

TRANSFER-FUNCTION

$$H(s) = \frac{a_m s^m + a_{m-1} s^{m-1} + \dots + a_0}{s^N + b_{N-1} s^{N-1} + \dots + b_0} \equiv \frac{N(s)}{D(s)}$$

(POLYNOMIAL FORM)

N => FILTER ORDER

$$M \leq N$$

$a_0, a_1, a_2, \dots, a_m, b_0, b_1, b_2, \dots, b_N \in \mathbb{R}$
 (REAL NUMBERS)

N(s) -> NUMERATOR OF H(s)

D(s) -> DENOMINATOR OF H(s)

CAN FACTOR N(s) + D(s) TO WRITE

$$H(s) = \frac{a_m (s-z_1)(s-z_2) \dots (s-z_m)}{(s-p_1)(s-p_2) \dots (s-p_n)}$$

(ROOT FORM)

z_1, z_2, \dots, z_m ARE ZEROS OF $H(S)$

p_1, p_2, \dots, p_n ARE POLES OF $H(S)$

AND ARE IN GENERAL COMPLEX NUMBERS

$H(z_i) = 0$ SINCE $N(z_i) = 0$

$H(p_i) \Rightarrow \infty$ SINCE $D(p_i) = 0$

POLES ALSO CALLED "NATURAL MODES"

ZEROS ALSO CALLED "TRANSMISSION ZEROS"

SINCE $b_{n-1}, b_{n-2}, \dots, b_0$ ALL REAL

THEN POLES OCCUR IN COMPLEX
CONJUGATE PAIRS

ie IF $-1 + j3$ IS A POLE
THEN $-1 - j3$ ALSO A POLE

SIMILAR FOR ZEROS.

CAN PLOT POLES & ZEROS IN S-PLANE

EX $P_1 = -0.1 + j0.1$

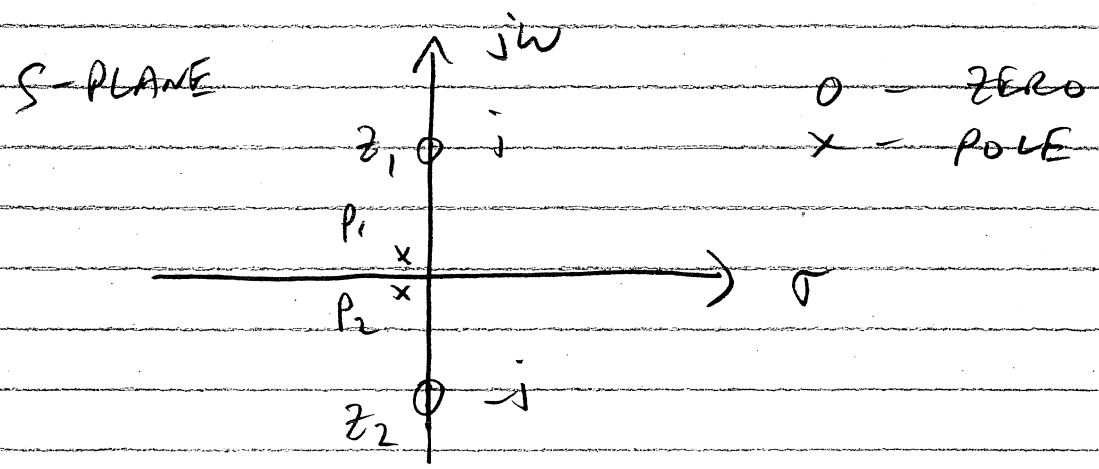
$P_2 = -0.1 - j0.1$

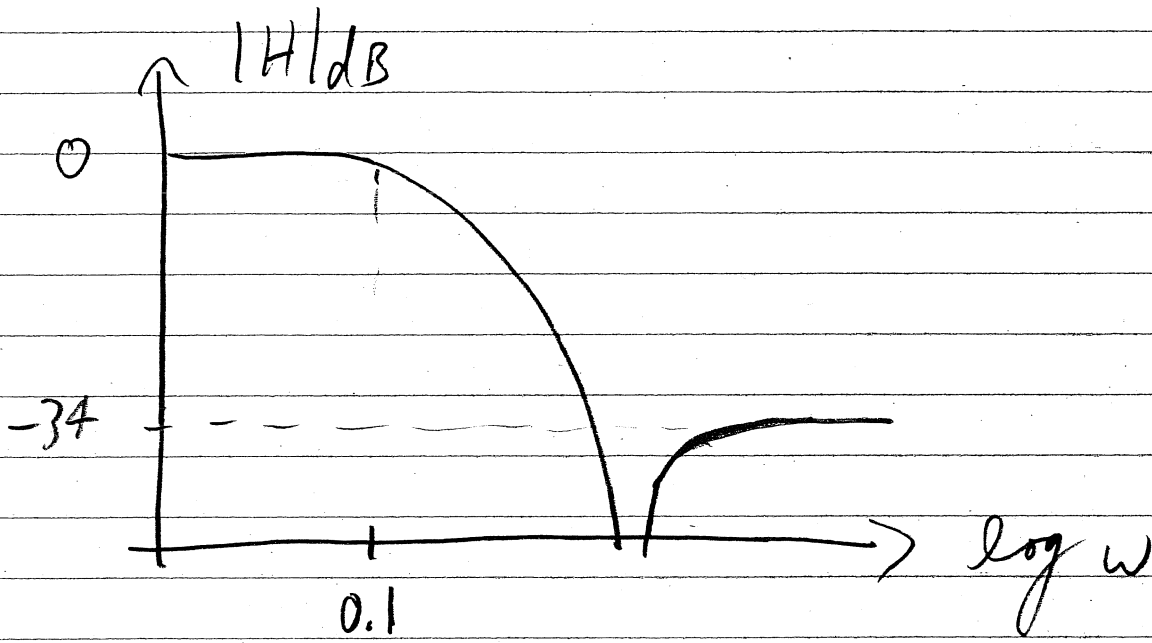
$z_1 = j$ $z_2 = -j$

$a_m = 0.02$

$H(s) = \frac{(0.02)(s-j)(s+j)}{(s+0.1-j0.1j)(s+0.1+0.1j)}$ ROOT FORM

$= \frac{(0.02s^2 + 0.02)}{(s^2 + 0.2s + 0.02)}$ POLY FORM





STABILITY

FOR $H(s)$ TO BE STABLE

$N \geq M$ OTHERWISE $|H| \rightarrow \infty$
AS $s \rightarrow \infty$

ALL POLES IN LEFT HAND PLANE

OTHERWISE IMPULSE RESPONSE $\rightarrow \infty$

ZEROS AT ∞

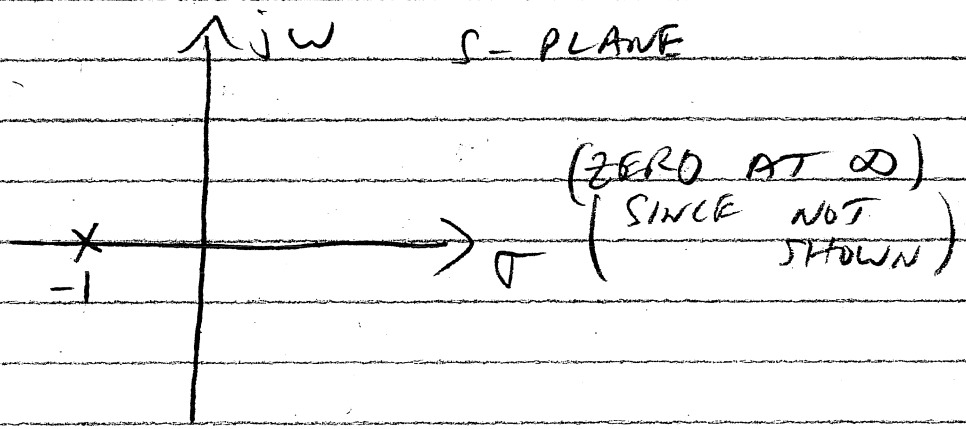
IF $M < N$ THEN FILTER IS SAID TO HAVE $N - M$ "ZEROS AT ∞ "

SINCE AS $s \rightarrow \infty$ $H(s) \rightarrow \frac{a_M}{s^{N-M}}$

DECAYS AT $\frac{1}{s^{N-M}}$

$H(s) = \frac{1}{s+1}$ HAS 1 ZERO AT ∞

↓ 1 POLE AT $s = -1$



$$H(s) = \frac{1}{s^2 + s + 1}$$

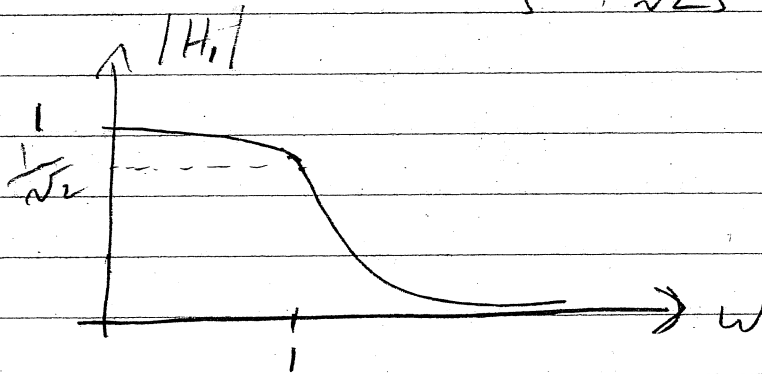
2 ZEROS AT ∞
POLES AT
 $-0.5 \pm j0.866$

FREQUENCY SCALING (NOT IN TEXTBOOK)

TO SCALE $H(s)$ BY k_f

MAKE SUBSTITUTION $s \rightarrow s/k_f$

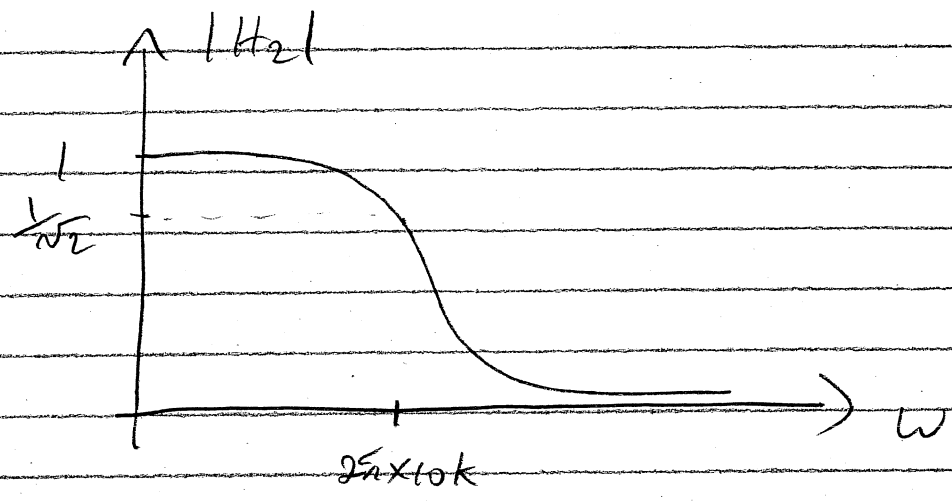
EXAMPLE $H_1(s) = \frac{1}{s^2 + \sqrt{2}s + 1}$



SCALE SO THAT 1 RAD/S \rightarrow $2\pi \times 10\text{kHz}$

$$H_2(s) = \frac{1}{\frac{s^2}{(2\pi \times 10k)^2} + \frac{\sqrt{2}s}{(2\pi \times 10k)} + 1}$$

$$H_2(s) = \frac{(25 \times 10^3)^2}{s^2 + (\sqrt{2})(25 \times 10^3)s + (25 \times 10^3)^2}$$



$$|H(j\omega)|^2 = H(s)H(-s) \Big|_{s=j\omega}$$

$$= H(j\omega)H(-j\omega)$$

Complex
conjugate

$$= H(j\omega)H^*(j\omega)$$

EASIER TO DEAL WITH $|H|^2$
RATHER THAN $|H|$

$|H(j\omega)|^2$ IS AN EVEN FUNCTION

AND IS RATIONAL IN ω^2

EXAMPLE $H(s) = \frac{s+2}{s^3+2s^2+2s+3}$

$$H(s)H(-s) = \frac{s^2-4}{s^6-8s^2-9}$$

$$|H(j\omega)|^2 = \frac{\omega^2+4}{\omega^6-8\omega^2+9}$$

GENERAL APPROACH FOR
LOWPASS IS TO LOOK AT

$$|H(j\omega)|^2 = \frac{A_0^2}{1 + F(\omega^2)}$$

SUCH THAT

$$F(\omega^2) \ll 1 \quad 0 < \omega < \omega_p$$

$$F(\omega^2) \gg 1 \quad \omega > \omega_s$$

THIS MAKES $|H(j\omega)|^2 \approx A_0^2 \quad 0 < \omega < \omega_p$

$$\downarrow \quad |H(j\omega)|^2 \ll A_0^2 \quad \omega > \omega_s$$