TRANSFER-FUNCTION

$$H(s) = \frac{a_M s^M + a_{M-1} s^{M-1} + \cdots + a_0}{s^N + b_{N-1} s^{N-1} + \cdots + b_0} \equiv \frac{N(s)}{D(s)}$$

(POLYNOMIAL FORM)

$N \Rightarrow$ FILTER ORDER

$M \leq N$

$a_0, a_1, a_2 \ldots a_M, b_0, b_1, b_2, \ldots b_N \in \mathbb{R}$ (REAL NUMBERS)

$N(s) \rightarrow$ NUMERATOR OF $H(s)$
$D(s) \rightarrow$ DENOMINATOR OF $H(s)$

CAN FACTOR $N(s)$ + $D(s)$ TO WRITE

$$H(s) = \frac{a_M (s-z_1)(s-z_2)\ldots(s-z_M)}{(s-p_1)(s-p_2)\ldots(s-p_N)}$$

(ROOT FORM)

$z_1, z_2, \ldots z_M$ ARE **ZEROS** OF $H(s)$

$p_1, p_2, \ldots p_N$ ARE **POLES** OF $H(s)$

AND ARE IN GENERAL COMPLEX NUMBERS

$H(z_i) = 0$ SINCE $N(z_i) = 0$

$H(p_i) \Rightarrow \infty$ SINCE $D(p_i) = 0$

POLES ALSO CALLED "NATURAL MODES"
ZEROS ALSO CALLED "TRANSMISSION ZEROS"

SINCE $b_{N-1}, b_{N-2}, \ldots b_0$ ALL REAL

THEN POLES OCCUR IN COMPLEX CONJUGATE PAIRS

ie IF $-1 + j3$ IS A POLE
THEN $-1 - j3$ ALSO A POLE

SIMILAR FOR ZEROS.

CAN PLOT POLES & ZEROS IN S-PLANE

EX

$$P_1 = -0.1 + j0.1$$

$$P_2 = -0.1 - j0.1$$

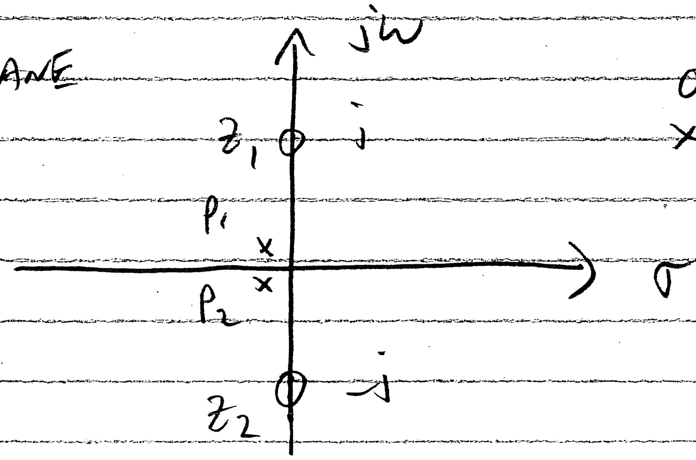
$$Z_1 = j \qquad Z_2 = -j$$

$$a_M = 0.02$$

$$H(s) = \frac{(0.02)(s-j)(s+j)}{(s+0.1-0.1j)(s+0.1+0.1j)} \qquad \text{ROOT FORM}$$

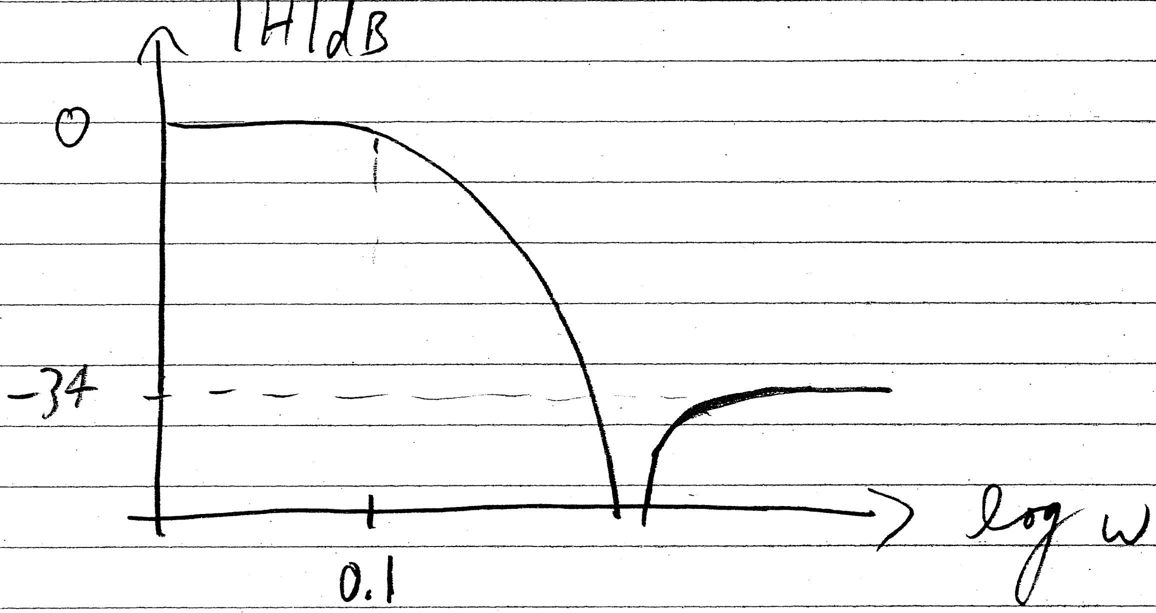
$$= \frac{(0.02s^2 + 0.02)}{(s^2 + 0.2s + 0.02)} \qquad \text{POLY FORM}$$

S-PLANE



o — ZERO

x — POLE



STABILITY

FOR $H(s)$ TO BE STABLE

$N \geq M$ OTHERWISE $|H| \to \infty$
AS $s \to \infty$

ALL POLES IN LEFT HAND PLANE

OTHERWISE IMPULSE RESPONSE $\to \infty$

ZEROS AT ∞

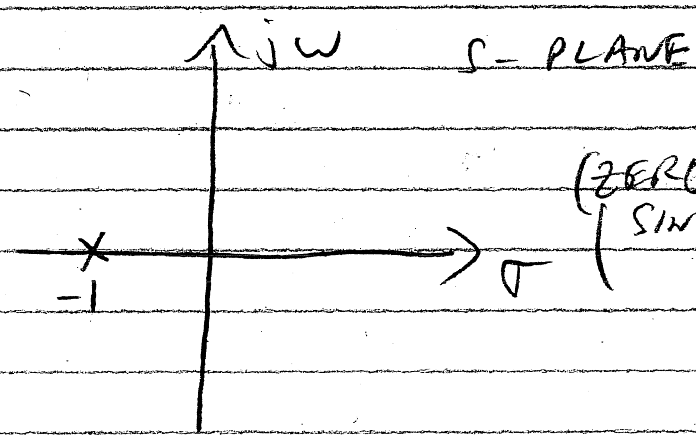
IF $M < N$ THEN FILTER IS SAID TO HAVE $N-M$ "ZEROS AT ∞"

SINCE AS $s \to \infty$ $\quad H(s) \to \dfrac{a_M}{s^{N-M}}$

DECAYS AT $\dfrac{1}{s^{N-M}}$

$H(s) = \dfrac{1}{s+1}$ HAS 1 ZERO AT ∞

& 1 POLE AT $s=-1$



(ZERO AT ∞)
(SINCE NOT SHOWN)

$$H(s) = \frac{1}{s^2+s+1}$$

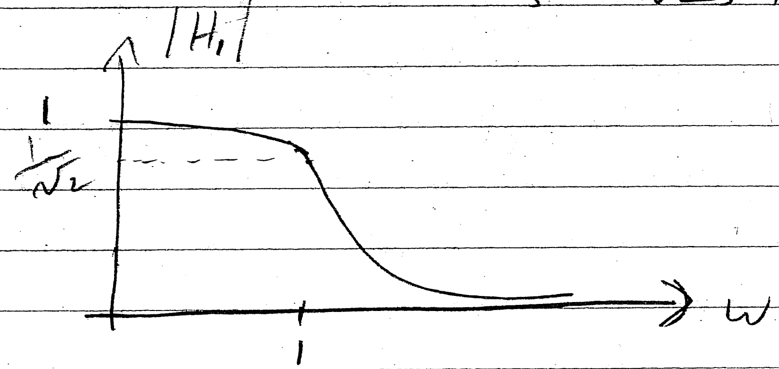
2 ZEROS AT ∞
POLES AT
$-0.5 \pm j\,0.866$

<u>FREQUENCY SCALING</u> (NOT IN TEXTBOOK)

TO SCALE $H(s)$ BY k_f

MAKE SUBSTITUTION $s \rightarrow s/k_f$

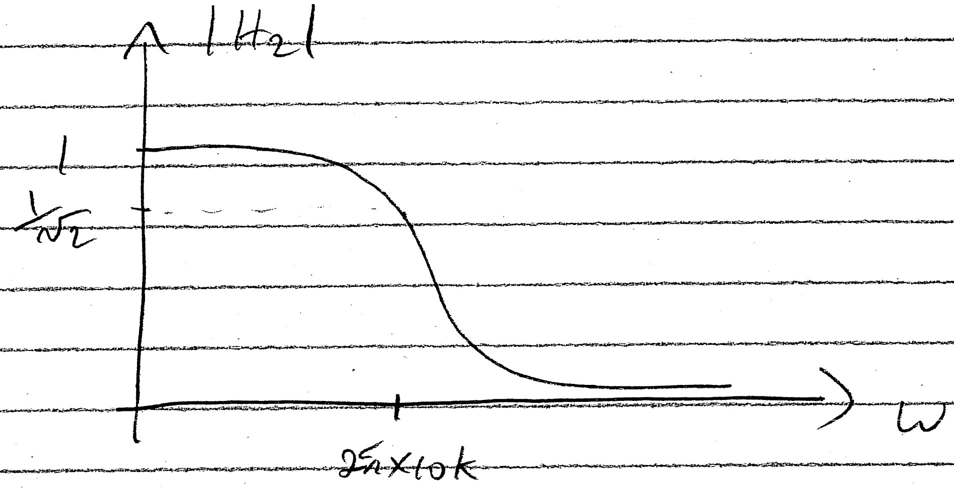
<u>EXAMPLE</u> $H_1(s) = \dfrac{1}{s^2+\sqrt{2}s+1}$



SCALE SO THAT 1 RAD/S $\rightarrow$ $2\pi \times 10\text{kHz}$

$$H_2(s) = \frac{1}{\dfrac{s^2}{(2\pi\times 10k)^2} + \dfrac{\sqrt{2}\,s}{(2\pi\times 10k)} + 1}$$

$$H_2(s) = \frac{(2\pi \times 10k)^2}{s^2 + (\sqrt{2})(2\pi \times 10k)s + (2\pi \times 10k)^2}$$



$$|H(j\omega)|^2 = H(s)\,H(-s)\Big|_{s=j\omega}$$

$$= H(j\omega)\,H(-j\omega)$$

COMPLEX CONJUGATE

$$= H(j\omega)\,H^*(j\omega)$$

EASIER TO DEAL WITH $|H|^2$

RATHER THAN $|H|$

$|H(j\omega)|^2$ IS AN EVEN FUNCTION

& IS RATIONAL IN ω^2

EXAMPLE $H(s) = \dfrac{s+2}{s^3+2s^2+2s+3}$

$$H(s)\,H(-s) = \frac{s^2-4}{s^6-8s^2-9}$$

$$|H(j\omega)|^2 = \frac{\omega^2+4}{\omega^6-8\omega^2+9}$$

GENERAL APPROACH FOR LOWPASS IS TO LOOK AT

$$|H(j\omega)|^2 = \frac{A_0^2}{1+F(\omega^2)}$$

SUCH THAT

$$F(\omega^2) \ll 1 \qquad 0 < \omega < \omega_p$$

$$F(\omega^2) \gg 1 \qquad \omega > \omega_s$$

THIS MAKES $|H(j\omega)|^2 \approx A_0^2 \qquad 0<\omega<\omega_p$

& $|H(j\omega)|^2 \ll A_0^2 \qquad \omega > \omega_s$