

FILTER REALIZATIONS

DIRECT-FORM REALIZATION ← SIMPLE BUT POOR PERFORMANCE FOR NARROW BAND FILTER

$$H(s) = \frac{V_o(s)}{V_I(s)} = \frac{a_M s^M + a_{M-1} s^{M-1} + \dots + a_1 s + a_0}{s^N + b_{N-1} s^{N-1} + \dots + b_1 s + b_0}$$

$$\triangleq \frac{P(s)}{E(s)}$$

CREATE N INTEGRATOR OUTPUTS, $X_0(s), X_1(s), \dots, X_{N-1}(s)$

SUCH THAT

$$\frac{X_0(s)}{V_I(s)} = \frac{1}{E(s)}, \quad \frac{X_1(s)}{V_I(s)} = \frac{s}{E(s)}, \quad \dots, \quad \frac{X_{N-1}(s)}{V_I(s)} = \frac{s^{N-1}}{E(s)}$$

AND ONE SUMMATION OUTPUT $X_N(s)$

SUCH THAT

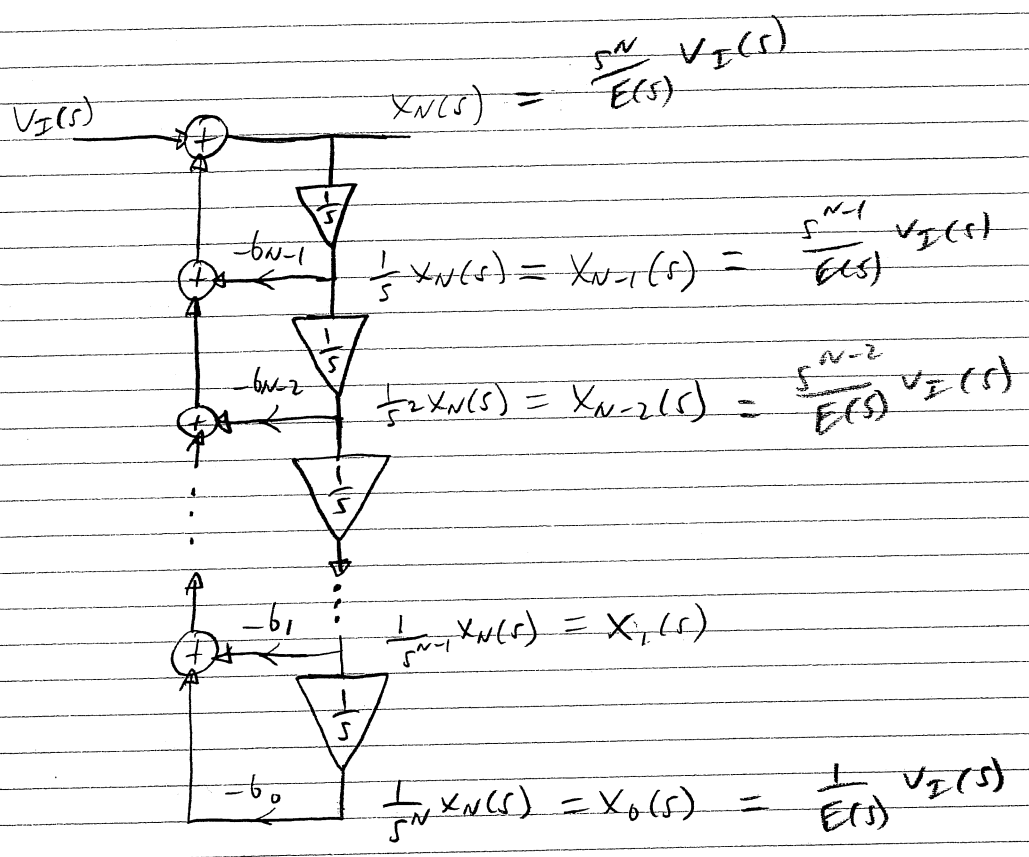
$$\frac{X_N(s)}{V_I(s)} = \frac{s^N}{E(s)}$$

HOW? WRITE OUT $X_N(s)$ AS A FUNCTION OF ITSELF AND THE INPUT $V_I(s)$

$$\frac{X_N(s)}{V_I(s)} = \frac{s^N}{s^N + b_{N-1} s^{N-1} + \dots + b_1 s + b_0}$$

$$X_N(s) s^N + X_N(s) b_{N-1} s^{N-1} + \dots + X_N(s) b_1 s + X_N(s) b_0 = s^N V_I(s)$$

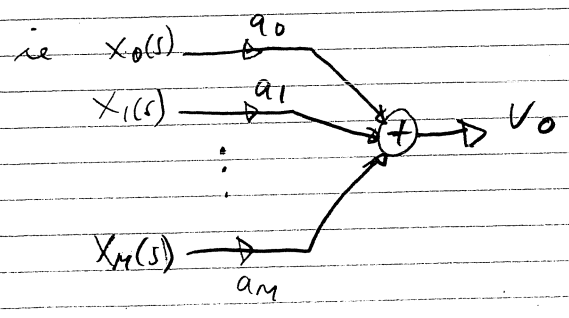
$$\Rightarrow \boxed{X_N(s) = -b_{N-1} \frac{1}{s} X_N(s) - b_{N-2} \frac{1}{s^2} X_N(s) - \dots - b_1 \frac{1}{s^{N-1}} X_N(s) - b_0 \frac{1}{s^N} X_N(s) + V_I(s)}$$



FINALLY,

$$\frac{V_0(s)}{V_I(s)} = a_m \frac{X_m(s)}{V_I(s)} + a_{m-1} \frac{X_{m-1}(s)}{V_I(s)} + \dots + a_1 \frac{X_1(s)}{V_I(s)} + a_0 \frac{X_0(s)}{V_I(s)}$$

$$\therefore V_0(s) = a_m X_m(s) + a_{m-1} X_{m-1}(s) + \dots + a_0 X_0(s)$$

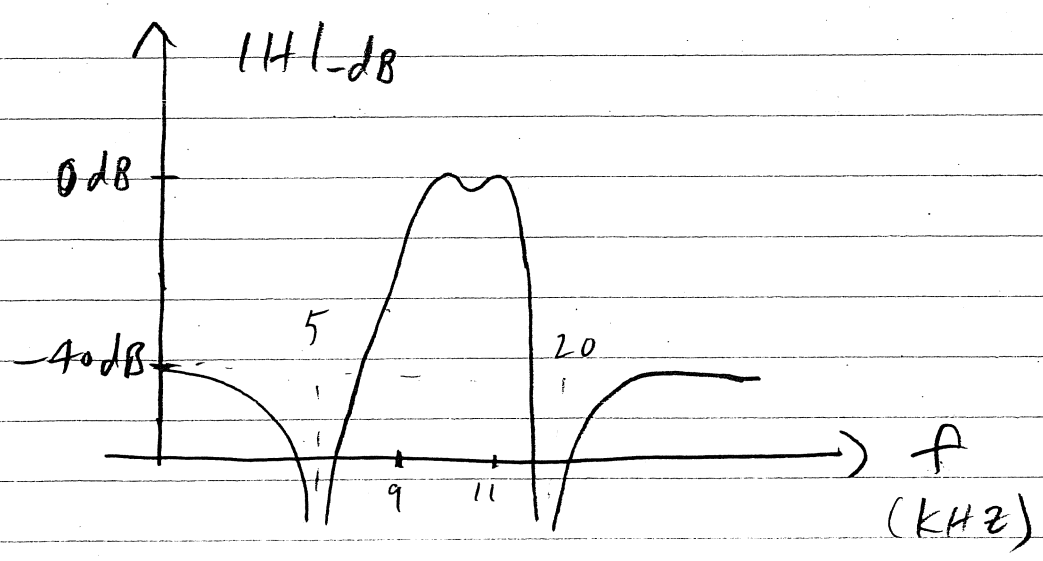
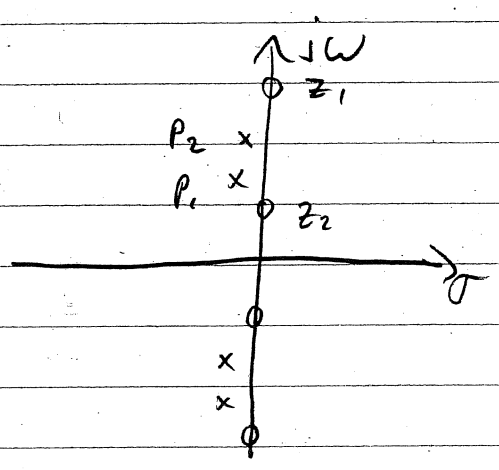


EXAMPLE

CONSIDER A BANDPASS FILTER
WITH PASSBAND GAIN ≈ 1
FROM 9-11 KHZ + STOPBAND
ATTENUATION ≈ -40 dB

ZEROS @ $z_1: \pm j(2\pi \times 20 \text{ KHZ})$
 $z_2: \pm j(2\pi \times 5 \text{ KHZ})$

POLES @ $P_1: \omega_{01} = 2\pi \times 9.5 \text{ KHZ} + Q_1 = 10$
 $P_2: \omega_{02} = 2\pi \times 10.5 \text{ KHZ} + Q_2 = 10$



$$H(s) = \frac{0.01 (s^2 + (2\pi \times 20k)^2) (s^2 + (2\pi \times 5k)^2)}{(s^2 + \frac{\omega_{01}}{Q_1} s + \omega_{01}^2) (s^2 + \frac{\omega_{02}}{Q_2} s + \omega_{02}^2)}$$

$$H(s) = \frac{0.01 s^4 + 1.6778 \times 10^8 s^2 + 1.559 \times 10^{17}}{s^4 + 1.26 \times 10^4 s^3 + 7.95 \times 10^9 s^2 + 4.95 \times 10^{13} s + 1.55 \times 10^{19}}$$

PROBLEM COEFFICIENT SPREAD OF $10^{19}!!!$

WHY? BECAUSE INTEGRATORS

$$H_I(s) = \frac{1}{s} \text{ HAS GAIN OF 1}$$

AT 1 RAD/S

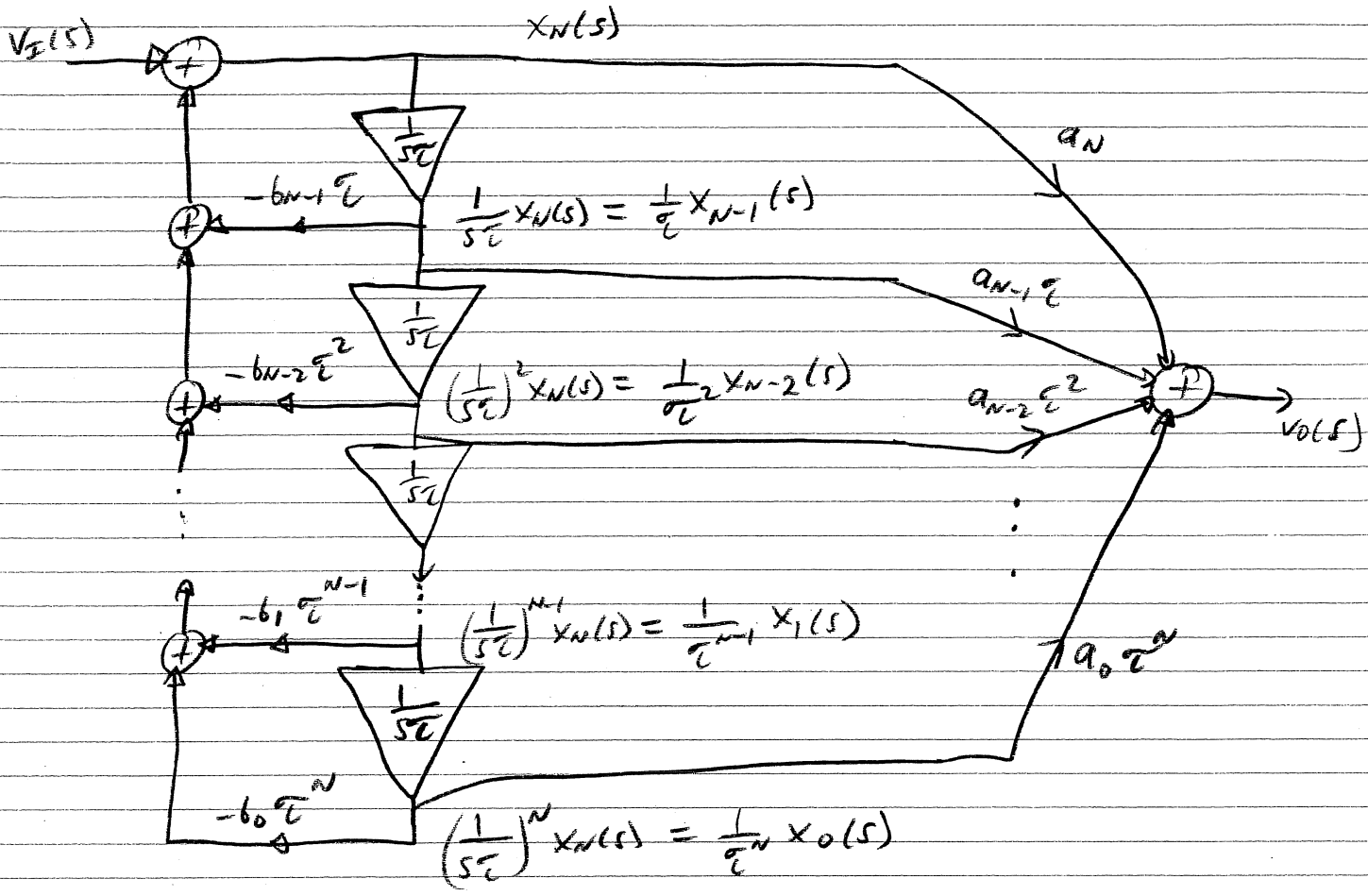
$$|H_I(j\omega)| = 1$$

BUT $|H_I(j(2\pi \times 10k))| = \frac{1}{(2\pi \times 10k)}$

GAIN $\ll 1$ NEAR PASSBAND EDGE OF FILTER

INSTEAD USE

$$H_I(s) \equiv \frac{1}{s\tau} \text{ WHERE } \tau = \frac{1}{\omega_0}$$



SO NOW COEFFICIENTS FOR EXAMPLE ARE :

$$-b_0 T^4 = -1.5508 \times 10^{19} \left(\frac{1}{25 \times 10^6} \right)^4 = -0.99503$$

$$-b_1 T^3 = -4.94856 \times 10^{17} \left(\frac{1}{25 \times 10^6} \right)^3 = -0.199498$$

$$-b_2 T^2 = -2.01497$$

$$-b_3 T = -0.2$$

SPREAD OF 10

$$a_4 = 0.01$$

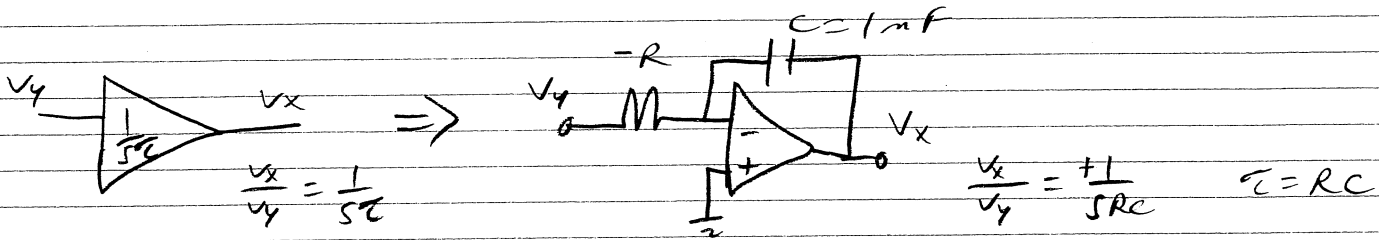
$$a_2 T^2 = 0.0425$$

$$a_0 T^4 = 0.01$$

SPREAD OF ~~10~~ 2.4

FIND AN ACTIVE-RC REALIZATION
IF NEGATIVE RESISTORS ALLOWED.

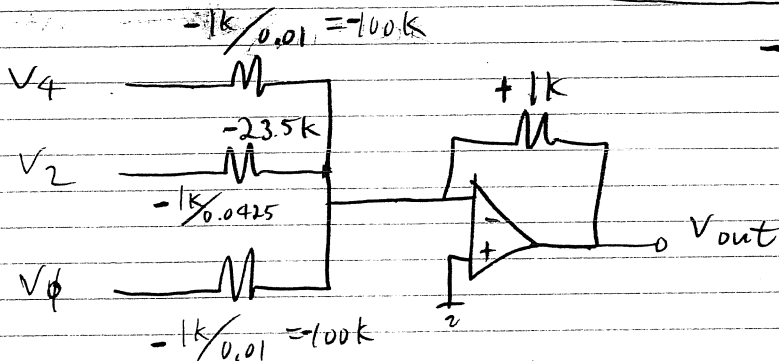
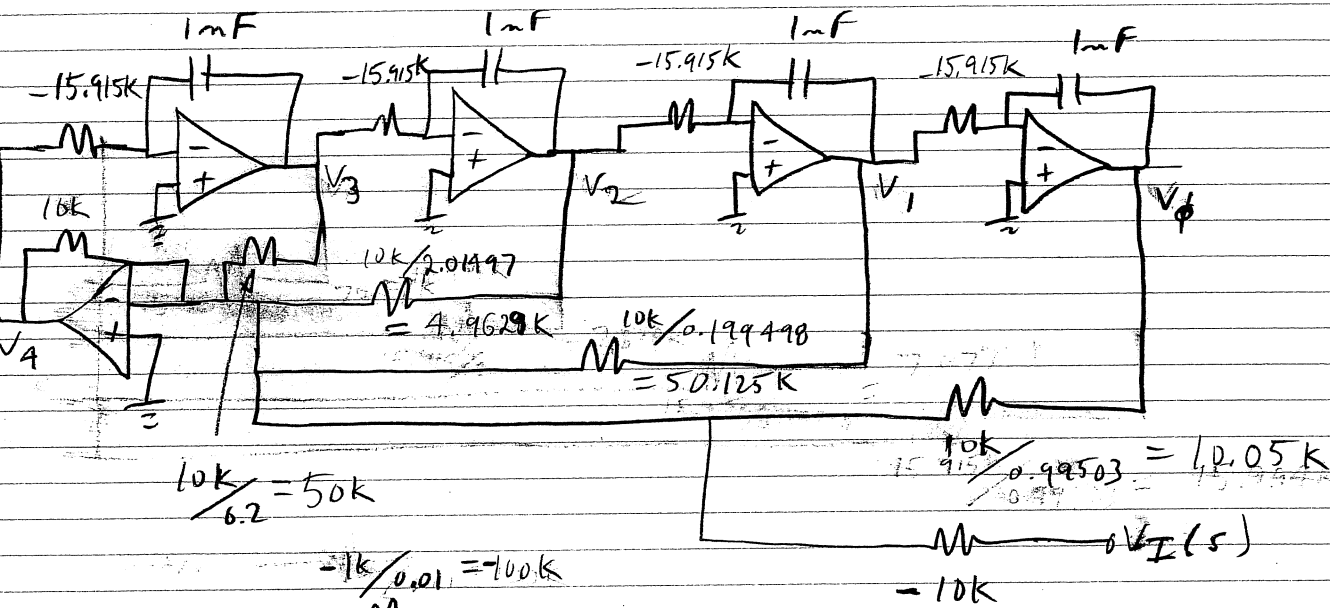
AND - ALL CAPACITORS ARE 1 mF



$$\tau = \frac{1}{25 \times 10^6}$$

$$R = \frac{1}{25 \times 10^6 \times 10^{-9}}$$

$$= 15.9155 \text{ k}\Omega$$



THERE ARE
 WAYS OF GETTING AROUND NEGATIVE RESISTORS
 BUT ALTHOUGH THIS APPROACH IS STRAIGHTFORWARD
 IT HAS POOR PERFORMANCE IN TERMS OF
 DYNAMIC RANGE AND SENSITIVITY, FOR HIGH Q FILTERS.