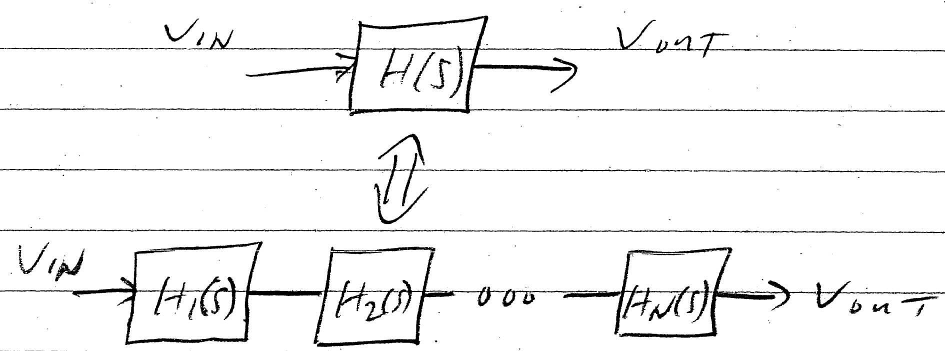
FIRST AND SECOND ORDER FILTERS

WHY SO IMPORTANT?

A N'TH ORDER H(S) CAN BE FACTORED INTO 1'st ORDER TRANSFER-FUNCTIONS

$$H(s) = H_1(s)\, H_2(s) \ldots H_N(s)$$

WHERE $H_i(s)$ ARE 1'st ORDER



BUT IF H(S) HAS COMPLEX ZEROS OR POLES THEN THOSE $H_i(s)$ HAVE COMPLEX COEFFICIENTS ⇒ NOT EASILY REALIZED

HOWEVER, IF WE GROUP COMPLEX-CONJUGATE PAIRS OF POLES AND ZEROS INTO 2nd ORDER SECTIONS THEN ALL COEFFICIENTS BECOME REAL

SPECIFICALLY, IF

$$H(s) = \frac{[s-(x_1+jy_1)][s-(x_1-jy_1)]\ldots}{\underbrace{\underbrace{[s-(x_2+jy_2)]}_{T_1(s)}\underbrace{[s-(x_2-jy_2)]}_{T_2(s)}}_{H_{BQ_1}(s)}\ldots}$$

x_1, y_1 ALL
x_2, y_2 REAL

$$H(s) = \frac{[s^2-2x_1s-(x_1^2+y_1^2)]\ldots}{\underbrace{[s^2-2x_2s-(x_2^2+y_2^2)]}_{H_{BQ_1}(s)}\ldots}$$

$H_{BQ_1}(s) \Leftarrow$ BIQUAD (BIQUADRATIC)

CAN ALWAYS WRITE $H(s)$ IN FACTORED FORM CONTAINING 2nd ORDER BIQUADS (WITH ONLY REAL COEFF) AND POSSIBLY ONE BILINEAR (1st ORDER WITH REAL COEFF)

(A) 1st ORDER

$$H(s) = \frac{a_1 s + a_0}{s + b_0} \triangleq \frac{a_1 s + a_0}{s + w_0}$$

$w_0 = b_0$

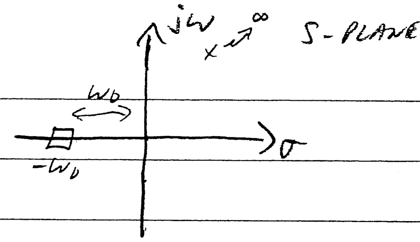
$w_0 \to$ POLE FREQUENCY

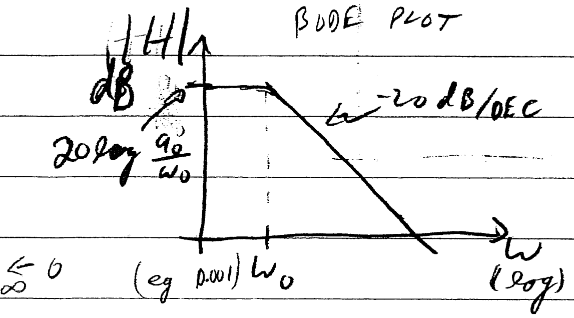
NATURAL POLE AT $s = -w_0$ $\therefore$ $w_0 > 0$ FOR STABILITY

TRANSMISSION ZERO AT $s = -\frac{a_0}{a_1}$

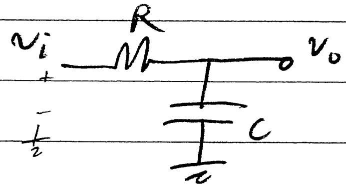
DC $s=0$ $\quad H(0) = \frac{a_0}{w_0}$ $\quad$ AT HF $s \to \infty$ $\quad H(\infty) = a_1$

a) LOW PASS $H(s) = \frac{a_0}{s+\omega_0}$





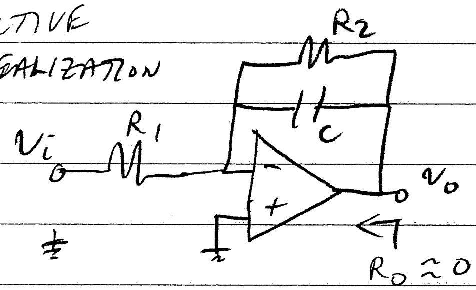
PASSIVE REALIZATION



$CR = \frac{1}{\omega_0}$

DC GAIN $= 1$

ACTIVE REALIZATION

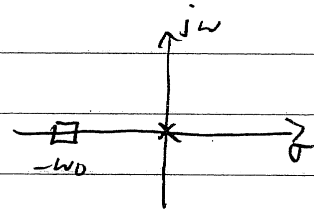


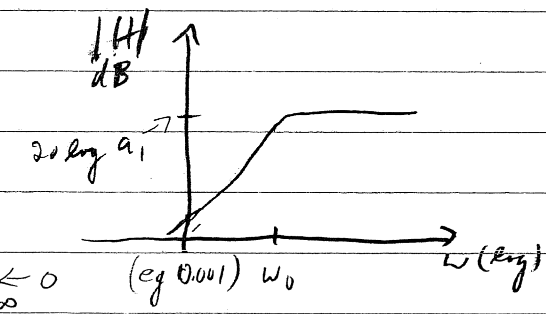
$CR_2 = \frac{1}{\omega_0}$

DC GAIN $= -\frac{R_2}{R_1}$

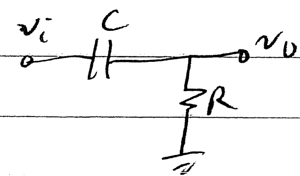
SO CAN CASCADE CIRCUITS WITH LITTLE INTERACTION

b) HIGH-PASS $H(s) = \frac{a_1 s}{s+\omega_0}$





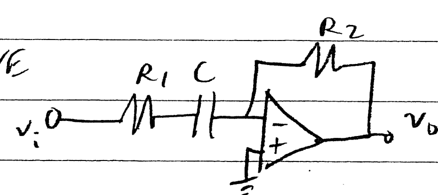
PASSIVE



$CR = \frac{1}{\omega_0}$

HF GAIN $= 1$

ACTIVE



$CR_1 = \frac{1}{\omega_0}$

HF GAIN $= -\frac{R_2}{R_1}$

SEE TEXT FOR GENERAL & ALL-PASS CASES.

(B) BIQUADRATIC $H(s) = \dfrac{a_2 s^2 + a_1 s + a_0}{s^2 + b_1 s + b_0} \triangleq \dfrac{a_2 s^2 + a_1 s + a_0}{s^2 + \left(\frac{\omega_0}{Q}\right) s + \omega_0^2}$

POLES

$\omega_0 = \sqrt{b_0}$ POLE FREQUENCY

$Q = \dfrac{\sqrt{b_0}}{b_1}$ POLE QUALITY FACTOR (OR Q FACTOR OR POLE-Q)

WRITING $s^2 + \frac{\omega_0}{Q} s + \omega_0^2 = (s - e_1)(s - e_2)$ (ie SOLVING FOR ROOTS)

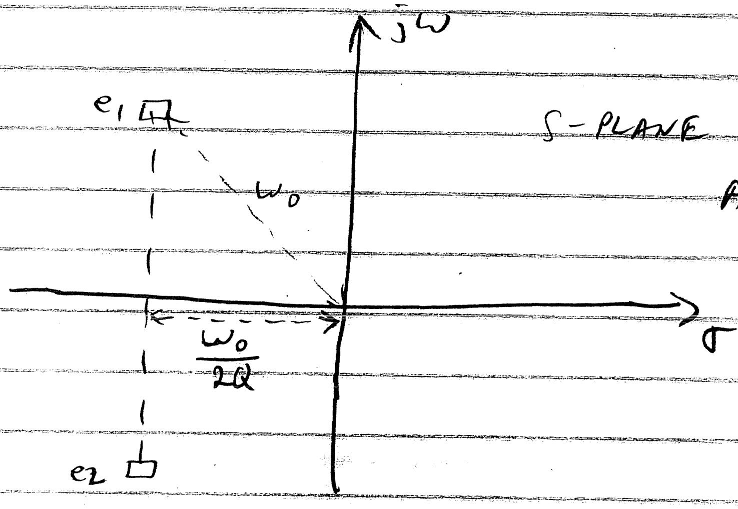
$e_1, e_2 = -\dfrac{\omega_0}{2Q} \pm j\omega_0 \sqrt{1 - \dfrac{1}{4Q^2}}$

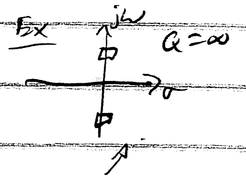
$\boxed{\text{FOR } \frac{1}{4Q^2} < 1}$ OR $\boxed{Q > 0.5}$ e_1 & e_2 ARE COMPLEX-CONJUGATE

& $|e_1| = |e_2| = \omega_0$ & $\text{REAL}[e_1] = \text{REAL}[e_2] = -\dfrac{\omega_0}{2Q}$

GRAPHICALLY

FOR $Q > 0.5$



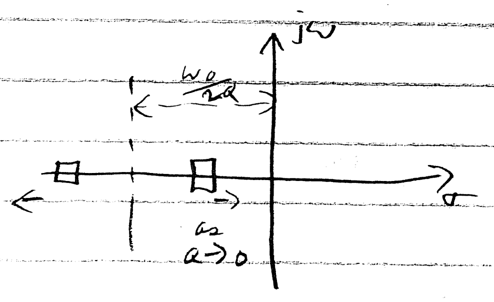


AS $Q \to \infty$ e_1 + e_2 APPROACH $j\omega$ AXIS

$Q \to 0.5$ e_1 & e_2 APPROACH REAL AXIS

$\boxed{\text{FOR } Q < 0.5 \quad Q > 0 \text{ FOR STABILITY}}$

e_1 & e_2 ARE ON THE REAL AXIS



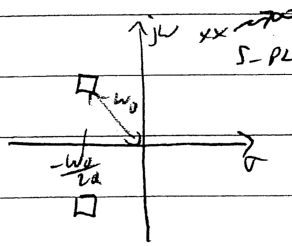
TRANSMISSION ZEROS

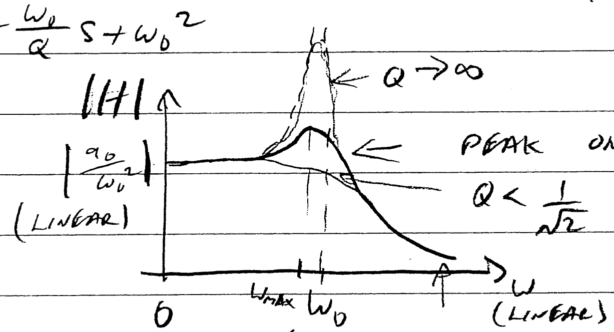
$a_2 s^2 + a_1 s + a_0$

$LP \Rightarrow a_0$ $\quad HP \Rightarrow a_2 s^2$

$BP \Rightarrow a_1 s$ $\quad NOTCH \Rightarrow a_2 s^2 + a_0$

a) LOW PASS $\quad H(s) = \dfrac{a_0}{s^2 + \frac{\omega_0}{Q} s + \omega_0^2}$ $\quad$ DC GAIN $\Rightarrow T(0) = \dfrac{a_0}{\omega_0^2}$





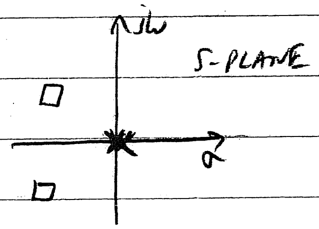
PEAK ONLY IF $Q > \frac{1}{\sqrt{2}}$

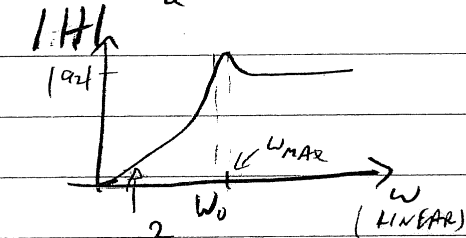
$Q < \frac{1}{\sqrt{2}}$

$\omega_{MAX} = \omega_0 \sqrt{1 - \frac{1}{2Q^2}}$

(ROUGHLY) $|H| \propto \frac{1}{\omega^2}$

b) HIGH PASS $\quad H(s) = \dfrac{a_2 s^2}{s^2 + \frac{\omega_0}{Q} s + \omega_0^2}$ $\quad$ HF GAIN $\Rightarrow H(\infty) = a_2$

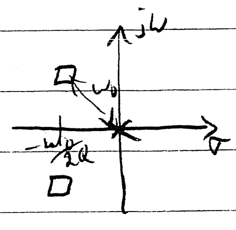


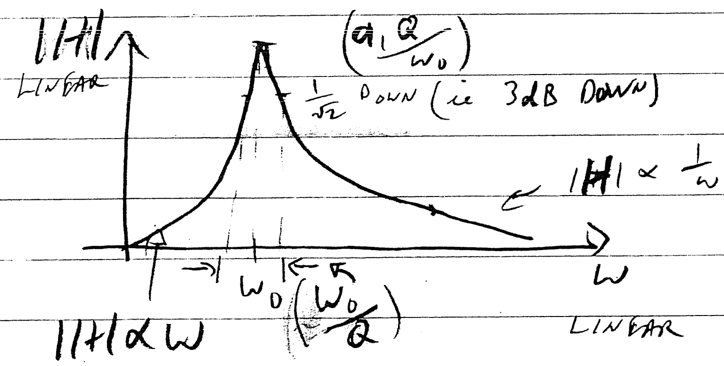


$\omega_{MAX} = \dfrac{\omega_0}{\sqrt{1 - \frac{1}{2Q^2}}}$

$|T| \propto \omega^2$

c) BAND PASS $\quad H(s) = \dfrac{a_1 s}{s^2 + \frac{\omega_0}{Q} s + \omega_0^2}$





$\frac{1}{\sqrt{2}}$ DOWN (ie 3dB DOWN)

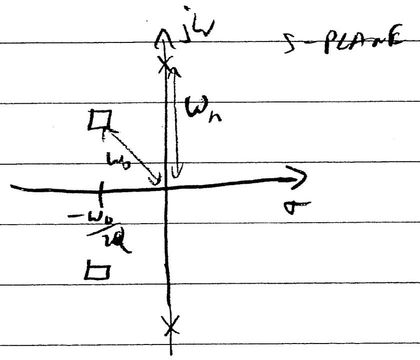
$|H| \propto \frac{1}{\omega}$

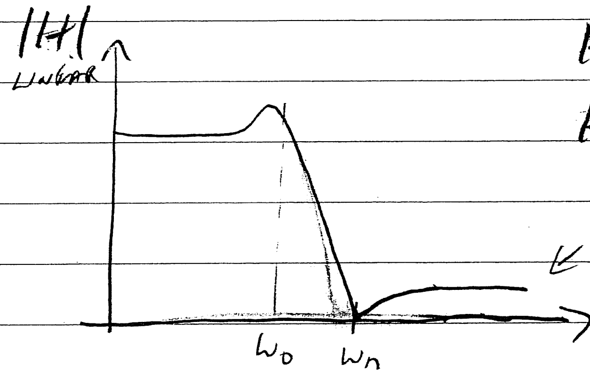
$|H| \propto \omega$

WIDTH OF 3dB BANDWIDTH $BW = \frac{\omega_0}{Q}$

[3dB BANDWIDTH $\triangleq \omega_2 - \omega_1$ WHERE $|H(\omega_2)| = |H(\omega_1)| = \frac{1}{\sqrt{2}}$ PEAK VALUE]

d) LOW PASS NOTCH (LPN) $\quad H(s) = a_2 \dfrac{s^2+\omega_n^2}{s^2+\frac{\omega_0}{Q}s+\omega_0^2} \quad \omega_n \geq \omega_0$



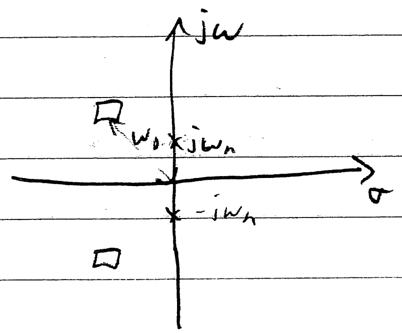


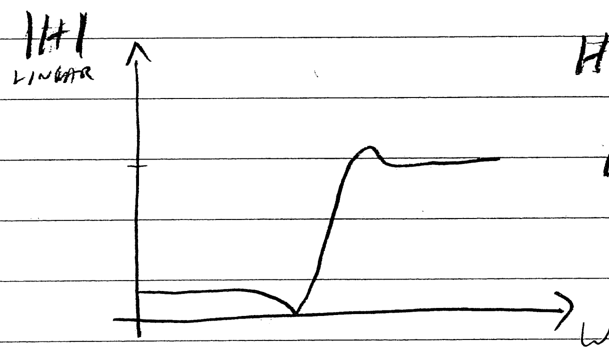
$H(0) = \dfrac{a_2\omega_n^2}{\omega_0^2}$

$H(\infty) = a_2$

NOTE $\neq 0$

e) HIGH PASS NOTCH (HPN) $\quad H(s) = a_2 \dfrac{s^2+\omega_n^2}{s^2+\frac{\omega_0}{Q}s+\omega_0^2} \quad \omega_n \leq \omega_0$

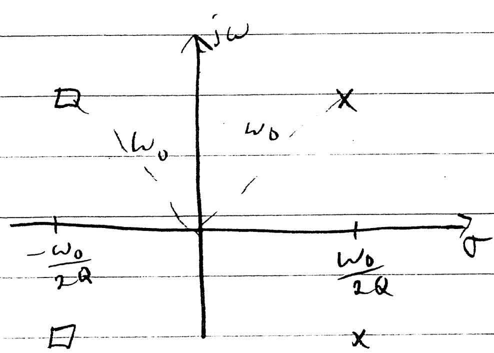


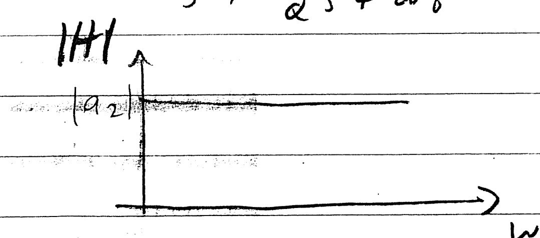


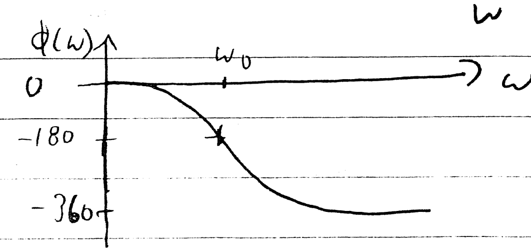
$H(0) = \dfrac{a_2\omega_n^2}{\omega_0^2}$

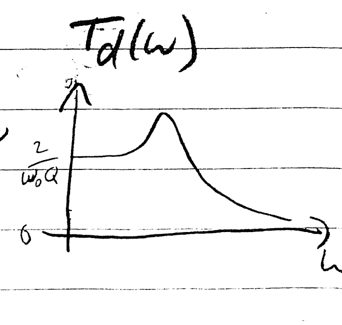
$H(\infty) = a_2$

f) ALL-PASS $\quad H(s) = a_2 \dfrac{s^2 - s\frac{\omega_0}{Q}+\omega_0^2}{s^2+\frac{\omega_0}{Q}s+\omega_0^2}$

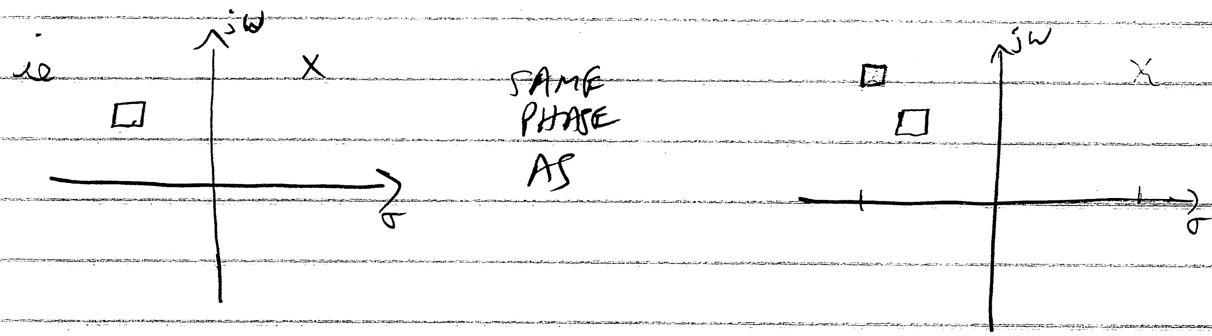






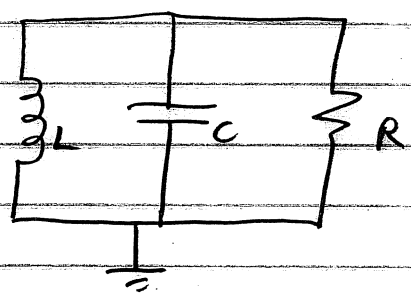


NOTE FOR PHASE – THE EFFECT OF A ZERO IN THE RHP IS THE SAME AS A MIRROR IMAGE POLE IN LHP



1.7 2ND ORDER LCR RESONATOR

THE **POLES** OF A BIQUAD CAN BE REALIZED WITH THE FOLLOWING LCR CIRCUIT



SHALL SEE THAT

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

$$Q = \omega_0 C R$$

IF THE INPUT IS APPLIED OR OUTPUT TAKEN SUCH THAT IT DOES NOT AFFECT THE NATURAL RESONANCE, THEN THE **POLES** CAN BE FOUND AS THE DENOMINATOR OF THE TRANSFER-FUNCTION POLYNOMIAL