BUTTERWORTH LOWPASS

LET $F(\omega^2) = \epsilon^2 \left(\frac{\omega}{\omega_p}\right)^{2N}$

N IS FILTER ORDER

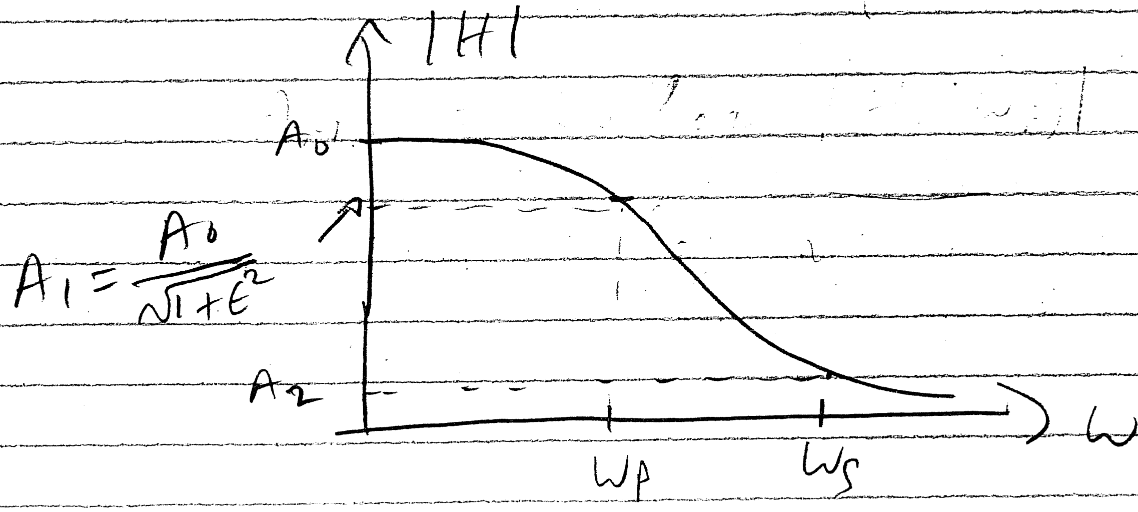
$$|H(j\omega)|^2 = \frac{A_0^2}{1 + \epsilon^2 \left(\frac{\omega}{\omega_p}\right)^{2N}}$$

AT $\omega = \omega_p$

$$|H(j\omega_p)|^2 = \frac{A_0^2}{1+\epsilon^2}$$

SO ϵ DETERMINES A_{MAX}

SINCE A_{MAX} IS MAX PASSBAND VARIATION



$$A_{MAX} = 20 \log \left(\frac{A_0}{A_1}\right) = 20 \log \sqrt{1+\epsilon^2}$$

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$$\epsilon = \sqrt{10^{\frac{A_{MAX}}{10}} - 1} \quad (1)$$

AT STOPBAND EDGE

$$A_2^2 = |H(j\omega_s)|^2$$

$$A_2^2 = \frac{A_0^2}{1 + \epsilon^2 \left(\frac{\omega_s}{\omega_p}\right)^{2N}}$$

$$A_{MIN} \leq 20 \log \left(\frac{A_0}{A_2}\right) \leq 10 \log \frac{A_0^2}{A_2^2}$$

$$A_{MIN} \leq 10 \log \left[1 + \epsilon^2 \left(\frac{\omega_s}{\omega_p}\right)^{2N}\right] \quad (2)$$

CAN USE (2) TO FIND

N THAT WILL MAKE

$A(\omega_s) = A_{min}$ IF N IS

A NON-INTEGER, CHOOSE NEXT LARGEST INTEGER & $A(\omega_s) > A_{min}$

POLES OF BUTTERWORTH FILTER

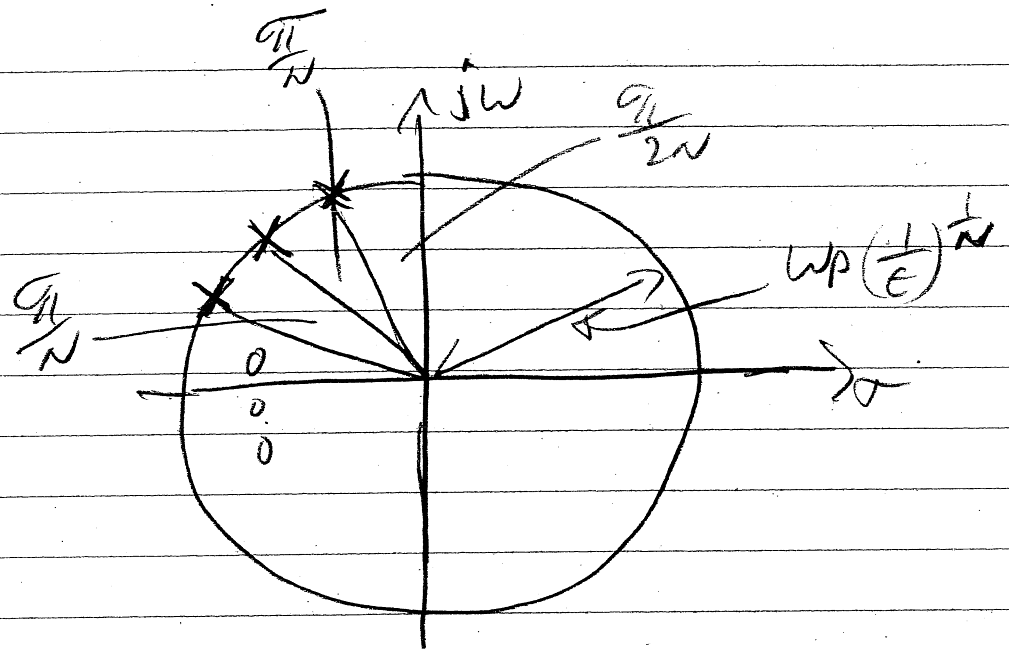
CAN BE SHOWN TO ALL LIE ON

A CIRCLE OF RADIUS $\omega_p\left(\frac{1}{\epsilon}\right)^{\frac{1}{N}}$

AND SPACED BY EQUAL ANGLES OF

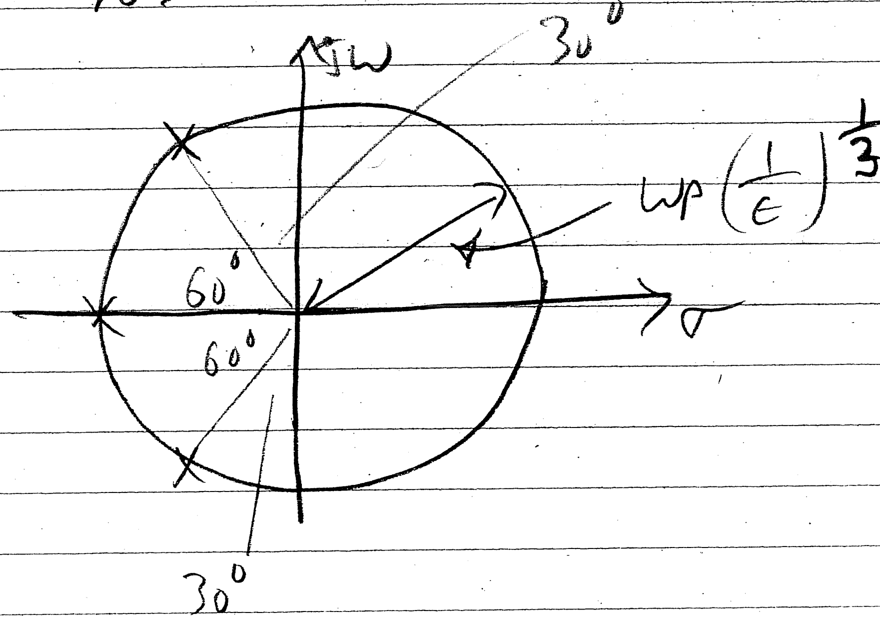
π/N WITH FIRST POLE AT $\frac{\pi}{2N}$

FROM $j\omega$ AXIS



π/N
$j\omega$
$\pi/2N$
$\omega_p\left(\frac{1}{\epsilon}\right)^{\frac{1}{N}}$
π/N
σ

Ex $N=3$



$j\omega$
30^0
$\omega_p\left(\frac{1}{\epsilon}\right)^{\frac{1}{3}}$
60^0
60^0
σ
30^0

TO FIND H(S) FOR BUTTERWORTH FILTER GIVEN A_{MAX}, A_{MIN}

1) FIND ϵ FROM (1)

2) FIND LOWEST INTEGER VALUE OF N THAT MEETS (2)

3) FIND POLES USING CIRCLE INFO

4)

$$H(S) = \frac{K\,\omega_0^{\,N}}{(S-P_1)(S-P_2)\ldots(S-P_N)}$$

WHERE $\omega_0 = \omega_P \left(\frac{1}{\epsilon}\right)^{\frac{1}{N}}$

K IS CONSTANT TO

SET dc GAIN (S=0)

BUTTERWORTH FILTER EXAMPLE

EXAMPLE FIND A 4'TH ORDER BUTTERWORTH FILTER WITH A 0.1 dB PASSBAND RIPPLE. $|H(j0)| = 1$

SOLN

$$0.1\,dB = 10 \log (1+\epsilon^2)$$

$$\epsilon^2 = 10^{0.01} - 1 = 23.29 \times 10^{-3} = 0.02329$$

$$\therefore \epsilon = 0.15261 \Rightarrow \frac{1}{\epsilon^{\frac{1}{4}}} = 1.59994$$

POLE ROOTS AT

$$\sigma_1 = 1.59994 \times \sin\left(\frac{3}{8}\pi\right) = \pm 1.4782$$

$$\omega_1 = 1.59994 \times \cos\left(\frac{3}{8}\pi\right) = \pm 0.61227$$

$$\sigma_2 = 1.59994 \times \sin\left(\frac{7}{8}\pi\right) = \pm 0.61227$$

$$\omega_2 = 1.59994 \times \cos\left(\frac{7}{8}\pi\right) = \pm 1.4782$$

(NOTE $\frac{5}{8}\pi$ IS COMPLEX CONJUGATE OF $\frac{3}{8}\pi$)

$$\therefore H(s) = \frac{k}{(s+1.4782+j0.61227)(s+1.4782-j0.61227) \times (s+0.61227+j1.4782)(s+0.61227-j1.4782)}$$

$$H(s) = \frac{k}{s^4 + 4.18099s^3 + 8.7399s^2 + 10.7025s + 6.5529}$$

$$|H(0)| = 1 \Rightarrow k = 6.5529$$