

CHEBYSHEV LOWPASS

LET $F(\omega^2) = \epsilon^2 C_N^2(\omega)$

WHERE ϵ SMALL NUMBER

$C_N(\omega) = \cos(N \cos^{-1}(\omega)) \quad |\omega| \leq 1$
 $C_N(\omega) = \cosh(N \cosh^{-1}(\omega)) \quad |\omega| \geq 1$

$C_N(\omega)$ IS CHEBYSHEV POLYNOMIAL OF N'TH ORDER + IS A POLYNOMIAL IN ω OF N'TH DEGREE

TO SEE THAT LET $\cos \theta = \omega$

$\theta = \cos^{-1} \omega$

$C_{N+1}(\omega) = \cos((N+1)\theta) = \cos(N\theta + \theta)$

① $C_{N+1}(\omega) = \cos(N\theta)\cos(\theta) - \sin(N\theta)\sin(\theta)$

② $C_{N-1}(\omega) = \cos(N\theta)\cos(\theta) + \sin(N\theta)\sin(\theta)$

①+② $C_{N+1}(\omega) + C_{N-1}(\omega) = 2 \cos(N\theta)\cos(\theta)$

$\therefore \cos N\theta = \cos(N \cos^{-1} \omega) = C_N(\omega)$

$\therefore \cos \theta = \omega$

$\therefore C_{N+1}(\omega) + C_{N-1}(\omega) = 2 \omega C_N(\omega)$

$$C_{N+1}(w) = 2wC_N(w) - C_{N-1}(w)$$

(SIMILAR FOR $\cosh(N \cosh^{-1}(w))$)

RECURSIVE FORMULA TO FIND $C_N(w)$

$$C_0(w) = \cosh(0) = 1$$

$$C_1(w) = \cosh(\cosh^{-1}(w)) = w$$

$$C_2(w) = 2wC_1(w) - C_0(w)$$

$$= 2w^2 - 1$$

$$C_3(w) = 4w^3 - 3w$$

$$C_4(w) = 8w^4 - 8w^2 + 1$$

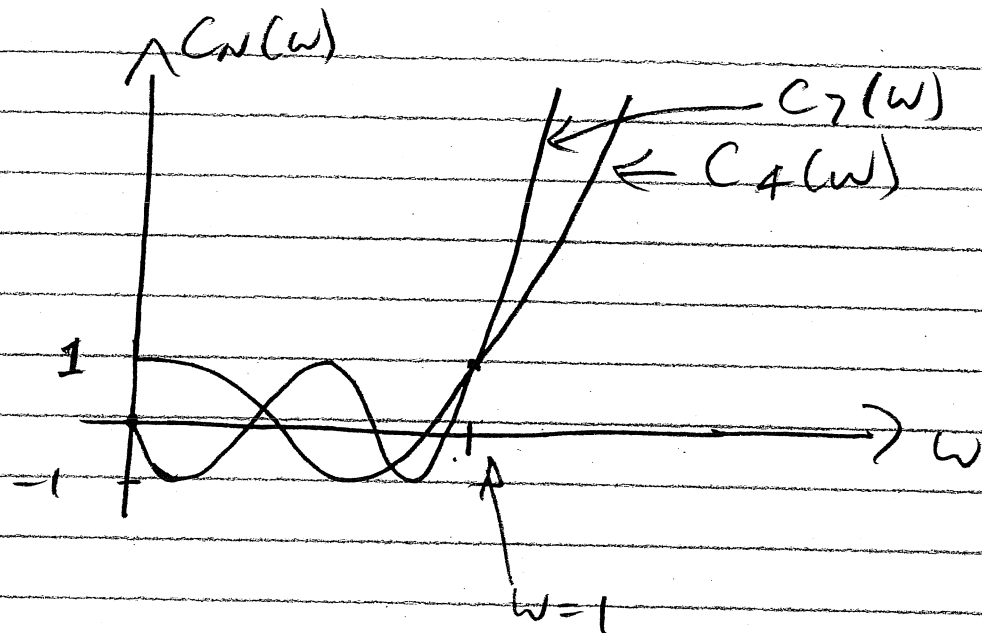
$$C_5(w) = 16w^5 - 20w^3 + 5w$$

$$C_6(w) = 32w^6 - 48w^4 + 18w^2 - 1$$

0

0

0



FOR $|\omega| \leq 1$ $|C_N(\omega)| \leq 1$

FOR $\omega > 1$ $C_N(\omega) > 1$
 & MONOTONICALLY RISING

③ FOR $\omega \gg 1$ $C_N(\omega) \approx 2^{N-1} \omega^N$

$$|H(j\omega)|^2 = \frac{A_0^2}{1 + \epsilon^2 \cos^2 \left[N \cos^{-1} \left(\frac{\omega}{\omega_p} \right) \right]} \quad \omega \leq \omega_p$$

$$|H(j\omega)|^2 = \frac{A_0^2}{1 + \epsilon^2 \cosh^2 \left[N \cosh^{-1} \left(\frac{\omega}{\omega_p} \right) \right]} \quad \omega \geq \omega_p$$

$$A_{max} = 10 \log (1 + \epsilon^2)$$

$$\epsilon = \sqrt{10^{\frac{A_{max}}{10}} - 1}$$

TO FIND ϵ

$$A_{min} \leq 10 \log \left[1 + \epsilon^2 \cosh^2 \left[N \cosh^{-1} \left(\frac{\omega_s}{\omega_p} \right) \right] \right]$$

TO FIND N

POLES ON AN ELLIPSE & LOCATED AT

$$P_k = -\omega_p \sin \left(\frac{2k-1}{N} \frac{\pi}{2} \right) \sinh \left(\frac{1}{N} \sinh^{-1} \frac{1}{\epsilon} \right) + j\omega_p \cos \left(\frac{2k-1}{N} \frac{\pi}{2} \right) \cosh \left(\frac{1}{N} \sinh^{-1} \frac{1}{\epsilon} \right)$$

$$k = 1, 2, \dots, N$$

$$H(s) = \frac{K \omega_p^N}{(s-p_1)(s-p_2) \dots (s-p_N) \epsilon^{2^{N-1}}}$$

K IS THE DC GAIN

(NOTE ω_p USED HERE WHILE ω_0 USED IN BUTTERWORTH)

COMPARISON OF BUTTERWORTH & CHEBYSHEV

BUTTERWORTH $F(\omega^2) = \omega^{2N}$ LET $\omega_p = 1$

$$|H(j\omega)|^2 = \frac{1}{1 + \epsilon^2 \omega^{2N}}$$

FOR $\omega \gg 1$

$$|H(j\omega)|^2 \approx \frac{1}{\epsilon^2 \omega^{2N}}$$

CHEBYSHEV $F(\omega^2) = \epsilon^2 C_N^2(\omega)$ LET $\omega_p = 1$

$$|H(j\omega)|^2 = \frac{1}{1 + \epsilon^2 C_N^2(\omega)}$$

FOR $\omega \gg 1$

$$|H(j\omega)| \approx \frac{1}{\epsilon^2 2^{\frac{2(N-1)}{N}} \omega^{2N}} \text{ FROM (3)}$$

WHICH IS $\frac{1}{2^{2(N-1)}}$ TIMES SMALLER

$$\text{OR } 10 \log 2^{2(N-1)} = 20(N-1) \log 2$$

= 6(N-1) dB MORE ATTENUATION

SO IF $N=7$ A CHEBYSHEV
 WOULD HAVE $\approx 6(7-1) = 36$ dB
 MORE ATTENUATION WHEN $\omega \gg \omega_p$
 THAN A BUTTERWORTH

EXAMPLE

- $A_{MAX} = 1$ dB
- $A_{MIN} = 25$ dB
- $\omega_p = 1$ RAD/S
- $\omega_s = 1.5$ RAD/S
- dc GAIN = 1

FIND N & $|H|^2$ FOR CHEBYSHEV

$$\epsilon = \sqrt{10^{\frac{A_{MAX}}{10}} - 1} = 0.5088$$

$$25 = 10 \log \left[1 + \epsilon^2 \cosh^2 \left[N \cosh^{-1} \left(\frac{\omega_s}{\omega_p} \right) \right] \right]$$

$$\frac{10^{2.5} - 1}{(0.5088)^2} = \cosh^2 \left[N \cosh^{-1} (1.5) \right]$$

$$34.89 = \cosh \left[N \cosh^{-1} (1.5) \right]$$

$$N = \frac{\cosh^{-1}[34.89]}{\cosh^{-1}[1.5]}$$

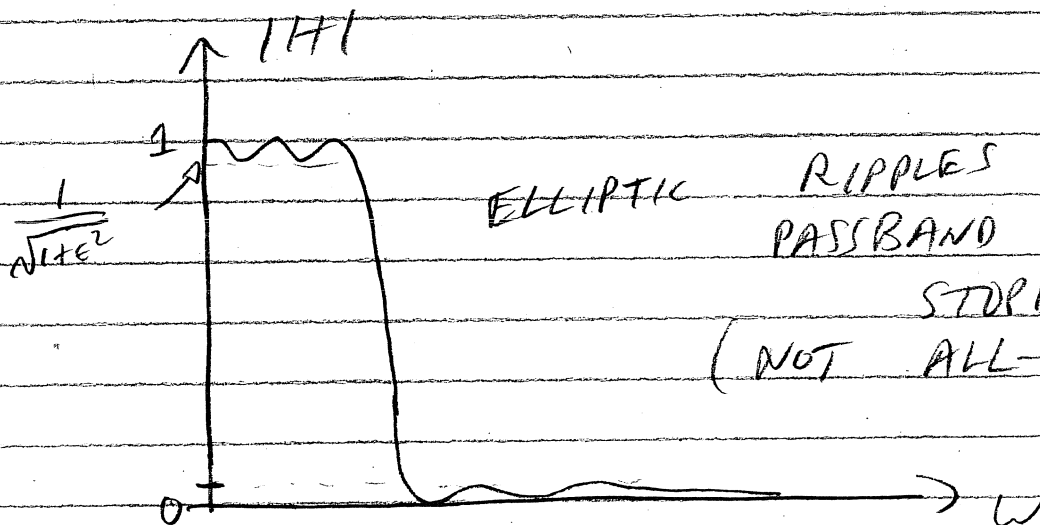
$$= 4.41$$

CHOOSE N = 5

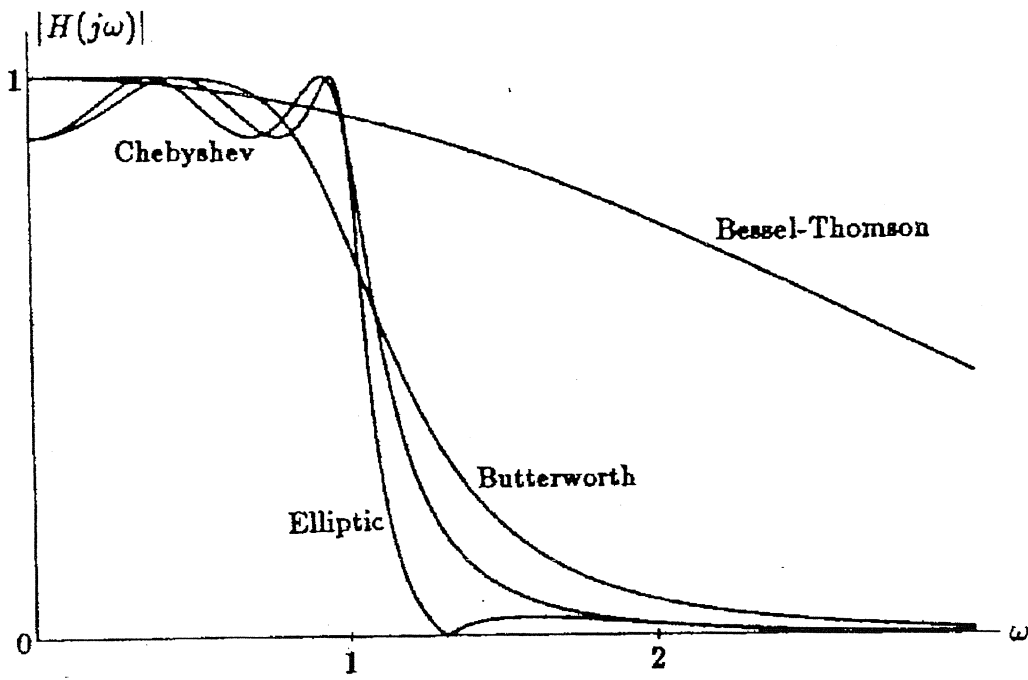
$$|H(j\omega)|^2 = \frac{1}{1 + (0.5088)^2 (16\omega^5 - 20\omega^3 + 5\omega)^2}$$

CAN ALSO BUILD

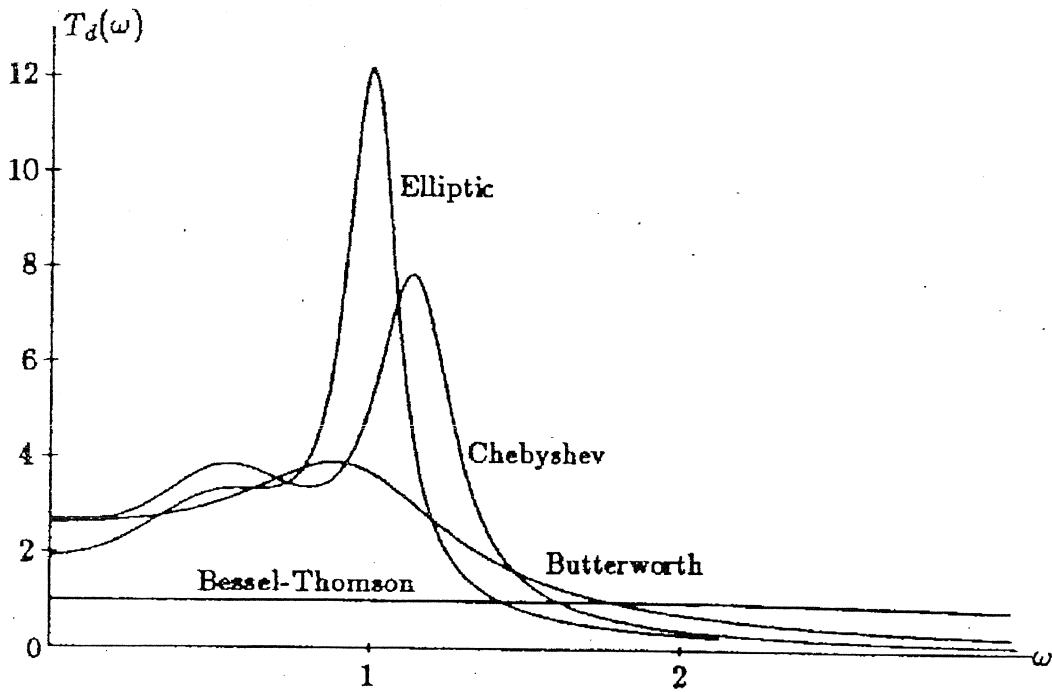
- ELLIPTIC FILTERS
- BESSEL-THOMSON



Some Filter plots - magnitude



Some Filter plots - group delay

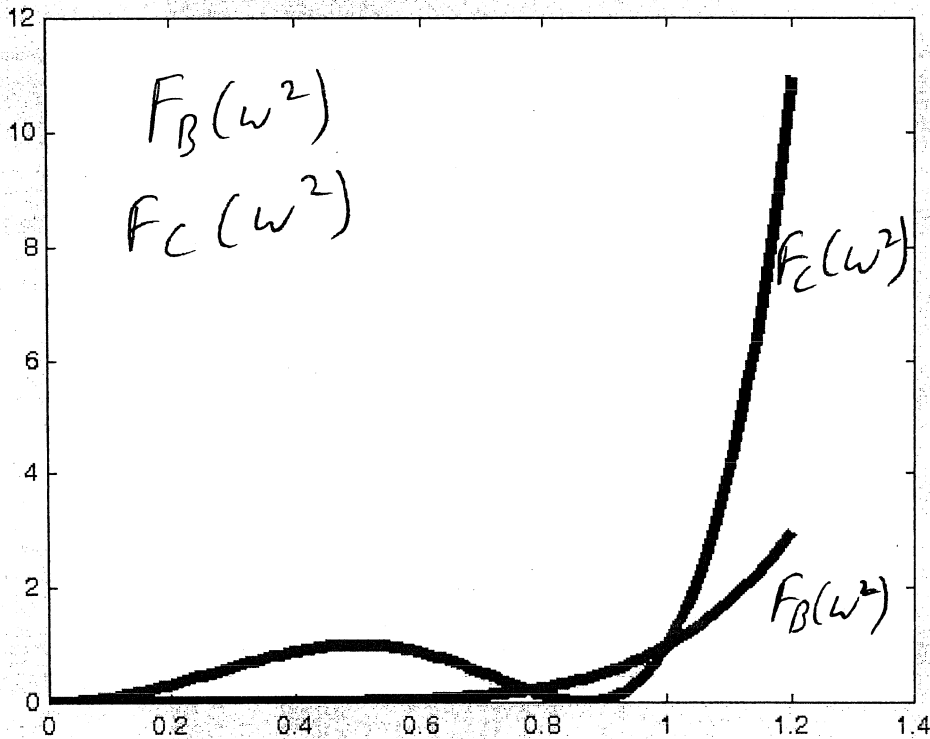


BRO ORDER ($N=3$), $\epsilon=1$, $\omega_p=1$

33A

BUTTERWORTH $F_B(\omega^2) = \omega^6$

CHEBYSHEV $F_C(\omega^2) = (4\omega^3 - 3\omega)^2$



$\Rightarrow \epsilon$ SCALES ABOVE CURVE
VERTICALLY

$\Rightarrow \omega_p$ SCALES ABOVE CURVE
HORIZONTALLY

$$|H(j\omega)|^2 = \frac{A_0^2}{1 + F(\omega^2)}$$