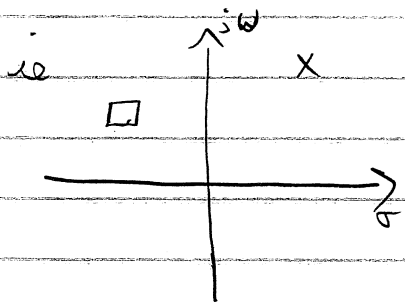
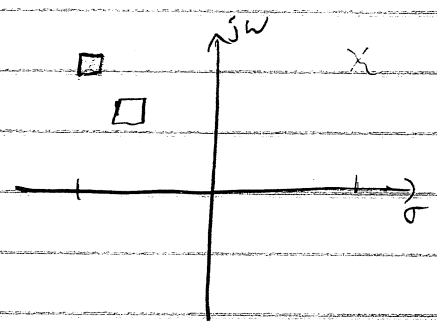


NOTE

FOR PHASE - THE EFFECT OF A ZERO IN THE RHP IS THE SAME AS A MIRROR IMAGE POLE IN LHP

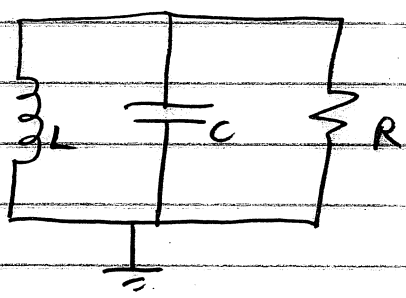


SAME PHASE AS



17 2ND ORDER LCR RESONATOR

THE POLES OF A BIQUAD CAN BE REALIZED WITH THE FOLLOWING LCR CIRCUIT



SHALL SEE THAT

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

$$Q = \omega_0 CR$$

IF THE INPUT IS APPLIED OR OUTPUT TAKEN SUCH THAT IT DOES NOT AFFECT THE NATURAL RESONANCE, THEN THE POLES CAN BE FOUND AS THE DENOMINATOR OF THE TRANSFER-FUNCTION POLYNOMIAL.

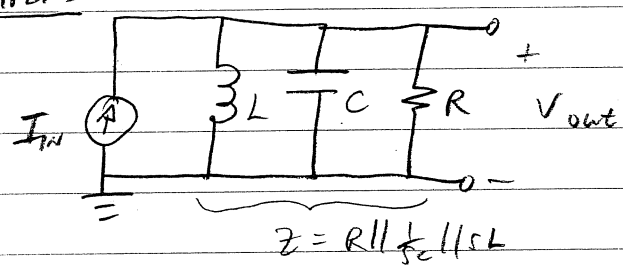
WHY CARE ABOUT LCR REALIZATIONS?

IT TURNS OUT THAT MANY FILTER REALIZATIONS SUCH AS ACTIVE-RC & DIGITAL FILTERS EMULATE THE OPERATION OF LCR FILTERS AS THEY ARE NEAR OPTIMAL.

TO APPLYING THE INPUT

- IF CURRENT SOURCE → OPENING SOURCE SHOULD GIVE ORIGINAL CIRCUIT
- IF VOLTAGE SOURCE → SHORTING " " " " " "

EXAMPLES



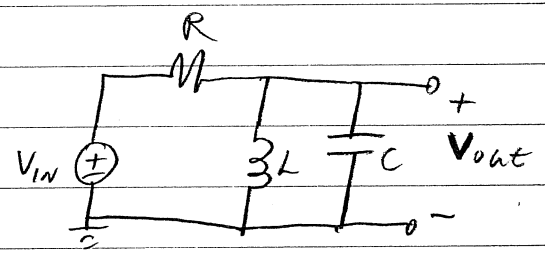
$$\frac{V_{out}}{I_{IN}} = Z = \left(\frac{1}{R} + sC + \frac{1}{sL} \right)^{-1}$$

$$= \frac{s/C}{s^2 + s \frac{1}{CR} + \frac{1}{LC}}$$

$$\approx \frac{s/C}{s^2 + \frac{\omega_0}{Q}s + \omega_0^2}$$

$$\boxed{\omega_0 = \frac{1}{\sqrt{LC}}} \quad \downarrow \quad \boxed{\frac{\omega_0}{Q} = \frac{1}{CR}}$$

$$\therefore \boxed{Q = \omega_0 CR}$$



$$\frac{V_{out}}{V_{IN}} = \frac{s \frac{1}{CR}}{s^2 + s \frac{1}{CR} + \frac{1}{LC}} \approx \frac{s \frac{1}{CR}}{s^2 + \frac{\omega_0}{Q}s + \omega_0^2}$$

$$\boxed{\omega_0 = \frac{1}{\sqrt{LC}}} \quad \boxed{Q = \omega_0 CR}$$

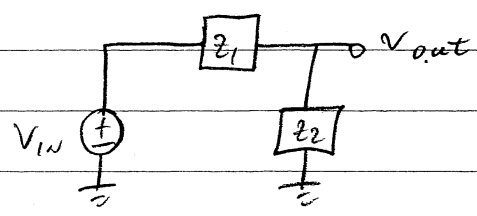
ALSO BANDPASS

CENTER FREQ GAIN OF 1

BANDPASS SINCE ZERO @ 0 & ∞
CENTER FREQ GAIN OF R

FOR TRANSMISSION ZEROS

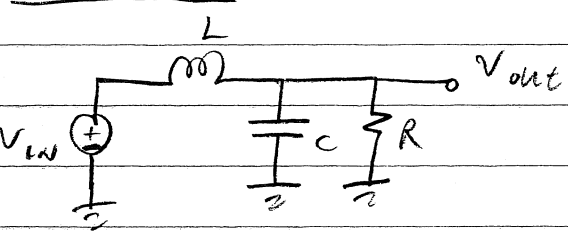
CONSIDER



$$H(s) \triangleq \frac{V_{out}}{V_{in}} = \frac{Z_2(s)}{Z_1(s) + Z_2(s)}$$

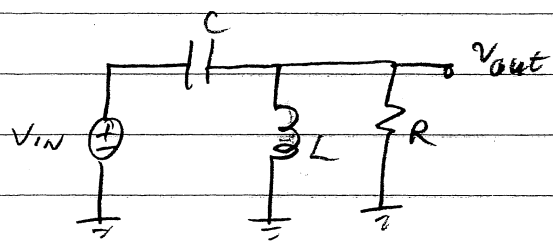
TRANSMISSION ZEROS OCCUR, WHERE $Z_1(s) = \infty$ & $Z_2(s) \neq \infty$
 & WHERE $Z_2(s) = 0$ & $Z_1(s) \neq 0$

LOWPASS



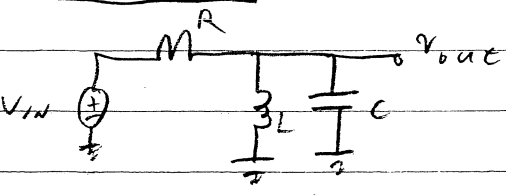
$$\frac{V_{out}}{V_{in}} = \frac{1/LC}{s^2 + s \frac{1}{CR} + \frac{1}{LC}}$$

HIGH PASS



$$\frac{V_{out}}{V_{in}} = \frac{s^2}{s^2 + s \frac{1}{CR} + \frac{1}{LC}}$$

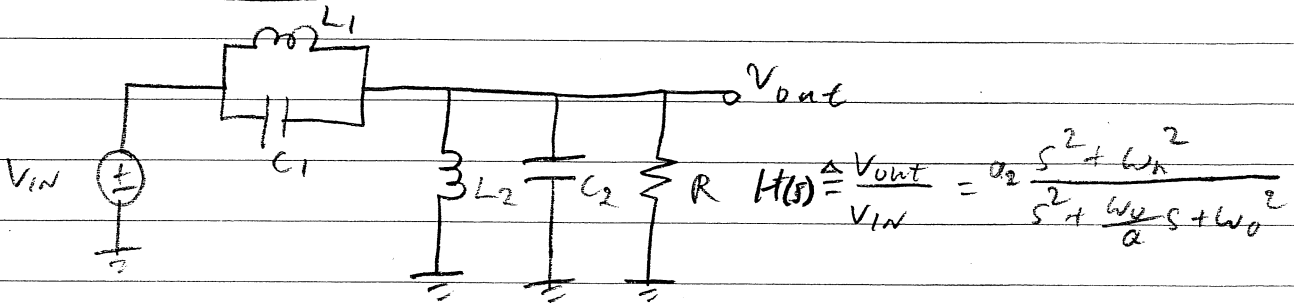
BAND PASS



$$\frac{V_{out}}{V_{in}} = \frac{s \frac{1}{CR}}{s^2 + s \frac{1}{CR} + \frac{1}{LC}}$$

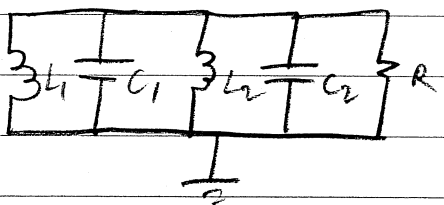
NOTE $Z_1 \neq 0$ OR ∞ FOR ANY ω
 BUT $Z_2 = 0$ AT $\omega = 0$ & ∞

GENERAL NOTCH ZERO PLACED AT $\pm j\omega_n$



$Z_1(s) \rightarrow \infty$ @ $\omega = \pm j\omega_n$ ie select $L_1 C_1 = \frac{1}{\omega_n^2}$

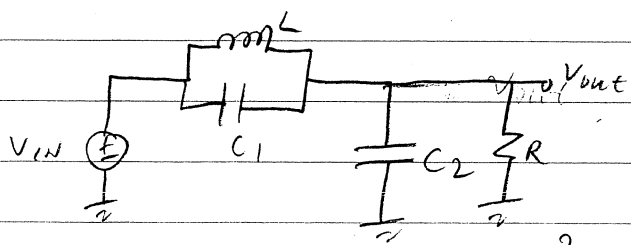
TO NOT AFFECT POLES : SHORT V_{IN} TO RECOGNIZE



$\therefore L_1 || L_2 = L$
 $\downarrow C_1 + C_2 = C$
 $\omega_0 = \frac{1}{\sqrt{LC}} \downarrow Q = \omega_0 CR$

NOTE THAT $H(0) = \frac{L_2}{L_1 + L_2} \downarrow H(\infty) = \frac{C_1}{C_1 + C_2}$

FOR LOW PASS NOTCH WITH $H(0) = 1 \Rightarrow$ LET $L_2 = \infty$

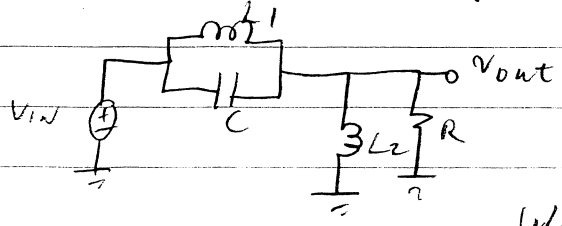


$\frac{V_{out}}{V_{in}} = a_2 \frac{s^2 + \omega_n^2}{s^2 + \frac{\omega_0}{Q}s + \omega_0^2}$

$\omega_0^2 = \frac{1}{L(C_1 + C_2)} \quad Q = \omega_0(C_1 + C_2)R \quad \boxed{\omega_n > \omega_0}$

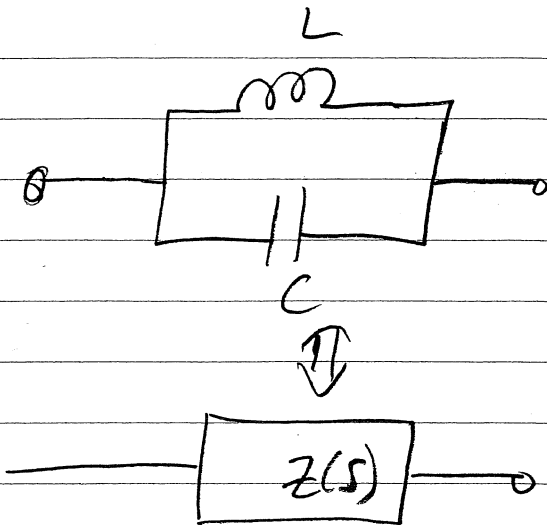
$\omega_n^2 = \frac{1}{L C_1} \quad a_2 = \frac{C_1}{C_1 + C_2} = H(\infty)$

HIGH PASS NOTCH $H(\infty) = 1$ so LET $C_2 = 0 \quad \boxed{\omega_n < \omega_0}$



$\frac{V_{out}}{V_{in}} = \frac{s^2 + \omega_n^2}{s^2 + \frac{\omega_0}{Q}s + \omega_0^2} \quad H(0) = \frac{L_2}{L_1 + L_2}$

$\omega_n^2 = \frac{1}{L_1 C} \quad \omega_0^2 = \frac{1}{(L_1 || L_2) C} \quad Q = \omega_0 C R$



$$Z(s) = sL \parallel \frac{1}{sC} = \left(\frac{1}{sL} + sC \right)^{-1} = \left(\frac{1 + s^2 CL}{sL} \right)^{-1}$$

$$Z(s) = \frac{sL}{1 + s^2 CL}$$

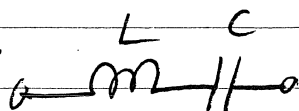
$$Z(j\omega) = \frac{j\omega L}{1 - \omega^2 CL}$$

$$Z(j\omega_r) \rightarrow \infty \quad \text{WHEN} \quad 1 - \omega_r^2 CL = 0$$

$$\omega_r = \frac{1}{\sqrt{LC}}$$

ω_r IS RESONANT FREQUENCY

SIMILAR



$$Z(j\omega_r) \rightarrow 0 \quad \text{WHEN}$$

$$\omega_r = \frac{1}{\sqrt{LC}}$$

- 93

ALL-PASS

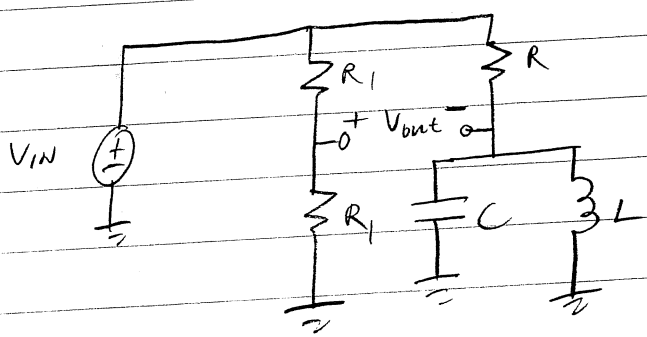
$$H(s) = \frac{s^2 - s \frac{\omega_0}{Q} + \omega_0^2}{s^2 + s \frac{\omega_0}{Q} + \omega_0^2} = 1 - \frac{s^2 \frac{\omega_0}{Q}}{s^2 + s \frac{\omega_0}{Q} + \omega_0^2}$$

BANDPASS WITH CENTER FREQ GAIN OF 2

ALREADY HAVE BANDPASS WITH CENTER FREQ GAIN OF 2
SO DESIGN ALL-PASS WITH MAGNITUDE GAIN OF 0.5

$$H(s) = 0.5 - \frac{s \frac{\omega_0}{Q}}{s^2 + \frac{\omega_0}{Q} s + \omega_0^2}$$

ONE REALIZATION

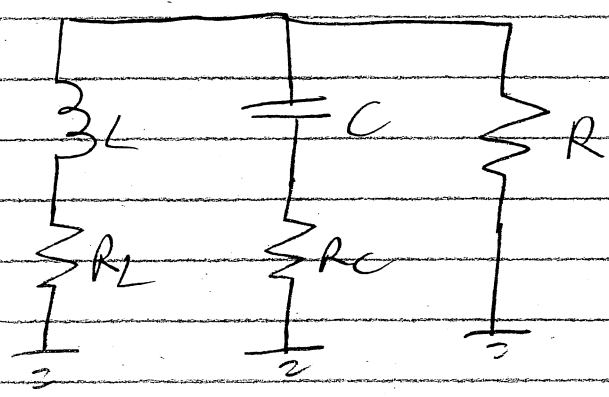


$$\omega_0^2 = \frac{1}{LC} \quad Q = \omega_0 CR$$

$$|H(j\omega)| = 0.5$$

GENERAL CASE

INCLUDE R_L & R_C



$$sL + R_L \parallel \frac{1}{sC} + R_C \parallel R$$

$$= \left(\frac{1}{sL + R_L} + \frac{sC}{1 + sCR_C} + \frac{1}{R} \right)^{-1}$$

$$= R(1 + sCR_C) + sCR(sL + R_L) + (sL + R_L)(1 + sCR_C)$$

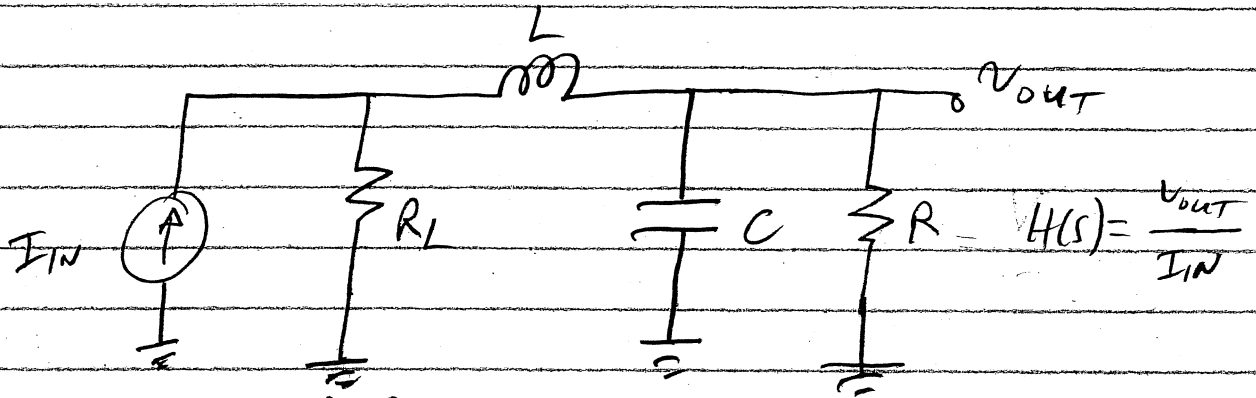
$$= R + sCR_C R + s^2 LCR + sCR_L R + sL + R_L + s^2 LCR_C + sCR_C R_L$$

$$= s^2 (LCR + LCR_C) + s(CR_C R + CR_L R + L + CR_C R_L) + R + R_L$$

$$\omega_0^2 = \frac{R + R_L}{LCR + LCR_C} = \frac{R + R_L}{LC(R + R_C)}$$

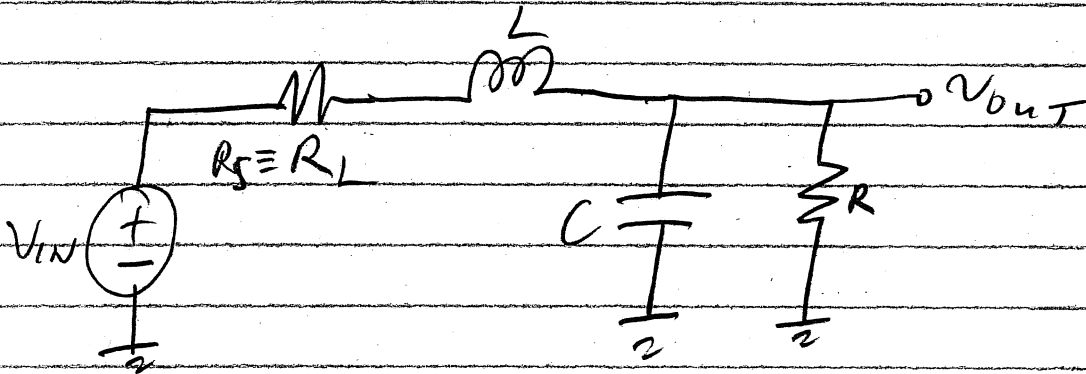
$$Q = \frac{\omega_0 LC(R + R_C)}{CR(R_C + R_C) + CR_C R_L + L}$$

LOWPASS CURRENT INPUT



$$H(s) = \frac{V_{OUT}}{I_{IN}}$$

$$H(j\omega) = \frac{R_1 R}{R + j\omega L}$$

LOWPASS WITH SOURCE RESISTANCE, R_S 

$$H(s) = \frac{V_{OUT}}{V_{IN}}$$

$$H(j\omega) = \frac{R}{R_S + R}$$