CHEBYSHEV LOWPASS

LET $F(W^2) = \epsilon^2 C_N^2(W)$

WHERE ϵ SMALL NUMBER

$$C_N(W) = \cos(N \cos^{-1}(W)) \qquad |W| \le 1$$
$$C_N(W) = \cosh(N \cosh^{-1}(W)) \qquad |W| \ge 1$$

$C_n(W)$ IS CHEBYSHEV POLYNOMIAL OF N'TH ORDER + IS A POLYNOMIAL IN W OF N'TH DEGREE

TO SEE THAT LET $\cos\theta = W$

$$\theta = \cos^{-1} W$$

$$C_{N+1}(W) = \cos((N+1)\theta) = \cos(N\theta + \theta)$$

① $C_{N+1}(W) = \cos(N\theta)\cos(\theta) - \sin(N\theta)\sin(\theta)$

② $C_{N-1}(W) = \cos(N\theta)\cos(\theta) + \sin(N\theta)\sin(\theta)$

①+② $C_{N+1}(W) + C_{N-1}(W) = 2\cos(N\theta)\cos(\theta)$

+ $\cos N\theta = \cos(N\cos^{-1} W) = C_N(W)$

+ $\cos\theta = W$

$\therefore$ $C_{N+1}(W) + C_{N-1}(W) = 2 W C_N(W)$

$$\boxed{C_{N+1}(w) = 2w\, C_N(w) - C_{N-1}(w)}$$

(SIMILAR FOR $\cosh(N \cosh^{-1}(w))$)

RECURSIVE FORMULA TO FIND $C_N(w)$

$C_0(w) = \cos(0) = 1$

$C_1(w) = \cos(\cos^{-1}(w)) = w$

$C_2(w) = 2w\, C_1(w) - C_0(w)$

$= 2w^2 - 1$

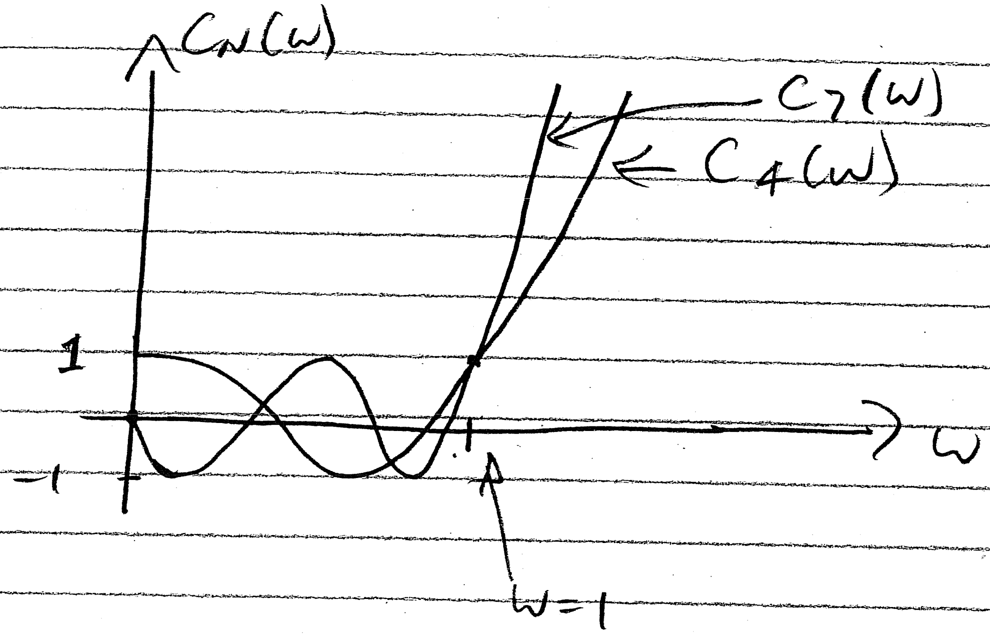
$C_3(w) = 4w^3 - 3w$

$C_4(w) = 8w^4 - 8w^2 + 1$

$C_5(w) = 16w^5 - 20w^3 + 5w$

$C_6(w) = 32w^6 - 48w^4 + 18w^2 - 1$

$\vdots$



FOR $|\omega| \leq 1$ $\quad |C_N(\omega)| \leq 1$

FOR $\omega > 1$ $\quad C_N(\omega) > 1$
& MONOTONICALLY RISING

(3) FOR $\omega \gg 1$ $\quad C_N(\omega) \approx 2^{N-1} \omega^N$

$$|H(j\omega)|^2 = \frac{A_0^2}{1 + \epsilon^2 \cos^2\left[N \cos^{-1}\left(\frac{\omega}{\omega_p}\right)\right]} \quad \omega \leq \omega_p$$

$$|H(j\omega)|^2 = \frac{A_0^2}{1 + \epsilon^2 \cosh^2\left[N \cosh^{-1}\left(\frac{\omega}{\omega_p}\right)\right]} \quad \omega \geq \omega_p$$

$$A_{MAX} = 10\log(1+\epsilon^2)$$

$$\epsilon = \sqrt{10^{\frac{A_{MAX}}{10}} - 1}$$

TO FIND ϵ

$$A_{MIN} \leq 10\log\left[1+\epsilon^2\cosh^2\left[N\cosh^{-1}\left(\frac{W_S}{W_P}\right)\right]\right]$$

TO FIND N

POLES ON AN ELLIPSE & LOCATED AT

$$P_k = -W_p \sin\left(\frac{2k-1}{N}\frac{\pi}{2}\right)\sinh\left(\frac{1}{N}\sinh^{-1}\frac{1}{\epsilon}\right)$$

$$+ jW_p\cos\left(\frac{2k-1}{N}\frac{\pi}{2}\right)\cosh\left(\frac{1}{N}\sinh^{-1}\frac{1}{\epsilon}\right)$$

$k = 1, 2, \ldots N$

$$H(s) = \frac{K\,W_p^N}{(s-P_1)(s-P_2)\cdots(s-P_N)\,\epsilon\,2^{N-1}}$$

K IS THE dc GAIN

(**NOTE** W_P USED HERE WHILE W_0 USED IN BUTTERWORTH)

COMPARISON OF BUTTERWORTH & CHEBYSHEV

BUTTERWORTH $F(\omega^2) = \omega^{2N}$ LET $\omega_p = 1$

$$|H(j\omega)|^2 = \frac{1}{1+\epsilon^2\omega^{2N}}$$

FOR $\omega \gg 1$

$$|H(j\omega)|^2 \approx \frac{1}{\epsilon^2\omega^{2N}}$$

CHEBSHEV $F(\omega^2) = \epsilon^2 C_N^2(\omega)$ LET $\omega_p = 1$

$$|H(j\omega)|^2 = \frac{1}{1+\epsilon^2 C_N^2(\omega)}$$

FOR $\omega \gg 1$

$$|H(j\omega)| \approx \frac{1}{\epsilon^2\, 2^{2(N-1)}\, \omega^{2N}}$$ FROM 3

WHICH IS $\frac{1}{2^{2(N-1)}}$ TIMES SMALLER

OR $10 \log 2^{2(N-1)} = 20(N-1)\log 2$

$= 6(N-1)$ dB MORE ATTENUATION

SO IF $N=7$ A CHEBYSHEV WOULD HAVE $\approx 6(7-1) = 36\,dB$ MORE ATTENUATION WHEN $\omega \gg \omega_p$ THAN A BUTTERWORTH

EXAMPLE

$A_{MAX} = 1\,dB$ dc GAIN = 1

$A_{MIN} = 25\,dB$

$\omega_p = 1$ RAD/S

$\omega_s = 1.5$ RAD/S

FIND N & $|H|^2$ FOR CHEBYSHEV

$$\epsilon = \sqrt{10^{\frac{A_{MAX}}{10}} - 1} = 0.5088$$

$$25 = 10 \log\left[1 + \epsilon^2 \cosh^2\left[N \cosh^{-1}\left(\frac{\omega_s}{\omega_p}\right)\right]\right]$$

$$\frac{10^{2.5} - 1}{(0.5088)^2} = \cosh^2\left[N \cosh^{-1}(1.5)\right]$$

$$34.89 = \cosh\left[N \cosh^{-1}(1.5)\right]$$

$$N = \frac{\cosh^{-1}[34.89]}{\cosh^{-1}[1.5]}$$

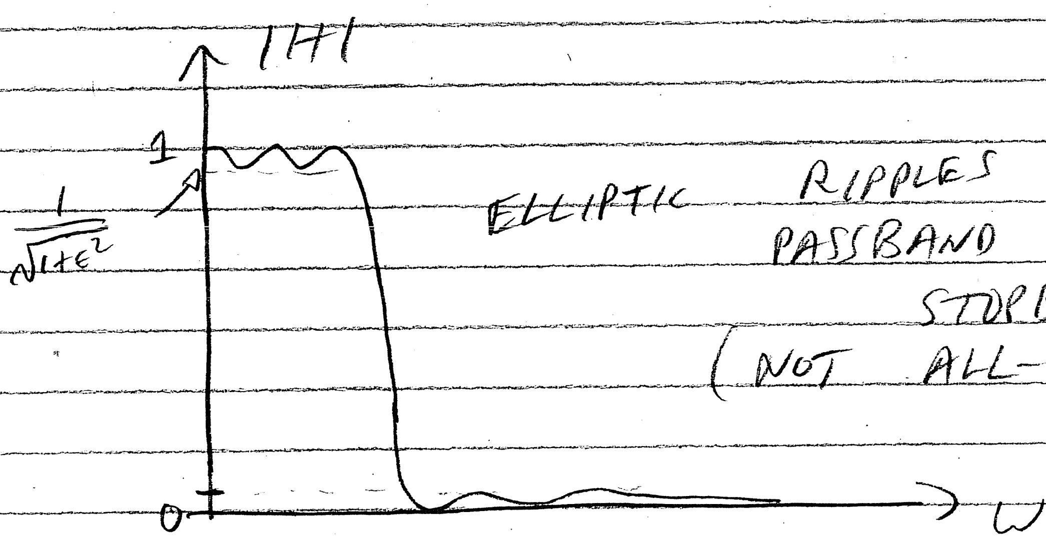
$$= 4.41$$

CHOOSE $\underline{\underline{N = 5}}$

$$|H(j\omega)|^2 = \frac{1}{1+(0.5088)^2(16\omega^5 - 20\omega^3 + 5\omega)^2}$$

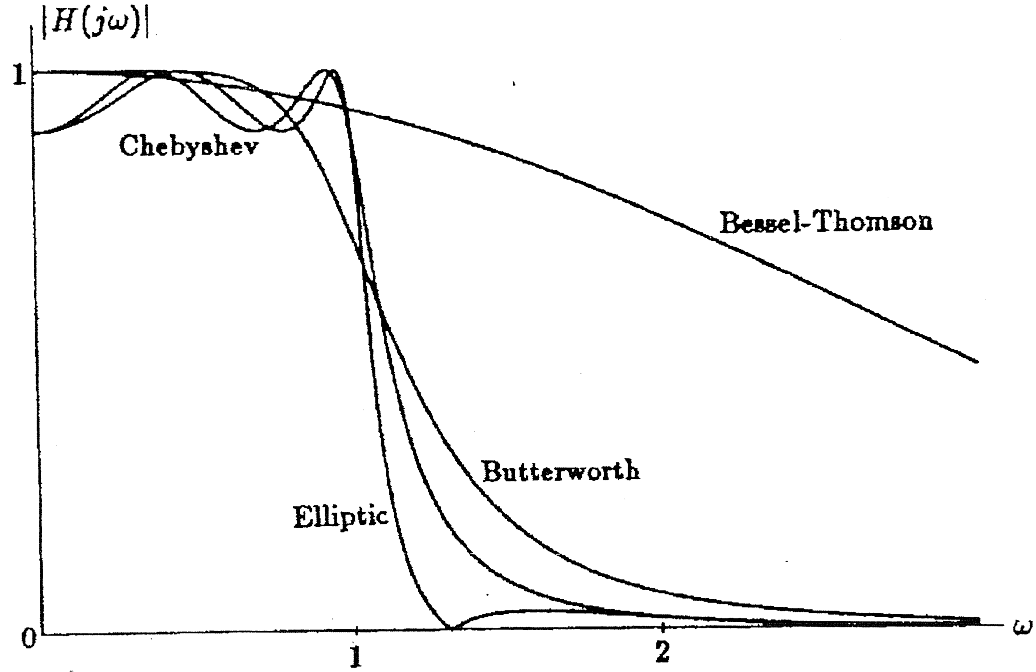
CAN ALSO BUILD

- ELLIPTIC FILTERS
- BESSEL-THOMSON

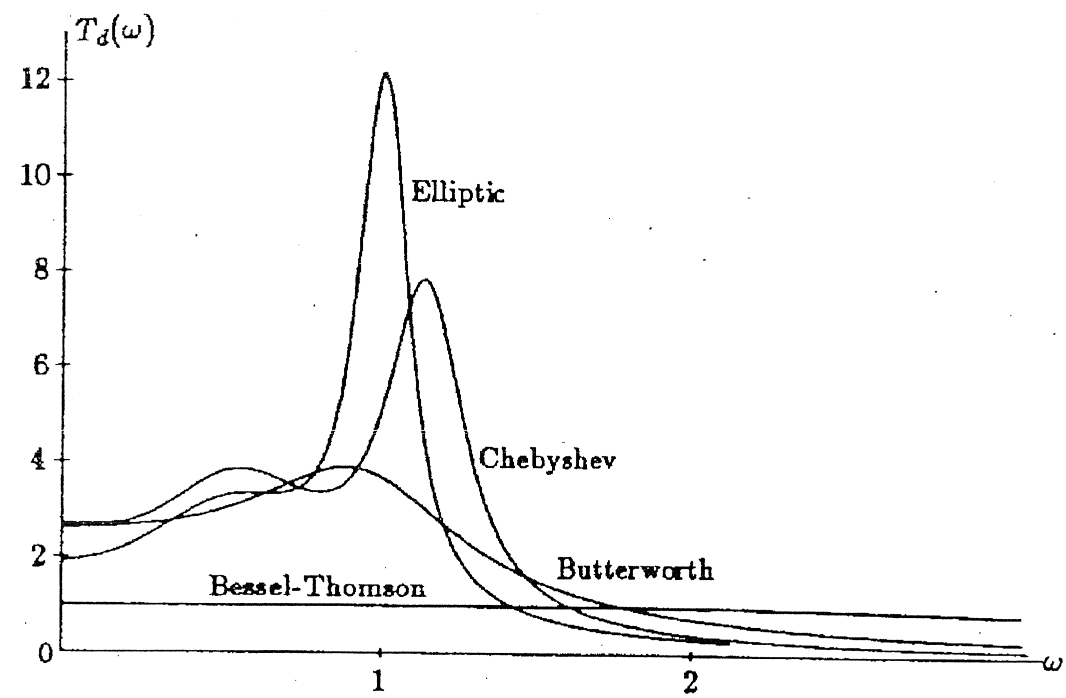


ELLIPTIC — RIPPLES IN PASSBAND & STOPBAND (NOT ALL-POLE)

Some Filter plots - magnitude



Some Filter plots - group delay

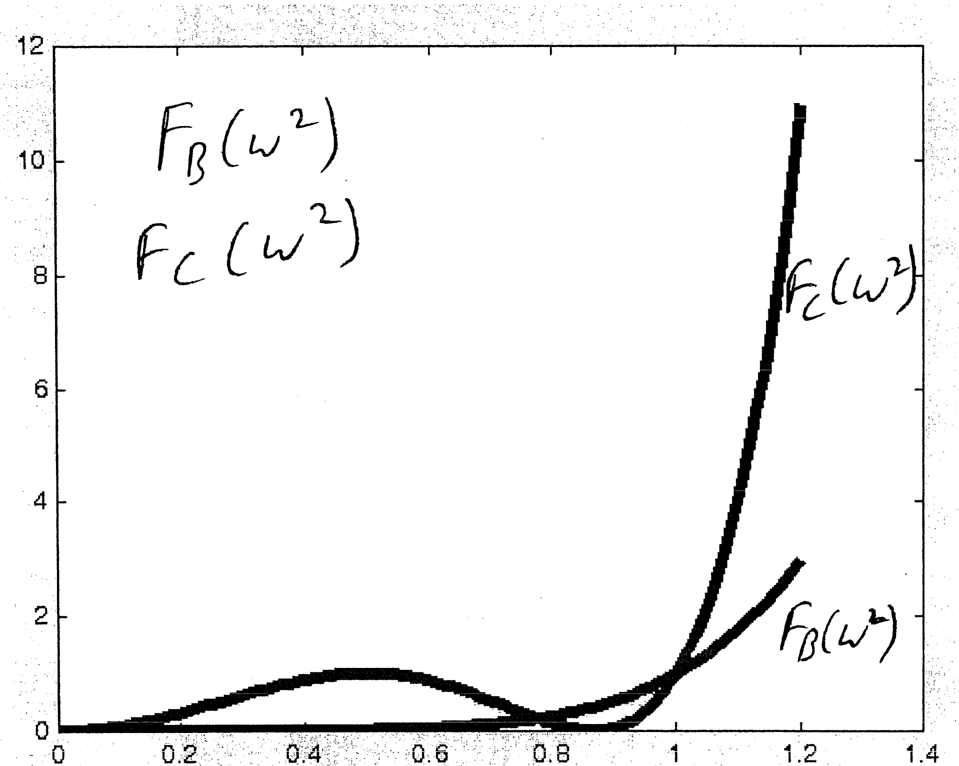


33A

3RD ORDER ($N=3$), $\epsilon = 1$, $\omega_p = 1$

BUTTERWORTH $\quad F_B(\omega^2) = \omega^6$

CHEBYSHEV $\quad F_C(\omega^2) = (4\omega^3 - 3\omega)^2$



$\Rightarrow$ ϵ SCALES ABOVE CURVE VERTICALLY

$\Rightarrow$ ω_p SCALES ABOVE CURVE HORIZONTALLY

$$|H(j\omega)|^2 = \frac{A_0^2}{1 + F(\omega^2)}$$