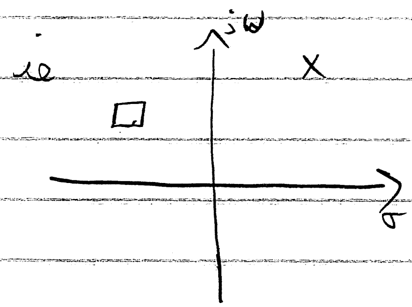
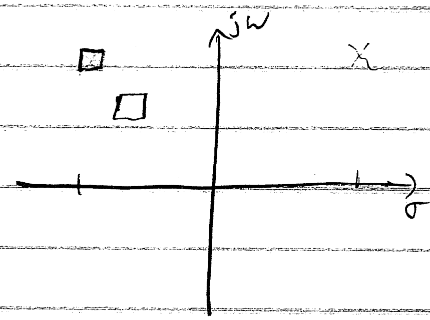
NOTE FOR PHASE – THE EFFECT OF A ZERO IN THE RHP IS THE SAME AS A MIRROR IMAGE POLE IN LHP

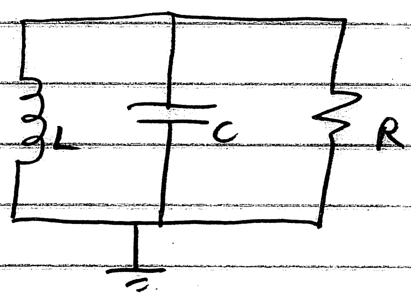


SAME PHASE AS



1.7 2ND ORDER LCR RESONATOR

THE **POLES** OF A BIQUAD CAN BE REALIZED WITH THE FOLLOWING LCR CIRCUIT



SHALL SEE THAT

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

$$Q = \omega_0 C R$$

IF THE INPUT IS APPLIED OR OUTPUT TAKEN SUCH THAT IT DOES NOT AFFECT THE NATURAL RESONANCE, THEN THE **POLES** CAN BE FOUND AS THE DENOMINATOR OF THE TRANSFER-FUNCTION POLYNOMIAL

WHY CARE ABOUT LCR REALIZATIONS?

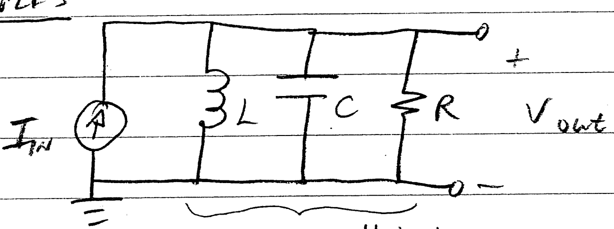
IT TURNS OUT THAT MANY FILTER REALIZATIONS SUCH AS ACTIVE-RC & DIGITAL FILTERS EMULATE THE OPERATION OF LCR FILTERS AS THEY ARE NEAR OPTIMAL.

APPLYING THE INPUT

IF CURRENT SOURCE → OPENING SOURCE SHOULD GIVE ORIGINAL CIRCUIT.

IF VOLTAGE SOURCE → SHORTING " " " " "

EXAMPLES



$$\underbrace{\qquad\qquad}_{Z = R \| \frac{1}{sC} \| sL}$$

$$\frac{V_{out}}{I_{IN}} = Z = \left(\frac{1}{R} + sC + \frac{1}{sL}\right)^{-1}$$

$$= \frac{s/C}{s^2 + s\frac{1}{CR} + \frac{1}{LC}}$$

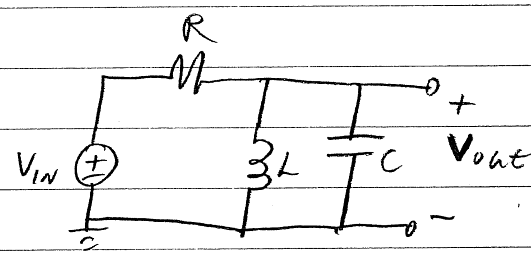
$$\triangleq \frac{s/C}{s^2 + \frac{\omega_0}{Q}s + \omega_0^2}$$

$$\therefore \boxed{\omega_0 = \frac{1}{\sqrt{LC}}} \;\&\; \frac{\omega_0}{Q} = \frac{1}{CR}$$

$$\therefore \boxed{Q = \omega_0 CR}$$

BANDPASS SINCE ZERO @ 0 & ∞

CENTER FREQ GAIN OF R



$$\frac{V_{out}}{V_{IN}} = \frac{s\frac{1}{CR}}{s^2 + s\frac{1}{CR} + \frac{1}{LC}} \triangleq \frac{s\frac{1}{CR}}{s^2 + \frac{\omega_0}{Q}s + \omega_0^2}$$

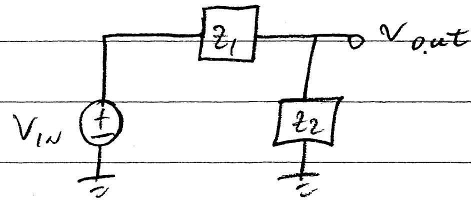
$$\boxed{\omega_0 = \frac{1}{\sqrt{LC}}} \qquad \boxed{Q = \omega_0 CR}$$

ALSO BANDPASS

CENTER FREQ GAIN OF 1

FOR TRANSMISSION ZEROS

CONSIDER

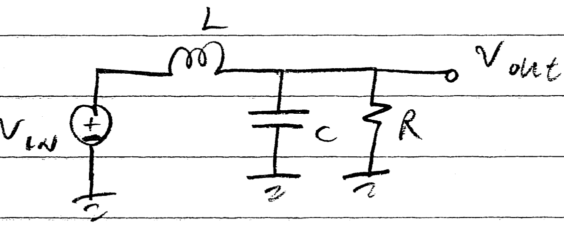


$$H(s) \triangleq \frac{v_{out}}{V_{IN}} = \frac{Z_2(s)}{Z_1(s)+Z_2(s)}$$

TRANSMISSION ZEROS OCCUR, WHERE $Z_1(s) = \infty$ & $Z_2(s) \neq \infty$

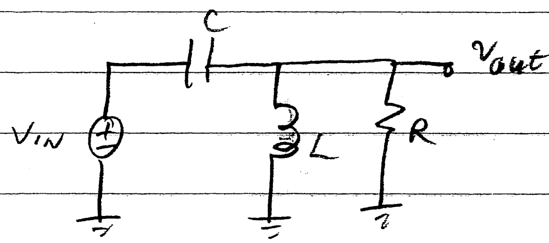
& WHERE $Z_2(s) = 0$ & $Z_1(s) \neq 0$

LOWPASS



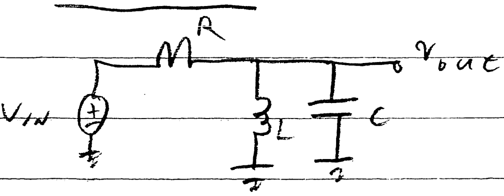
$$\frac{V_{out}}{V_{IN}} = \frac{1/LC}{s^2 + s\frac{1}{CR} + \frac{1}{LC}}$$

HIGH PASS



$$\frac{V_{out}}{V_{IN}} = \frac{s^2}{s^2 + s\frac{1}{CR} + \frac{1}{LC}}$$

BAND PASS

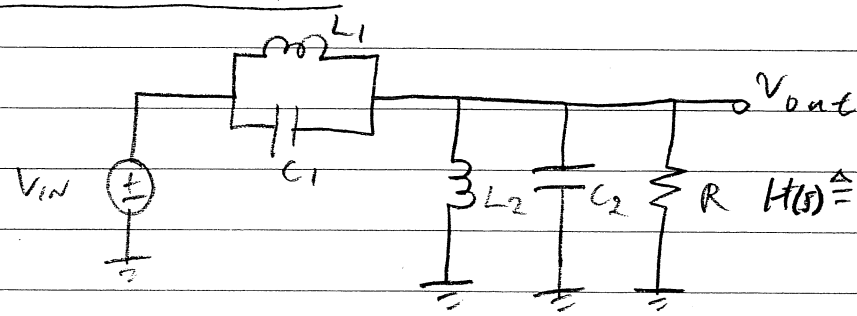


$$\frac{V_{out}}{V_{IN}} = \frac{s\frac{1}{CR}}{s^2 + s\frac{1}{CR} + \frac{1}{LC}}$$

NOTE $Z_1 \neq 0$ OR ∞ FOR ANY ω

BUT $Z_2 = 0$ AT $\omega = 0$ & ∞

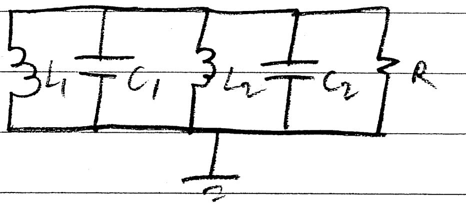
GENERAL NOTCH ZERO PLACED AT $\pm j\omega_n$



$$H(s) \triangleq \frac{V_{out}}{V_{IN}} = a_2 \frac{s^2 + \omega_n^2}{s^2 + \frac{\omega_0}{Q}s + \omega_0^2}$$

$Z_1(s) \to \infty$ @ $\omega = \pm j\omega_n$ ie SELECT $L_1 C_1 = \frac{1}{\omega_n^2}$

TO NOT AFFECT POLES : SHORT V_{IN} TO RECOGNIZE



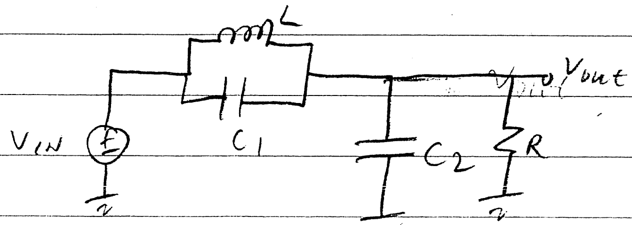
$\therefore$ $L_1 \| L_2 = L$

& $C_1 + C_2 = C$

& $\omega_0 = \frac{1}{\sqrt{LC}}$ & $Q = \omega_0 CR$

NOTE THAT $\boxed{H(0) = \frac{L_2}{L_1 + L_2}}$ & $\boxed{H(\infty) = \frac{C_1}{C_1 + C_2}}$

FOR LOW PASS NOTCH WITH $H(0) = 1$ $\Rightarrow$ LET $L_2 = \infty$

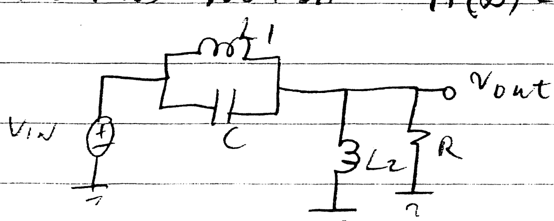


$$\frac{V_{out}}{V_{IN}} = a_2 \frac{s^2 + \omega_n^2}{s^2 + \frac{\omega_0}{Q}s + \omega_0^2}$$

$\omega_0^2 = \frac{1}{L(C_1 + C_2)}$ $Q = \omega_0 (C_1 + C_2) R$ $\boxed{\omega_n > \omega_0}$

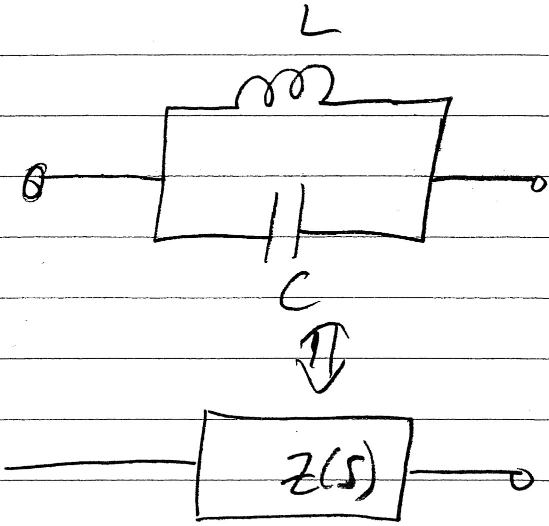
$\omega_n^2 = \frac{1}{LC_1}$ $a_2 = \frac{C_1}{C_1 + C_2} = H(\infty)$

HIGH PASS NOTCH $H(\infty) = 1$ SO LET $C_2 = 0$ $\boxed{\omega_n < \omega_0}$



$$\frac{V_{out}}{V_{IN}} = \frac{s^2 + \omega_n^2}{s^2 + \frac{\omega_0}{Q}s + \omega_0^2}$$ $H(0) = \frac{L_2}{L_1 + L_2}$

$\omega_n^2 = \frac{1}{L_1 C}$ $\omega_0^2 = \frac{1}{(L_1 \| L_2) C}$ $Q = \omega_0 CR$



$$Z(S) = SL \,||\, \frac{1}{SC} = \left(\frac{1}{SL} + SC\right)^{-1} = \left(\frac{1+S^2CL}{SL}\right)^{-1}$$

$$Z(S) = \frac{SL}{1+S^2CL}$$

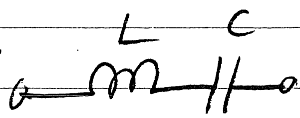
$$Z(j\omega) = \frac{j\omega L}{1-\omega^2 CL}$$

$Z(j\omega_r) \to \infty$ WHEN $1-\omega_r^2 CL = 0$

$$\boxed{\omega_r = \frac{1}{\sqrt{LC}}}$$

ω_r IS RESONANT FREQUENCY

SIMILAR



$Z(j\omega_r) \to 0$ WHEN

$$\omega_r = \frac{1}{\sqrt{LC}}$$

— 93

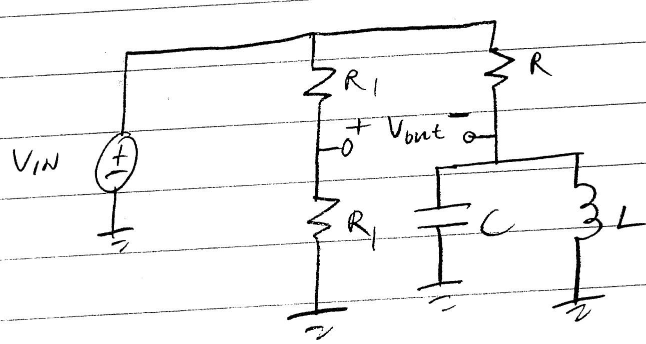
ALL-PASS

$$H(s) = \frac{s^2 - s\frac{\omega_0}{Q} + \omega_0^2}{s^2 + s\frac{\omega_0}{Q} + \omega_0^2} = 1 - \underbrace{\frac{s\, 2\frac{\omega_0}{Q}}{s^2 + s\frac{\omega_0}{Q} + \omega_0^2}}_{\text{BANDPASS WITH CENTER FREQ GAIN OF 2}}$$

ALREADY HAVE BANDPASS WITH CENTER FREQ GAIN OF 1
SO DESIGN ALL-PASS WITH MAGNITUDE GAIN OF 0.5

$$H(s) = 0.5 - \frac{s\frac{\omega_0}{Q}}{s^2 + \frac{\omega_0}{Q}s + \omega_0^2}$$

ONE REALIZATION



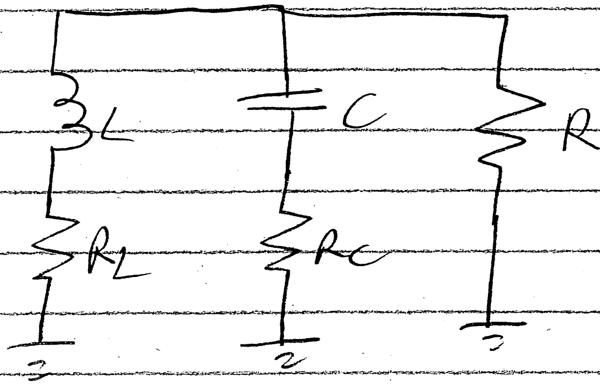
$$\omega_0^2 = \frac{1}{LC} \qquad Q = \omega_0 CR$$

$$|H(j\omega)| = 0.5$$

53A

GENERAL CASE

INCLUDE R_L & R_C



$$SL + R_L \parallel \frac{1}{SC} + R_C \parallel R$$

$$= \left(\frac{1}{SL+R_L} + \frac{SC}{1+SCR_C} + \frac{1}{R} \right)^{-1}$$

$$= R(1+SCR_C) + SCR(SL+R_L) + (SL+R_L)(1+SCR_C)$$

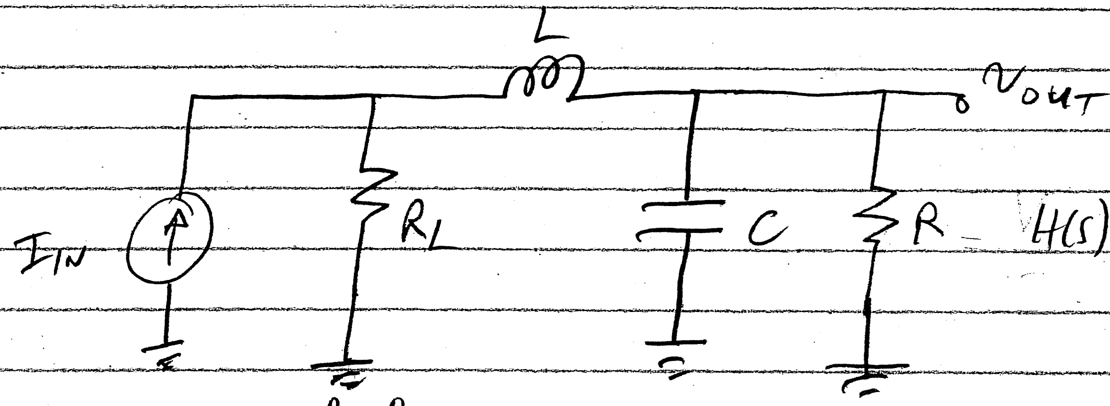
$$= R + SCR_CR + S^2LCR + SCR_LR + SL + R_L + S^2LCR_C + SCR_CR_L$$

$$= S^2(LCR + LCR_C) + S(CR_CR + CR_LR + L + CR_CR_L) + R + R_L$$

$$\omega_0^2 = \frac{R+R_L}{LCR+LCR_C} = \frac{R+R_L}{LC(R+R_C)}$$

$$Q = \frac{\omega_0 LC(R+R_C)}{CR(R_C+R_L)+CR_CR_L+L}$$

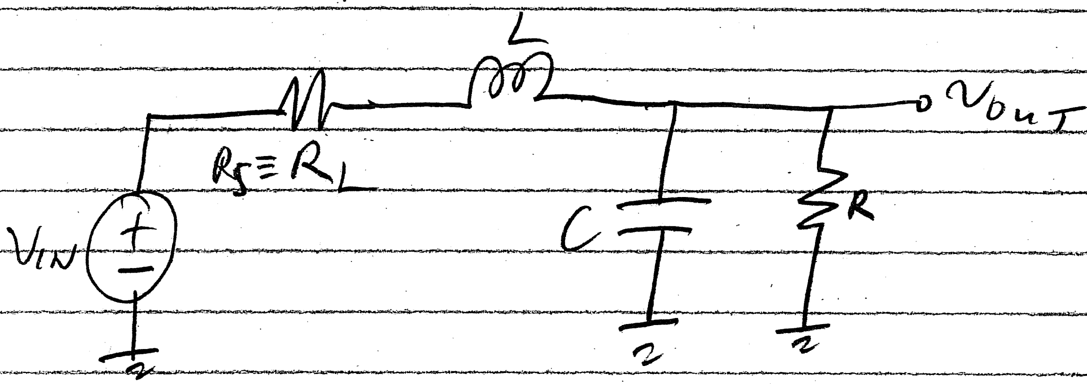
LOWPASS CURRENT INPUT



$$H(s) = \frac{v_{OUT}}{I_{IN}}$$

$$H(j0) = \frac{R_L R}{R + R_L}$$

LOWPASS WITH SOURCE RESISTANCE, R_S



$$H(s) = \frac{v_{OUT}}{V_{IN}}$$

$$H(j0) = \frac{R}{R_S + R}$$