FILTER REALIZATIONS

DIRECT-FORM REALIZATION $\Leftarrow$ SIMPLE BUT POOR PERFORMANCE FOR NARROW BAND FILTER

$$H(s) = \frac{V_o(s)}{V_I(s)} = \frac{a_M s^M + a_{M-1} s^{M-1} + \ldots + a_1 s + a_0}{s^N + b_{N-1} s^{N-1} + \ldots + b_1 s + b_0}$$

$$\triangleq \frac{P(s)}{E(s)}$$

CREATE N INTEGRATOR OUTPUTS, $X_0(s)$, $X_1(s)$, ..., $X_{N-1}(s)$

SUCH THAT $\dfrac{X_0(s)}{V_I(s)} = \dfrac{1}{E(s)}$, $\dfrac{X_1(s)}{V_I(s)} = \dfrac{s}{E(s)}$, ... $\dfrac{X_{N-1}(s)}{V_I(s)} = \dfrac{s^{N-1}}{E(s)}$

AND ONE SUMMATION OUTPUT $X_N(s)$

SUCH THAT $\dfrac{X_N(s)}{V_I(s)} = \dfrac{s^N}{E(s)}$

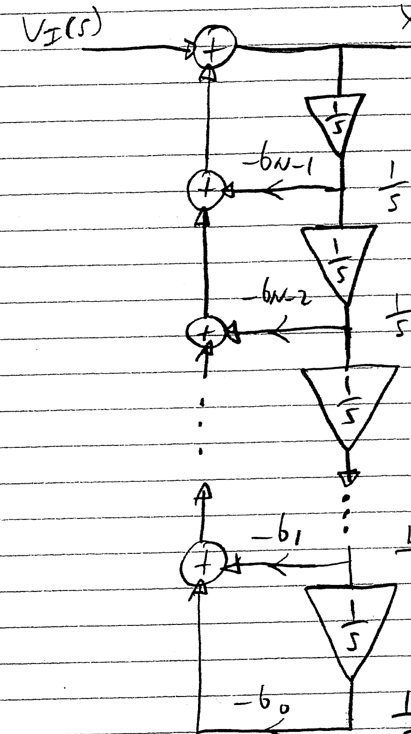
HOW? WRITE OUT $X_N(s)$ AS A FUNCTION OF ITSELF AND THE INPUT $V_I(s)$

$$\frac{X_N(s)}{V_I(s)} = \frac{s^N}{s^N + b_{N-1} s^{N-1} + \ldots + b_1 s + b_0}$$

$$X_N(s)\, s^N + X_N(s)\, b_{N-1} s^{N-1} + \ldots + X_N(s)\, b_1 s + X_N(s)\, b_0 = s^N V_I(s)$$

$\div s^N \Rightarrow$

$$\boxed{X_N(s) = -b_{N-1}\frac{1}{s}X_N(s) - b_{N-2}\frac{1}{s^2}X_N(s) - \ldots - b_1\frac{1}{s^{N-1}}X_N(s) - b_0\frac{1}{s^N}X_N(s) + V_I(s)}$$



$$X_N(s) = \frac{s^N}{E(s)} V_I(s)$$

$$\frac{1}{s} X_N(s) = X_{N-1}(s) = \frac{s^{N-1}}{E(s)} V_I(s)$$

$$\frac{1}{s^2} X_N(s) = X_{N-2}(s) = \frac{s^{N-2}}{E(s)} V_I(s)$$

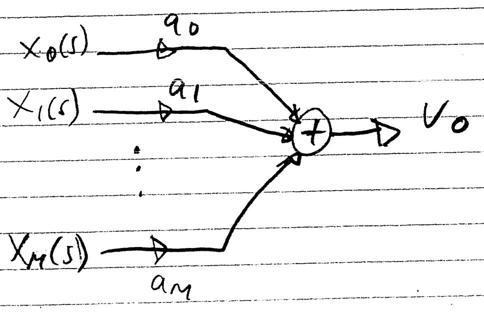
$$\frac{1}{s^{N-1}} X_N(s) = X_1(s)$$

$$\frac{1}{s^N} X_N(s) = X_0(s) = \frac{1}{E(s)} V_I(s)$$

FINALLY,
$$\frac{V_o(s)}{V_I(s)} = a_M \frac{X_M(s)}{V_I(s)} + a_{M-1} \frac{X_{M-1}(s)}{V_I(s)} + \ldots + a_1 \frac{X_1(s)}{V_I(s)} + a_0 \frac{X_0(s)}{V_I(s)}$$

$$\therefore \quad V_o(s) = a_M X_M(s) + a_{M-1} X_{M-1}(s) + \ldots + a_0 X_0(s)$$

ie

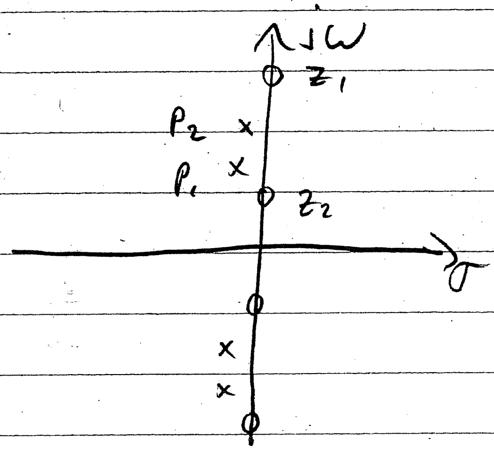


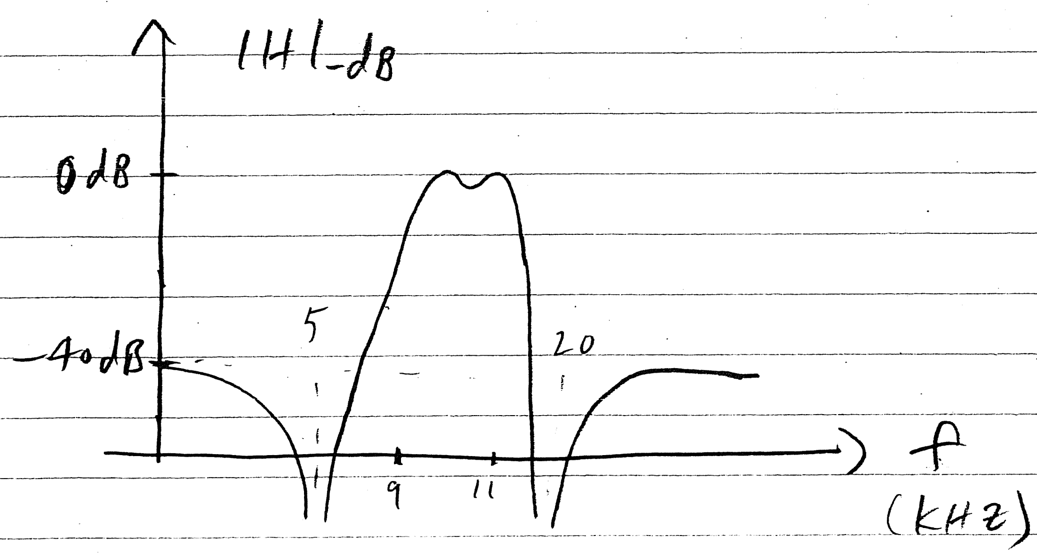
EXAMPLE

CONSIDER A BANDPASS FILTER WITH PASSBAND GAIN ≈ 1 FROM 9–11 KHZ + STOPBAND ATTENUATION ≈ -40 dB

ZEROS @ z_1: $\pm j(2\pi \times 20\text{KHZ})$
z_2: $\pm j(2\pi \times 5\text{KHZ})$

POLES @ P_1: $\omega_{01} = 2\pi \times 9.5\text{KHZ}$ + $Q_1 = 10$
P_2: $\omega_{02} = 2\pi \times 10.5\text{KHZ}$ & $Q_2 = 10$





$$H(s) = \frac{0.01\,(s^2 + (2\pi \times 20k)^2)(s^2 + (2\pi \times 5k)^2)}{\left(s^2 + \frac{\omega_{01}}{Q_1}s + \omega_{01}^2\right)\left(s^2 + \frac{\omega_{02}}{Q_2}s + \omega_{02}^2\right)}$$

$$H(s) = \frac{0.01\,s^4 + 1.6778 \times 10^8 s^2 + 1.559 \times 10^{17}}{s^4 + 1.26 \times 10^4 s^3 + 7.95 \times 10^9 s^2 + 4.95 \times 10^{13} s + 1.55 \times 10^{19}}$$

PROBLEM COEFFICIENT SPREAD OF 10^{19}!!!

WHY? BECAUSE INTEGRATORS

$H_I(s) = \frac{1}{s}$ HAS GAIN OF 1 AT 1 RAD/S

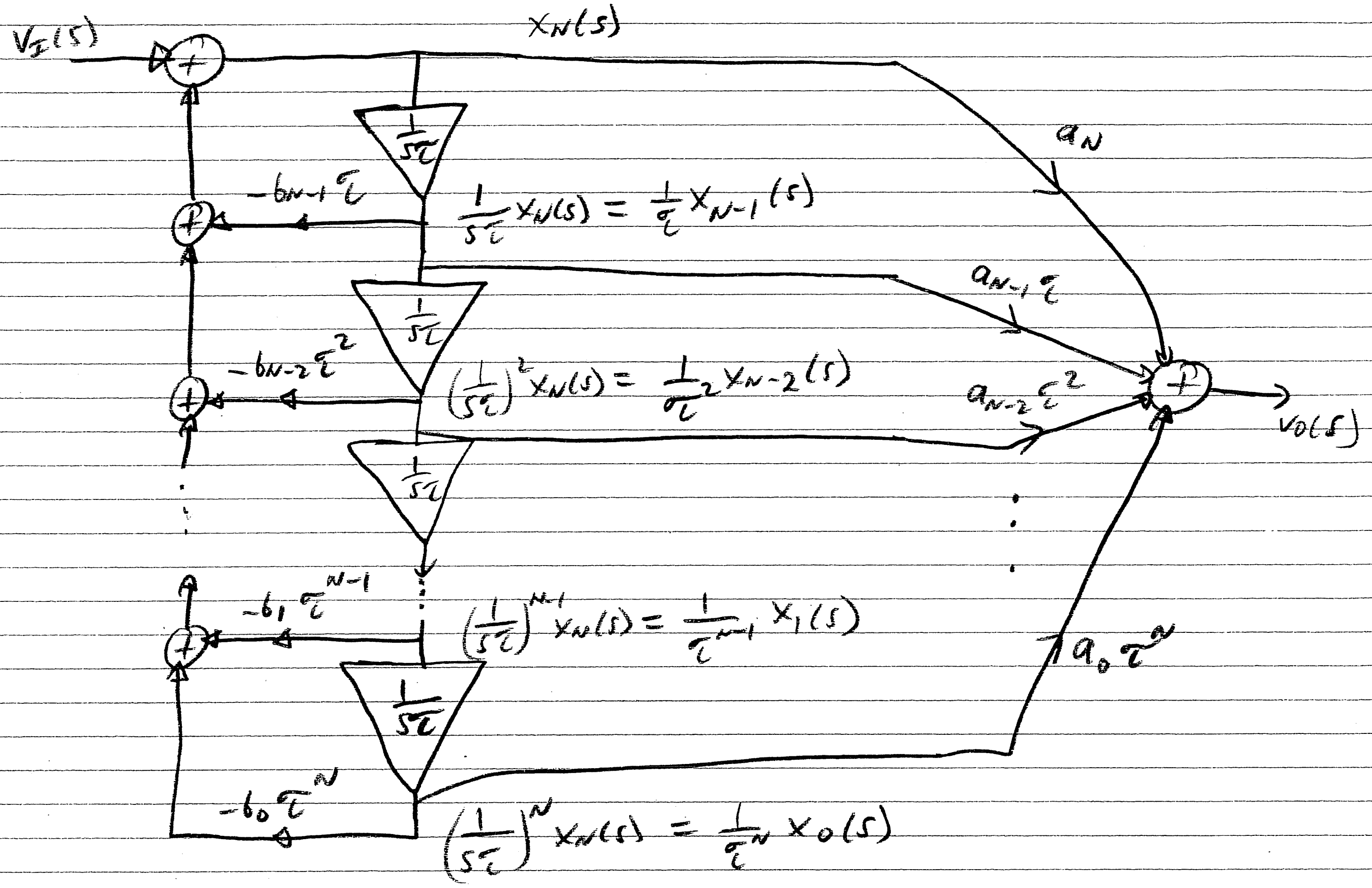
$|H_I(j1)| = 1$

BUT $|H_I(j(2\pi \times 10k))| = \frac{1}{(2\pi \times 10k)}$

GAIN $\ll 1$ NEAR PASSBAND EDGE OF FILTER

INSTEAD USE

$H_I(s) \equiv \frac{1}{s\tau}$ WHERE $\tau = \frac{1}{\omega_0}$



SO NOW COEFFICIENTS FOR EXAMPLE ARE:

$$-b_0\tau^4 = -1.5508\times10^{19}\left(\frac{1}{2\pi\times10k}\right)^4 = -0.99503$$

$$-b_1\tau^3 = -4.94856\times10^{13}\left(\frac{1}{2\pi\times10k}\right)^3 = -0.199498$$

$$-b_2\tau^2 = -2.01497$$

$$-b_3\tau = -0.2$$

SPREAD OF 10

$$a_4 = 0.01$$

$$a_2\tau^2 = 0.0425$$

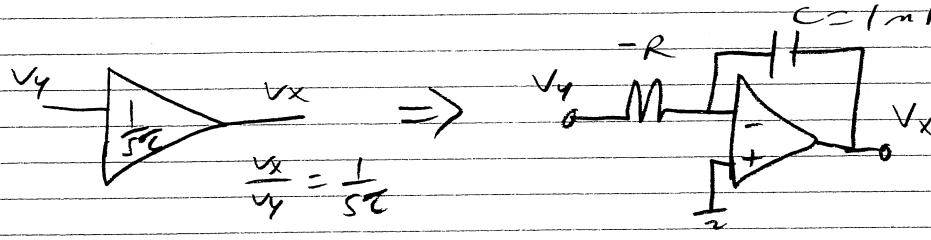
$$a_0\tau^4 = 0.01$$

SPREAD OF 2.4

FIND AN ACTIVE-RC REALIZATION

IF NEGATIVE RESISTORS ALLOWED.

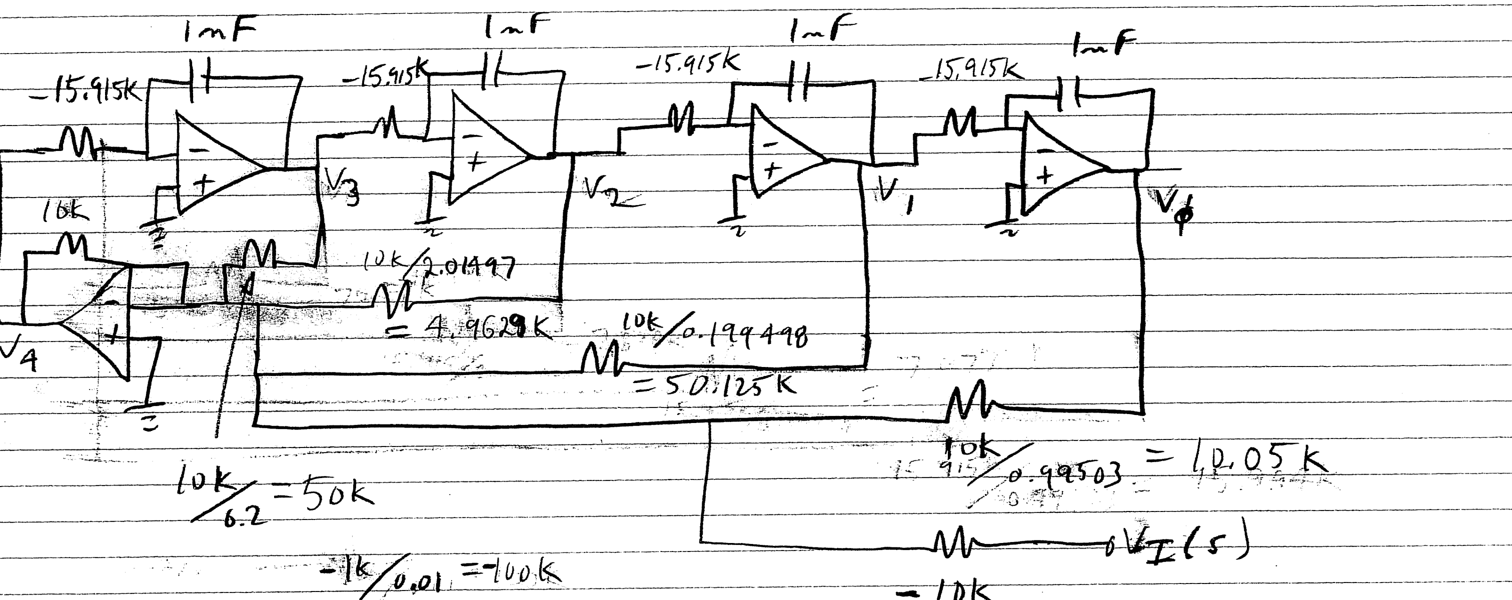
AND — ALL CAPACITORS ARE 1 nF

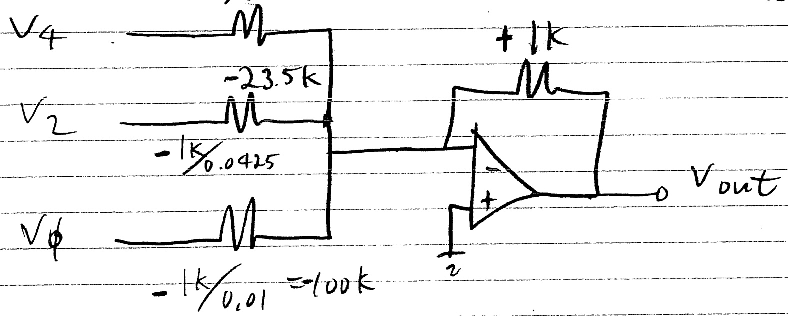


$$\frac{V_x}{V_y} = \frac{1}{s\tau} \qquad \frac{V_x}{V_y} = \frac{+1}{sRC} \qquad \tau = RC$$

$$\tau = \frac{1}{2\pi \times 10k} \qquad \therefore\ R = \frac{1}{2\pi \times 10k \times 1 \times 10^{-9}}$$

$$= 15.9155\ k\Omega$$





~~THERE ARE~~

WAYS OF GETTING AROUND NEGATIVE RESISTORS

BUT ALTHOUGH THIS APPROACH IS STRAIGHTFORWARD

IT HAS POOR PERFORMANCE IN TERMS OF

DYNAMIC RANGE AND SENSITIVITY. FOR HIGH Q FILTERS.