

BUTTERWORTH LOWPASS

$$\text{LET } F(\omega^2) = \epsilon^2 \left(\frac{\omega}{\omega_p} \right)^{2N}$$

N IS FILTER ORDER

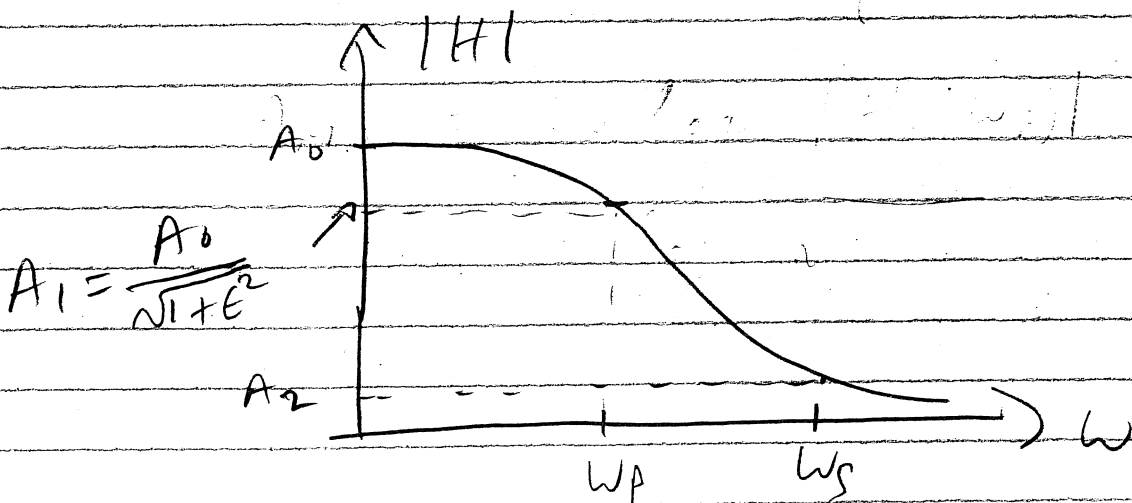
$$|H(j\omega)|^2 = \frac{A_0^2}{1 + \epsilon^2 \left(\frac{\omega}{\omega_p} \right)^{2N}}$$

AT $\omega = \omega_p$

$$|H(j\omega_p)|^2 = \frac{A_0^2}{1 + \epsilon^2}$$

SO ϵ DETERMINES A_{MAX}

SINCE A_{MAX} IS MAX PASSBAND VARIATION



$$A_{MAX} = 20 \log \left(\frac{A_0}{A_1} \right) = 20 \log \sqrt{1 + \epsilon^2}$$

$A_{MAX} = 20 \log \sqrt{1 + \epsilon^2}$
$\epsilon = \sqrt{10^{\frac{A_{MAX}}{10}} - 1}$

①

AT STOP BAND EDGE

$$A_2^2 = |H(j\omega_s)|^2$$

$$A_2^2 = \frac{A_0^2}{1 + \epsilon^2 \left(\frac{\omega_s}{\omega_p} \right)^{2N}}$$

$$A_{MIN} \leq 20 \log \left(\frac{A_0}{A_2} \right) \leq 10 \log \frac{A_0^2}{A_2^2}$$

$A_{MIN} \leq 10 \log \left[1 + \epsilon^2 \left(\frac{\omega_s}{\omega_p} \right)^{2N} \right]$
--

②

CAN USE (2) TO FIND

N THAT WILL MAKE

$$A(\omega_s) = A_{min} \text{ IF } N \text{ IS}$$

A NON-INTEGER, CHOOSE NEXT LARGEST
INTEGER & $A(\omega_s) > A_{min}$

POLES OF BUTTERWORTH FILTER

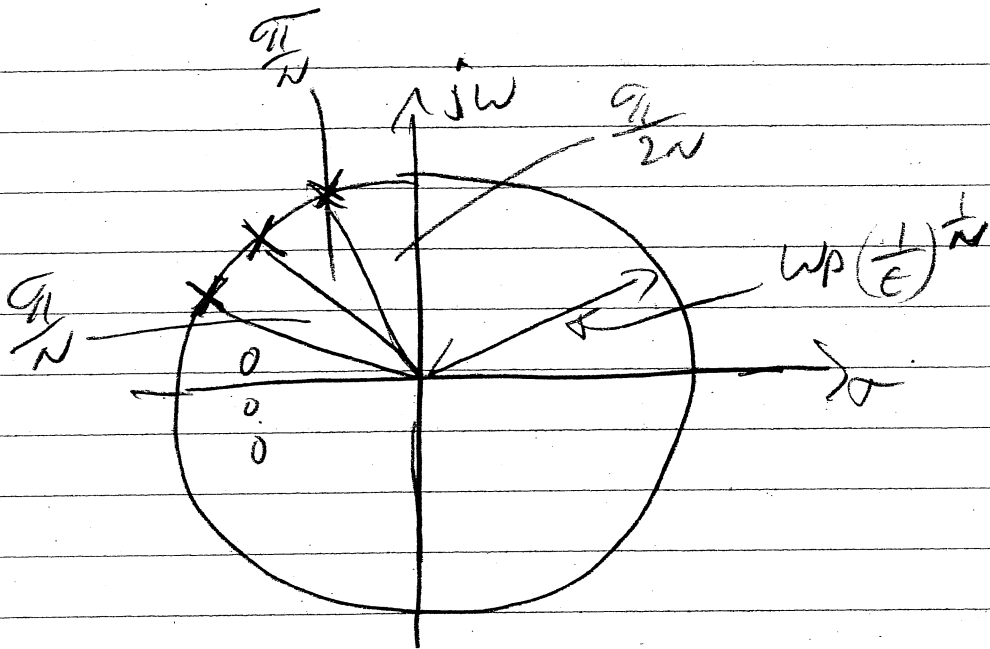
CAN BE SHOWN TO ALL LIE ON

A CIRCLE OF RADIUS $\omega_p \left(\frac{L}{\epsilon}\right)^{\frac{1}{2N}}$

AND SPACED BY EQUAL ANGLES OF

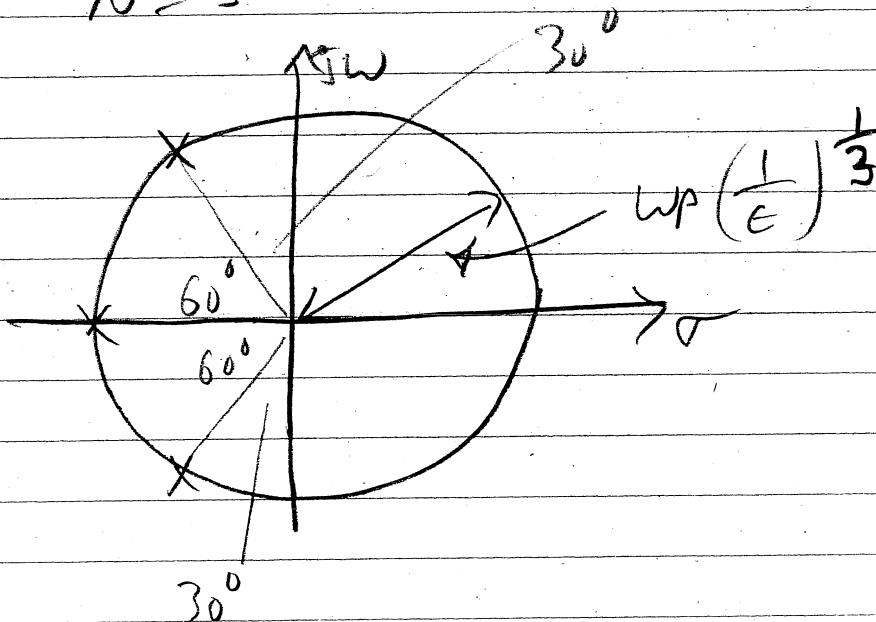
$\frac{\pi}{N}$ WITH FIRST POLE AT $\frac{\pi}{2N}$

FROM $j\omega$ AXIS



Ex

$N=3$



TO FIND $H(s)$ FOR BUTTERWORTH FILTER GIVEN A_{MAX} , A_{MIN}

- 1) FIND ϵ FROM (1)
- 2) FIND LOWEST INTEGER VALUE OF N THAT MEETS (2)
- 3) FIND POLES USING CIRCLE INFO
- 4)

$$H(s) = \frac{k \omega_0^N}{(s-p_1)(s-p_2)\dots(s-p_N)}$$

WHERE $\omega_0 = \omega_p \left(\frac{1}{\epsilon}\right)^{\frac{1}{N}}$

k IS CONSTANT TO

SET dc GAIN ($s=0$)

BUTTERWORTH FILTER EXAMPLE

EXAMPLE FIND A 4TH ORDER BUTTERWORTH FILTER WITH A 0.1 dB PASSBAND RIPPLE, $|H(j\omega)| = 1$

SOLN

$$0.1 \text{ dB} = 10 \log(1 + \epsilon^2)$$

$$\epsilon^2 = 10^{0.01} - 1 = 23.29 \times 10^{-3} = 0.02329$$

$$\therefore \epsilon = 0.15261 \Rightarrow \frac{1}{\epsilon^{\frac{1}{4}}} = 1.59994$$

POLE ROOTS AT

$$\sigma_1 = 1.59994 \times \sin\left(\frac{3}{8}\pi\right) = \pm 1.4782$$

$$\omega_1 = 1.59994 \times \cos\left(\frac{3}{8}\pi\right) = \pm 0.61227$$

$$\sigma_2 = 1.59994 \times \sin\left(\frac{7}{8}\pi\right) = \pm 0.61227$$

$$\omega_2 = 1.59994 \times \cos\left(\frac{7}{8}\pi\right) = \pm 1.4782$$

(NOTE $\frac{5}{8}\pi$ IS COMPLEX CONJUGATE OF $\frac{3}{8}\pi$)

$$\therefore H(s) = \frac{k}{(s + 1.4782 + j0.61227)(s + 1.4782 - j0.61227) \times (s + 0.61227 + j1.4782)(s + 0.61227 - j1.4782)}$$

$$H(s) = \frac{k}{s^4 + 4.1809s^3 + 9.7399s^2 + 10.7025s + 6.5529}$$

$$|H(0)| = 1 \Rightarrow k = 6.5529$$