|H(j\omega)| = \left| \frac{1}{j\omega+1} \right| = \sqrt{\frac{1}{\omega^2+1}}$$

$$\begin{aligned} \phi(\omega) &= \angle(1) - \angle(j\omega+1) \\ &= 0^\circ - \tan^{-1}(\omega) \end{aligned}$$

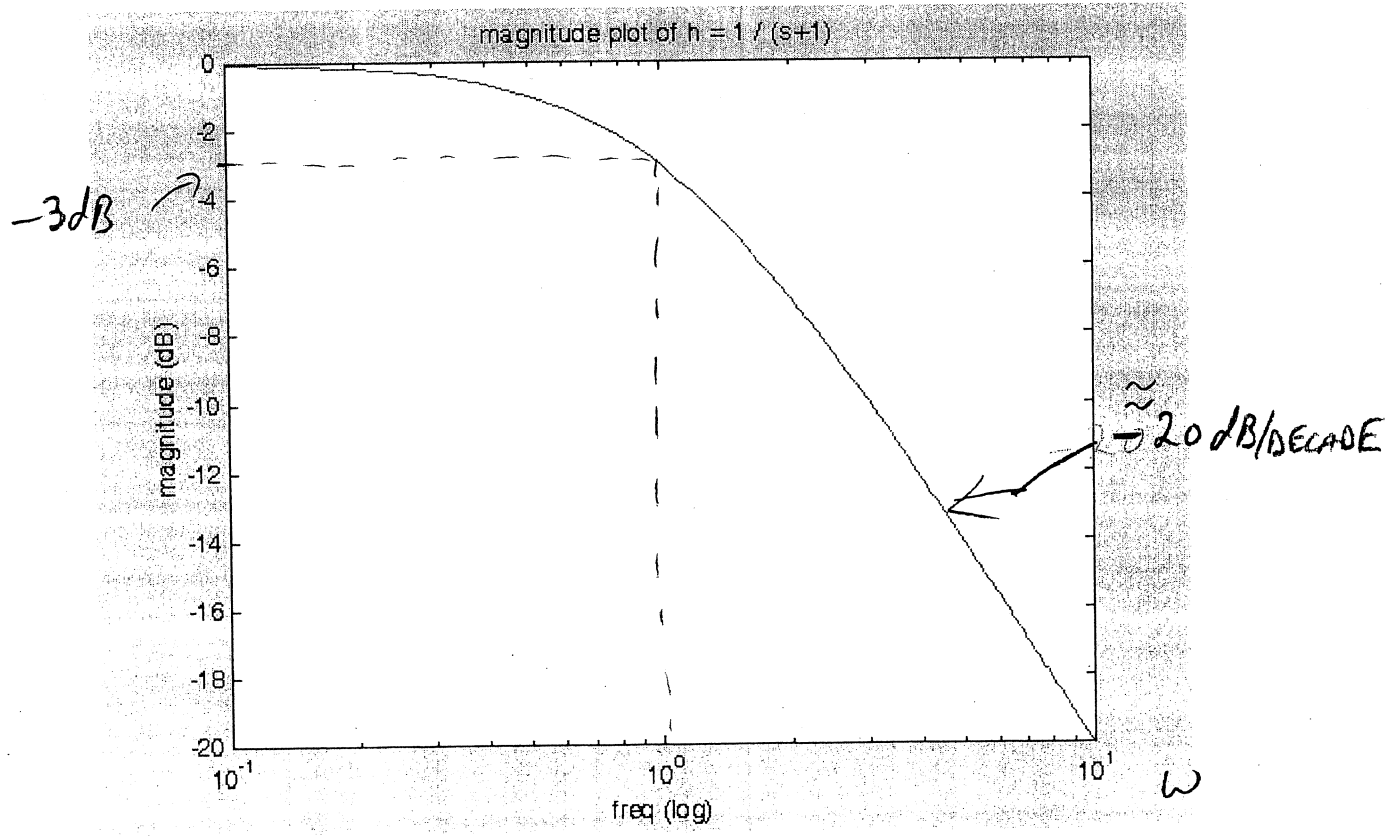
$$\phi(\omega) = -\tan^{-1}(\omega)$$



Linear - linear plot



Log - log plot

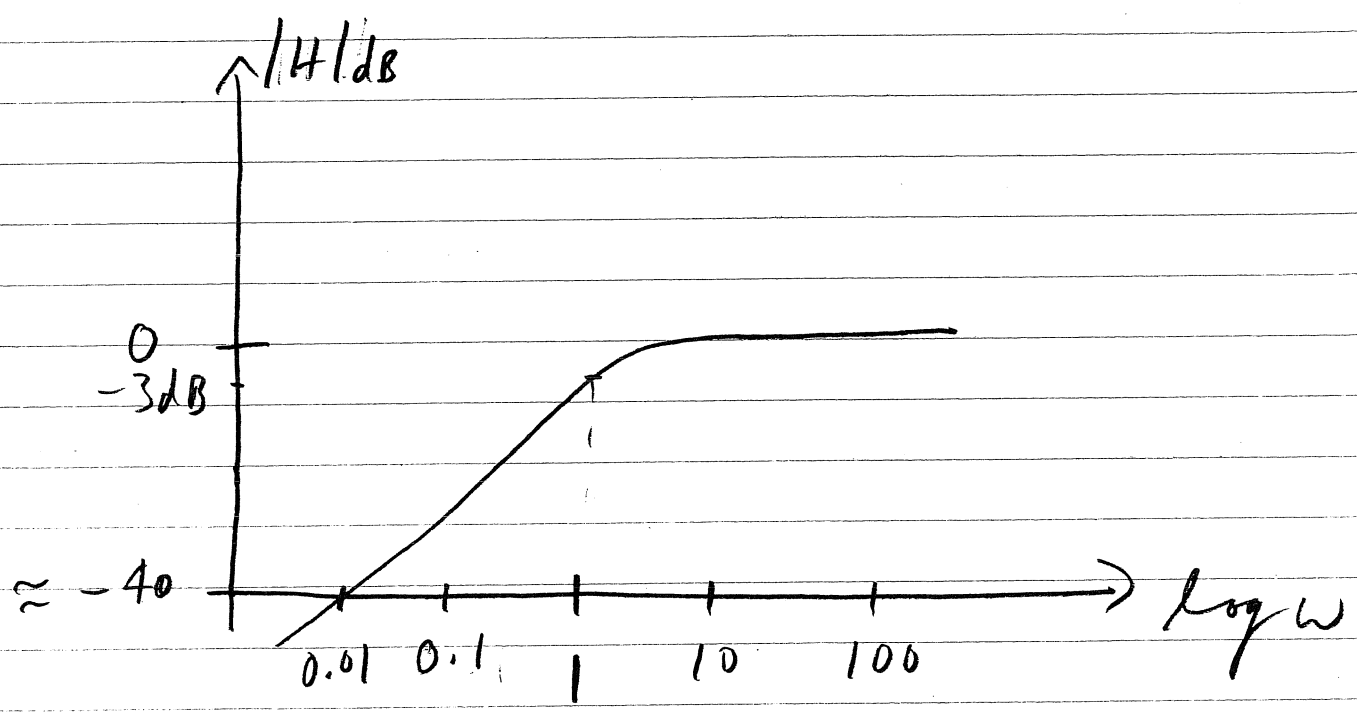


EXAMPLE

HIGH PASS

$$H(s) = \frac{s}{s+1}$$

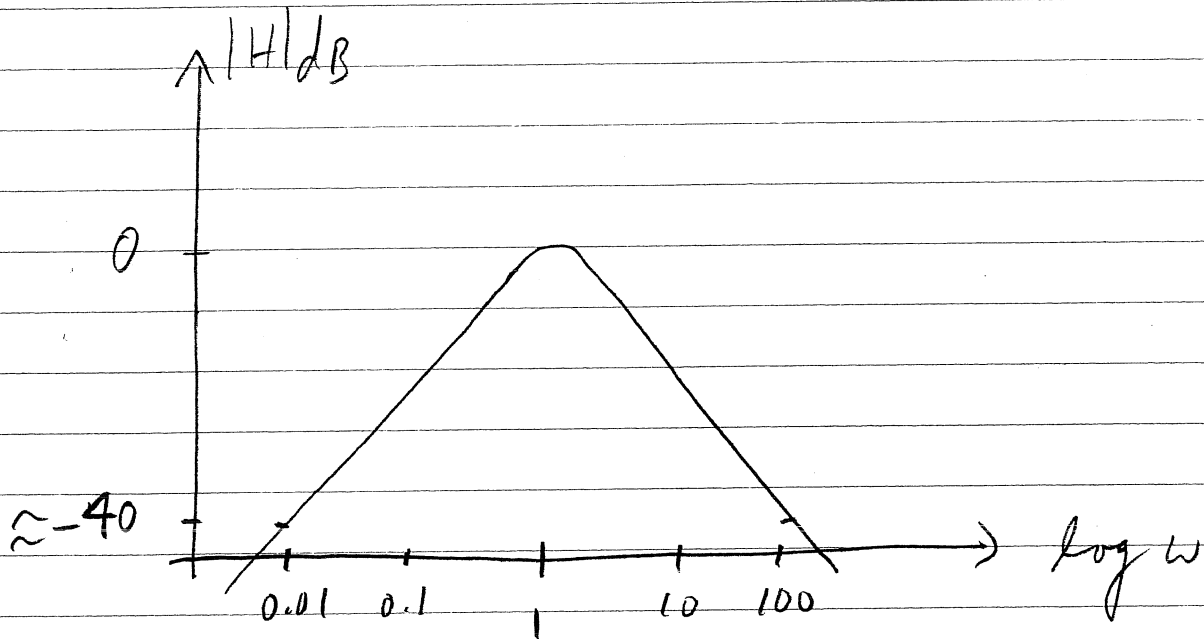
$$|H(j\omega)| = \frac{|j\omega|}{|j\omega+1|} = \frac{\omega}{\sqrt{\omega^2+1}}$$
$$= \sqrt{\frac{\omega^2}{\omega^2+1}}$$



EXAMPLE

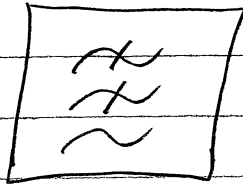
BAND PASS

$$H(s) = \frac{s}{s^2 + s + 1}$$

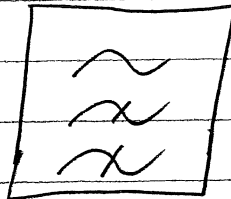


# FILTER NOTATION SYMBOL

LOW PASS



HIGH PASS



BAND PASS

