FIRST AND SECOND ORDER FILTERS

WHY SO IMPORTANT?

A N'TH ORDER H(S) CAN BE FACTORED INTO 1'st ORDER TRANSFER-FUNCTIONS

$$H(s) = H_1(s)\, H_2(s) \ldots H_N(s)$$

WHERE $H_i(s)$ ARE 1'st ORDER

$$V_{IN} \longrightarrow \boxed{H(s)} \longrightarrow V_{OUT}$$

$$\Updownarrow$$

$$V_{IN} \longrightarrow \boxed{H_1(s)} - \boxed{H_2(s)} - \cdots - \boxed{H_N(s)} \longrightarrow V_{OUT}$$

BUT IF H(S) HAS COMPLEX ZEROS **OR** POLES THEN THOSE $H_i(s)$ HAVE COMPLEX COEFFICIENTS $\Rightarrow$ **NOT** EASILY REALIZED

HOWEVER, IF WE <u>GROUP</u> COMPLEX-CONJUGATE PAIRS OF POLES AND ZEROS INTO 2nd ORDER SECTIONS THEN ALL COEFFICIENTS BECOME REAL

SPECIFICALLY, IF $H(s) = \dfrac{[s-(x_1+jy_1)][s-(x_1-jy_1)]\ldots}{\underbrace{\underbrace{[s-(x_2+jy_2)]}_{T_1(s)}\underbrace{[s-(x_2+jy_2)]}_{T_2(s)}}_{H_{BQ_1}(s)}\ldots}$ x_1, y_1 / x_2, y_2 ALL REAL

$$H(s) = \frac{[s^2-2x_1 s-(x_1^2+y_1^2)]\ldots}{\underbrace{[s^2-2x_2 s-(x_2^2+y_2^2)]}_{H_{BQ_1}(s)}\ldots}$$

$H_{BQ_1}(s) \leftarrow$ BIQUAD (BIQUADRATIC)

CAN <u>ALWAYS</u> WRITE $H(s)$ IN FACTORED FORM CONTAINING BIQUADS (WITH 2nd ORDER ONLY REAL COEFF) AND POSSIBLY ONE BILINEAR (1st ORDER WITH REAL COEFF)

(A) 1st

$$H(s) = \frac{a_1 s + a_0}{s + b_0} \triangleq \frac{a_1 s + a_0}{s + \omega_0}$$

$\omega_0 = b_0$

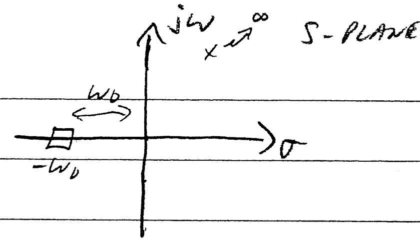
$\omega_0 \rightarrow$ POLE FREQUENCY

POLE AT $s = -\omega_0$ $\therefore$ $\omega_0 > 0$ FOR STABILITY

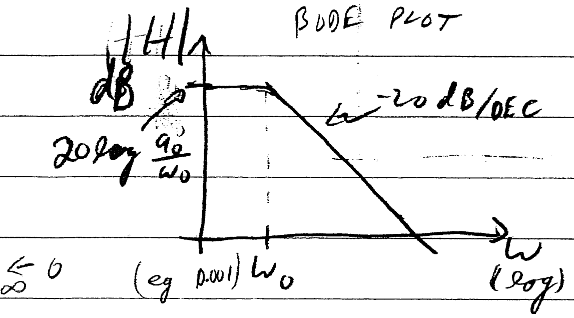
TRANSMISSION ZERO AT $s = -\dfrac{a_0}{a_1}$

DC $s=0$ $H(0) = \dfrac{a_0}{\omega_0}$ AT HF $s \rightarrow \infty$ $H(\infty) = a_1$

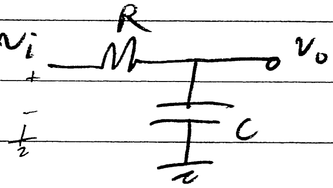
a) LOW PASS $H(s) = \dfrac{a_0}{s+\omega_0}$



BODE PLOT



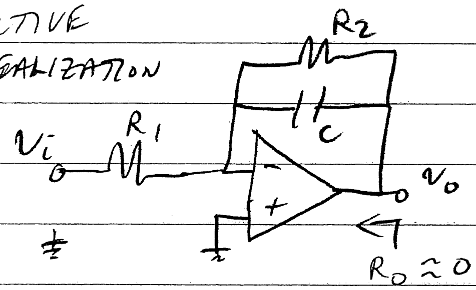
PASSIVE REALIZATION



$CR = \frac{1}{\omega_0}$

DC GAIN = 1

ACTIVE REALIZATION

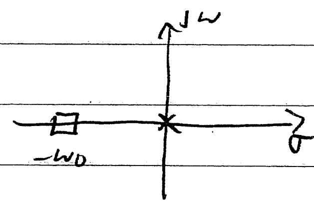


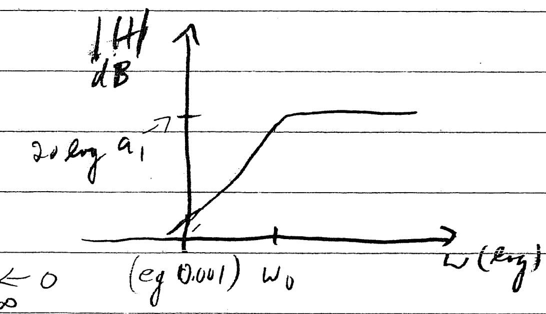
$CR_2 = \frac{1}{\omega_0}$

DC GAIN = $-\dfrac{R_2}{R_1}$

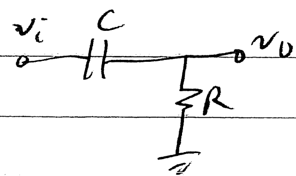
SO CAN CASCADE CIRCUITS WITH LITTLE INTERACTION

b) HIGH-PASS $H(s) = \dfrac{a_1 s}{s+\omega_0}$





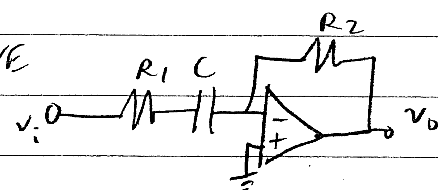
PASSIVE



$CR = \frac{1}{\omega_0}$

HF GAIN = 1

ACTIVE



$CR_1 = \frac{1}{\omega_0}$

HF GAIN = $-\dfrac{R_2}{R_1}$

SEE TEXT FOR GENERAL & ALL-PASS CASES.

(B) BIQUADRATIC $H(s) = \dfrac{a_2 s^2 + a_1 s + a_0}{s^2 + b_1 s + b_0} \triangleq \dfrac{a_2 s^2 + a_1 s + a_0}{s^2 + \left(\frac{\omega_0}{Q}\right) s + \omega_0^2}$

POLES

$\omega_0 = \sqrt{b_0}$ POLE FREQUENCY

$Q = \dfrac{\sqrt{b_0}}{b_1}$ POLE QUALITY FACTOR (OR Q FACTOR OR POLE-Q)

WRITING $s^2 + \frac{\omega_0}{Q} s + \omega_0^2 = (s - e_1)(s - e_2)$ (ie SOLVING FOR ROOTS)

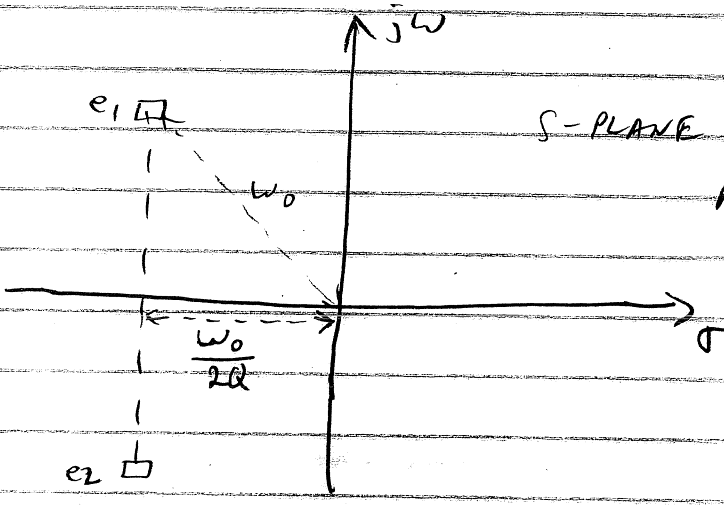
$$e_1, e_2 = -\frac{\omega_0}{2Q} \pm j\omega_0 \sqrt{1 - \frac{1}{4Q^2}}$$

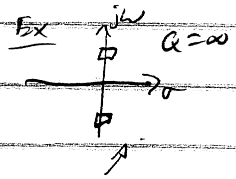
$\boxed{\text{FOR } \frac{1}{4Q^2} < 1}$ OR $\boxed{Q > 0.5}$ e_1 & e_2 ARE COMPLEX-CONJUGATE

& $|e_1| = |e_2| = \omega_0$ & $\text{REAL}[e_1] = \text{REAL}[e_2] = -\frac{\omega_0}{2Q}$

GRAPHICALLY

FOR $Q > 0.5$

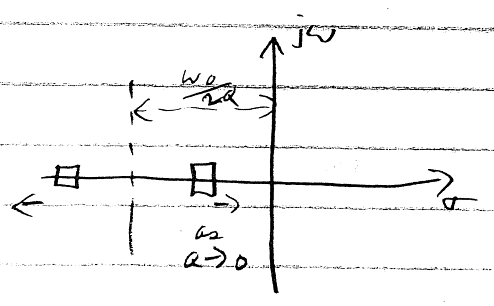




AS $Q \to \infty$ e_1 + e_2 APPROACH $j\omega$ AXIS

$Q \to 0.5$ e_1 & e_2 APPROACH REAL AXIS

$\boxed{\text{FOR } Q < 0.5 \quad Q > 0 \text{ FOR STABILITY}}$ e_1 & e_2 ARE ON THE REAL AXIS



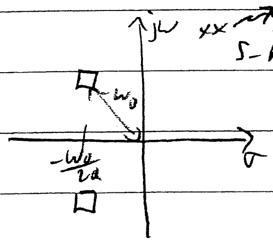
<u>TRANSMISSION ZEROS</u>

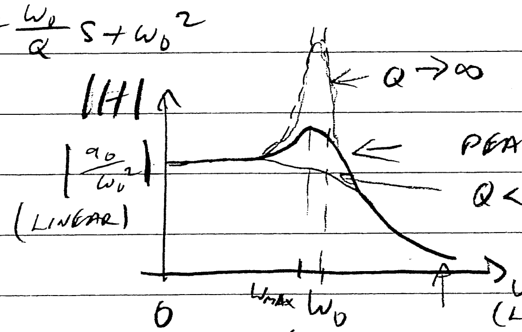
$a_2 s^2 + a_1 s + a_0$

$LP \Rightarrow a_0$ $\qquad HP \Rightarrow a_2 s^2$

$BP \Rightarrow a_1 s$ $\qquad NOTCH \Rightarrow a_2 s^2 + a_0$

a) LOW PASS $\qquad H(s) = \dfrac{a_0}{s^2 + \frac{\omega_0}{Q} s + \omega_0^2}$ $\qquad$ DC GAIN $\Rightarrow T(0) = \dfrac{a_0}{\omega_0^2}$





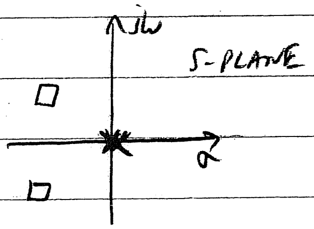
PEAK ONLY IF $Q > \frac{1}{\sqrt{2}}$

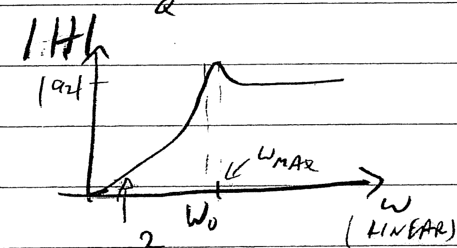
$Q < \frac{1}{\sqrt{2}}$

$\omega_{MAX} = \omega_0 \sqrt{1 - \frac{1}{2Q^2}}$

(ROUGHLY) $|H| \propto \frac{1}{\omega^2}$

b) HIGH PASS $\qquad H(s) = \dfrac{a_2 s^2}{s^2 + \frac{\omega_0}{Q} s + \omega_0^2}$ $\qquad$ HF GAIN $\Rightarrow H(\infty) = a_2$

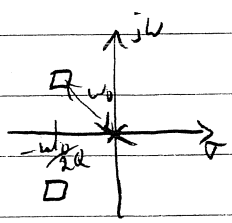


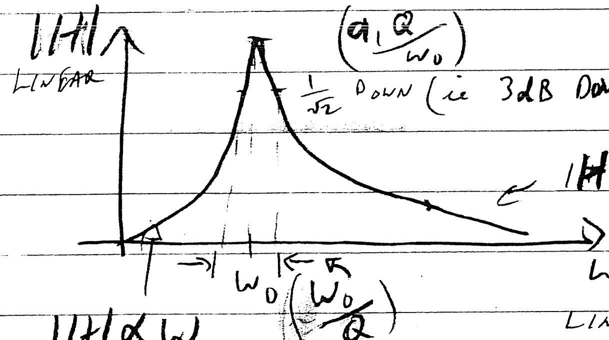


$\omega_{MAX} = \dfrac{\omega_0}{\sqrt{1 - \frac{1}{2Q^2}}}$

$|T| \propto \omega^2$

c) BAND PASS $\qquad H(s) = \dfrac{a_1 s}{s^2 + \frac{\omega_0}{Q} s + \omega_0^2}$





$\frac{1}{\sqrt{2}}$ DOWN (ie 3dB DOWN)

$|H| \propto \frac{1}{\omega}$

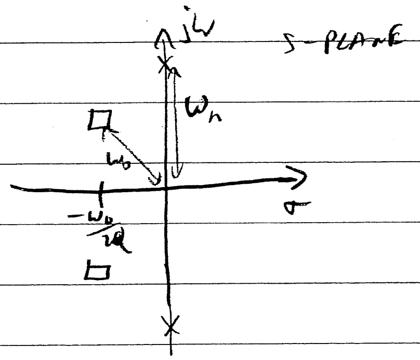
$|H| \propto \omega$

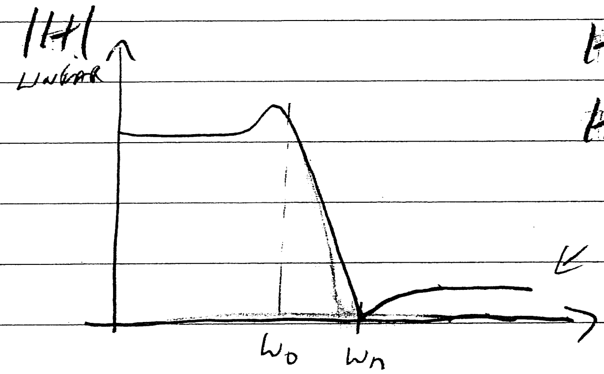
WIDTH OF 3dB BANDWIDTH $BW = \frac{\omega_0}{Q}$

[3dB BANDWIDTH $\triangleq \omega_2 - \omega_1$ WHERE $|H(\omega_2)| = |H(\omega_1)| = \frac{1}{\sqrt{2}}$ PEAK VALUE]

d) LOW PASS NOTCH (LPN)

$$H(s) = a_2 \frac{s^2 + \omega_n^2}{s^2 + \frac{\omega_0}{Q}s + \omega_0^2} \qquad \omega_n \geq \omega_0$$





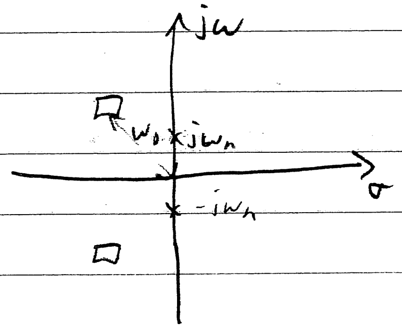
$$H(0) = \frac{a_2 \omega_n^2}{\omega_0^2}$$

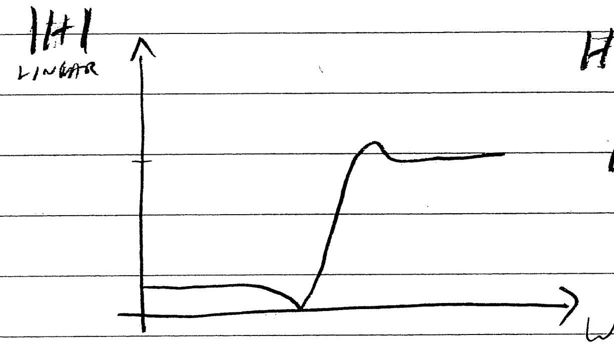
$$H(\infty) = a_2$$

NOTE $\neq 0$

e) HIGH PASS NOTCH (HPN)

$$H(s) = a_2 \frac{s^2 + \omega_n^2}{s^2 + \frac{\omega_0}{Q}s + \omega_0^2} \qquad \omega_n \leq \omega_0$$



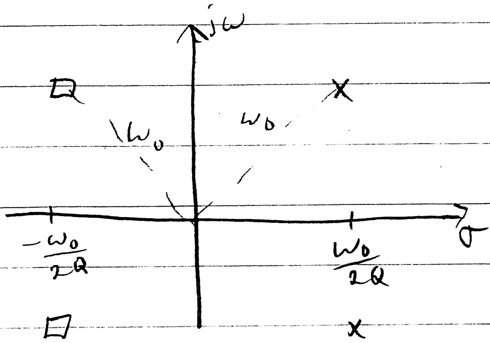


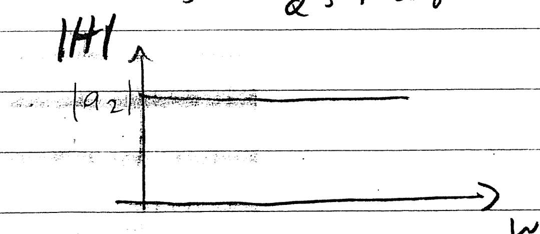
$$H(0) = \frac{a_2 \omega_n^2}{\omega_0^2}$$

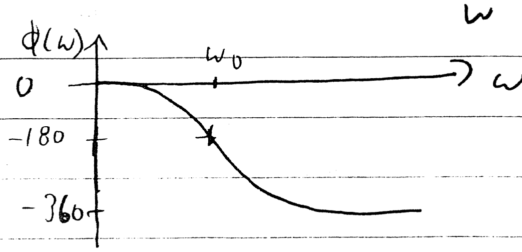
$$H(\infty) = a_2$$

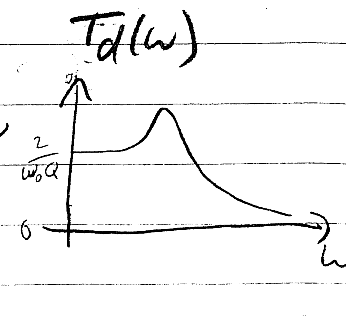
f) ALL-PASS

$$H(s) = a_2 \frac{s^2 - s\frac{\omega_0}{Q} + \omega_0^2}{s^2 + \frac{\omega_0}{Q}s + \omega_0^2}$$

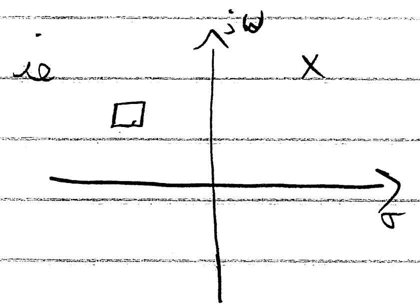




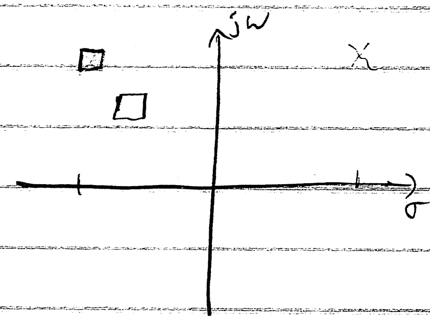




NOTE FOR PHASE – THE EFFECT OF A ZERO IN THE RHP IS THE SAME AS A MIRROR IMAGE POLE IN LHP

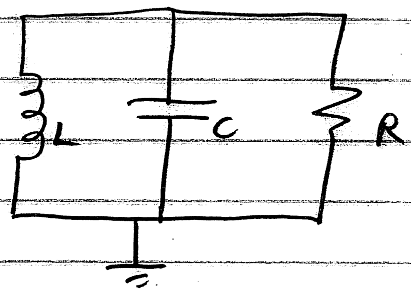


SAME PHASE AS



1.7 2ND ORDER LCR RESONATOR

THE POLES OF A BIQUAD CAN BE REALIZED WITH THE FOLLOWING LCR CIRCUIT



SHALL SEE THAT $\omega_0 = \frac{1}{\sqrt{LC}}$

$Q = \omega_0 C R$

IF THE INPUT IS APPLIED OR OUTPUT TAKEN SUCH THAT IT DOES NOT AFFECT THE NATURAL RESONANCE, THEN THE POLES CAN BE FOUND AS THE DENOMINATOR OF THE TRANSFER-FUNCTION POLYNOMIAL