BIQUAD DESIGN

CASCADES-OF-BIQUADS ARE THE MOST COMMON DESIGN APPROACH FOR BOTH ANALOG AND DIGITAL FILTERS.

WHY? $\Rightarrow$ PERFORMANCE IS GOOD (THOUGH NOT EXCELLENT) BUT TESTING IS SIMPLE (TEST INDIVIDUAL BIQUADS SEPARATELY). DESIGN IS ALSO RELATIVELY STRAIGHTFORWARD.

STEPS IN DESIGNING A CASCADE-OF-BIQUADS.

1) CHOOSE APPROPRIATE POLE-ZERO PAIRING

2) CHOOSE A SUITABLE CASCADE-ORDERING

3) REALIZE EACH BIQUAD SECTION IN DESIRED TECHNOLOGY (ACTIVE-RC, DIGITAL FILTERS, SWITCHED-CAPA

CAN BE DONE IN OPPOSITE ORDER

4) SCALE APPROPRIATE SIGNAL NODES TO OBTAIN MAXIMUM DYNAMIC RANGE.

LETS LOOK AT EACH OF THESE STEPS USING ACTIVE-RC FOR EXAMPLES AND THE FOLLOWING TRANSFER-FUNCTION:

8TH ORDER BANDPASS FILTER $\rightarrow$ EQUIRIPPLE PASSBAND FROM 1 KHZ TO 1.4 KHZ (0.4 dB RIPPLE) PEAK GAIN OF 0 dB

$T(s)$

POLES: P_1, P_1^* $\quad \omega_{01} = 0.7106 \quad Q = 15.256$

P_2, P_2^* $\quad \omega_{02} = 0.7911 \quad Q = 6.192$

P_3, P_3^* $\quad \omega_{03} = 0.9188 \quad Q = 6.743$

P_4, P_4^* $\quad \omega_{04} = 1.0048 \quad Q = 19.113$

NORMALIZED WITH RESPECT TO $2\pi \times 1.4$ k RAD/S

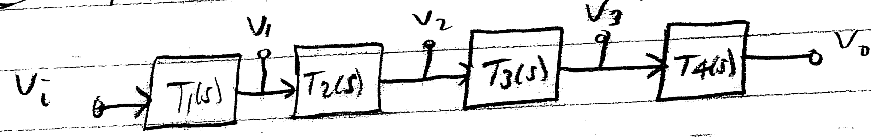
P_1^* IS COMPLEX CONJUGATE OF P_1

ZEROS: z_1, z_1^* $\pm j\,0.3996$
z_2, z_2^* $\pm j\,1.2561$
z_3, z_4 $0, 0$
z_5, z_6 ∞, ∞

NORMALIZED WRT $2\pi \times 1.4$ K RAD/S.

DYNAMIC RANGE IN GENERAL

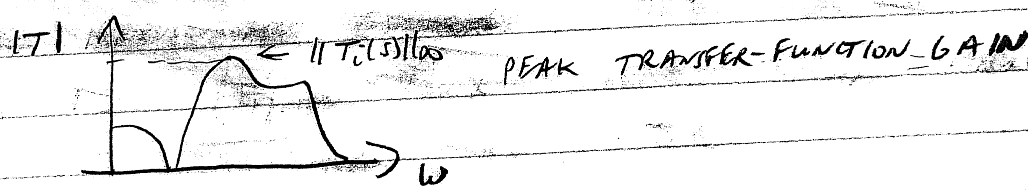
FOR THIS 8TH ORDER BANDPASS FILTER, THERE WILL BE 4 BIQUAD FILTERS; $T_1(s), T_2(s), T_3(s), T_4(s)$



$$\frac{V_1}{V_i} = T_1(s) \qquad \frac{V_2}{V_i} = T_1(s)\,T_2(s) \qquad \frac{V_3}{V_i} = T_1(s)\,T_2(s)\,T_3(s)$$

$$\frac{V_o}{V_i} = T(s) = T_1(s)\,T_2(s)\,T_3(s)\,T_4(s)$$

DEFINE $\|T_i(s)\|_\infty = \max(|T_i(j\omega)|, 0 < \omega < \infty)$ ← CALLED AN L_∞ NORM



ASSUMING V_1, V_2, V_3, V_o ALL CLIP AT THE SAME LEVEL THEN WE SHOULD SCALE $\|T_1\|_\infty$, $\|T_1 T_2\|_\infty$, $\|T_1 T_2 T_3\|_\infty$ SUCH THAT

$$\|T\|_\infty = \|T_1\|_\infty = \|T_1 T_2\|_\infty = \|T_1 T_2 T_3\|_\infty$$

THAT WAY, FOR A SWEPT SINUSOID INPUT OF SAY 1 VPP, EACH OUTPUT, V_1, V_2, V_3, V_o WILL ATTAIN THE SAME PEAK VOLTAGE LEVEL (THOUGH AT DIFFERENT FREQUENCIES PERHAPS). THUS DYNAMIC RANGE IS OPTIMIZED IN AN L_∞ SENSE, FOR A GIVEN SET OF T_1, T_2, T_3, T_4

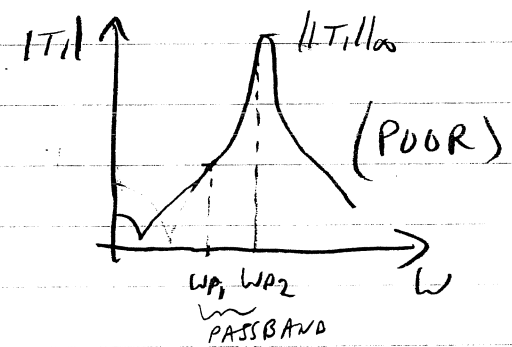
WHY? DYNAMIC RANGE IS DETERMINED BY THE RATIO OF LARGEST USEFUL SIGNAL TO THE SMALLEST USEFUL SIGNAL.

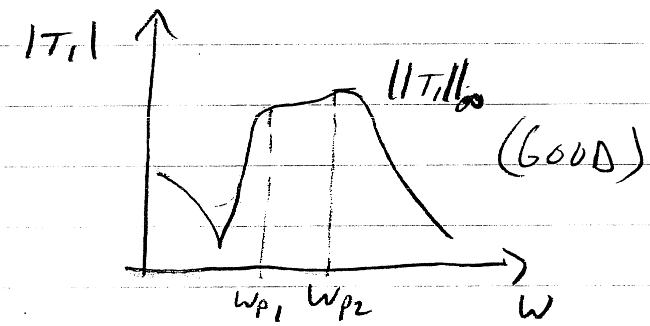
LARGEST USEFUL SIGNAL → DETERMINED BY MAXIMUM VOLTAGE LEVELS POSSIBLE WITHOUT SEVERE DISTORTION. (ie CLIPPING)

SMALLEST USEFUL SIGNAL → DETERMINED BY NOISE CONSIDERATIONS THERMAL NOISE, INTERFERENCE, ETC.

TO MAXIMIZE DYNAMIC RANGE, KEEP ALL INTERNAL SIGNALS AS LARGE AS POSSIBLE WITHOUT DISTORTING.

WHILE ~~STEP A)~~ SCALES FOR DYNAMIC RANGE FOR A GIVEN T_1, T_2, T_3, T_4 THE CHOICE OF T_1, T_2, T_3 T_4 & THEIR ORDERING
ALSO AFFECT DYNAMIC RANGE PERFORMANCE.
FOR FLAT PASSBANDS,
TO SEE THIS CONSIDER TWO POSSIBLE CHOICES FOR $T_1(s)$





NOT VERY FLAT

FLAT NEAR PASSBAND OF FILTER

DYNAMIC RANGE POOR
FOR FREQ NEAR w_{p1}
(A GAIN LATER ON MUST COMPENSATE FOR THIS LOSS NEAR w_{p1})

IN SUMMARY, CHOOSE T_1, T_2, T_3, T_4 TO RESULT IN AS FLAT TRANSFER-FUNCTIONS AS POSSIBLE AROUND FILTER'S PASSBAND. (FOR FLAT PASSBANDS)

POLE - ZERO PAIRING → EACH BIQUAD NEEDS A PAIR OF POLES AND A PAIR OF ZEROS

NEED 4 BIQUADS ⇒ $\frac{k_1(s-z_1)(s-z_1^*)}{(s-p_1)(s-p_1^*)}$ OR $\frac{k_1(s-z_2)(s-z_2^*)}{(s-p_1)(s-p_1^*)}$, etc

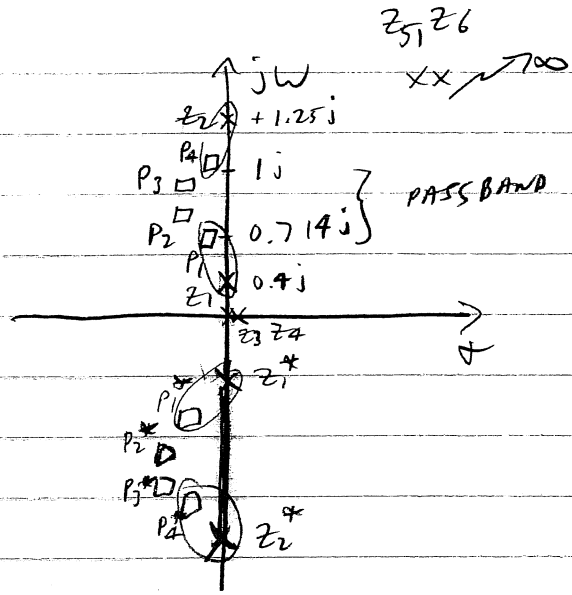
(KEEP COMPLEX-CONJUGATES TOGETHER)

IN GENERAL, $\left(\frac{N}{2}\right)!$ DIFFERENT POLE-ZERO PAIRING POSSIBILITIES

TO GET AS FLAT TRANSFER-FUNCTIONS AS POSSIBLE NOTE HIGH Q POLES CAUSE MOST DEVIATION IN PASSBAND & HAVING ZEROS CLOSE BY POLES IMPLY CANCELLATION AS $|T(j\omega)|$ MOVE AWAY FROM PAIR. SYSTEMATIC PROCEDURES ARE AVAILABLE BUT A GOOD APPROXIMATION IS:

RULE-OF-THUMB FOR POLE-ZERO PAIRING

POLES SHOULD BE PAIRED WITH THEIR NEAREST ZEROS WITH HIGH POLE Q's TAKING PRIORITY. OTHER CHOICES DEPEND ON REALIZATION (e.g. OFTEN BP PREFERRED OVER HP & LP DUE TO EASE OF TUNING)



P_4 IS HIGHEST $Q \Rightarrow z_2$ & $P_4 \Rightarrow T_1 = \dfrac{k_1 (s^2 + 1.2501^2)}{s^2 + \frac{1.0048}{19.113} s + 1.0048^2}$

P_1 IS NEXT HIGHEST $Q \Rightarrow z_1$ & $P_1 \Rightarrow T_2 = \dfrac{k_2 (s^2 + 0.3996^2)}{s^2 + \frac{0.7106}{15.256} s + 0.7106^2}$

LAST TWO BP

$$T_3 = \frac{k_3\ s}{s^2 + \frac{0.7911}{6.192} s + 0.7911^2} \qquad T_4 = \frac{k_4\ s}{s^2 + \frac{0.9188}{6.743} s + 0.9188^2}$$

OR LP & HP

$$T_3 = \frac{k_3\ s^2}{s^2 + \frac{0.7911}{6.192} s + 0.7911^2} \qquad T_4 = \frac{k_4}{s^2 + \frac{0.9188}{6.743} s + 0.9188^2}$$

CHOICE #1

NOTE: $T_1 \Rightarrow$ LPN
$T_2 \Rightarrow$ HPN
$T_3, T_4 \Rightarrow$ BP

OR

CHOICE #2

$T_1 \Rightarrow$ LPN
$T_2 \Rightarrow$ HPN
$T_3 \Rightarrow$ HP
$T_4 \Rightarrow$ LP

CASCADE-ORDERING → NEED TO FIND A GOOD ORDERING FOR CASCADE-OF-BIQUADS.

$T_1 \rightarrow T_2 \rightarrow T_3 \rightarrow T_4$ OR $T_3 \rightarrow T_2 \rightarrow T_4 \rightarrow T_1$ OR etc.

IN GENERAL $\left(\frac{N}{2}\right)!$ DIFFERENT ORDERING POSSIBILITIES

GOAL ⇒ KEEP TRANSFER-FUNCTIONS FLAT.

SYSTEMATIC PROCEDURE AVAILABLE BUT ...

RULE-OF-THUMB FOR CASCADE ORDERING

ALTERNATE HP & LP SECTIONS AND PLACE HIGH-Q SECTIONS IN THE MIDDLE WITH LOW Q AT EITHER END.

HP & LP ALTERNATING ⇒ ~~FLATTENS~~ OUT RESPONSE
HIGH-Q SECTIONS TOGETHER ⇒ FLATTEN OUT EACH OTHER
IN MIDDLE FOR SYMMETRY.

IN ABOVE EXAMPLE CHOICE # 1

ONE OF : $T_3\ T_1\ T_2\ T_4$ ← SINCE T_3 PEAKS ~~AT~~ 0.7911 - THEN CHOOSE T_1 NEXT WHICH PEAKS AROUND 1.0048 (T_2 PEAKS AROUND 0.7106 + BUMPS WOULD ADD)

$T_4\ T_2\ T_1\ T_3$

CHOICE #2

$T_3\ T_1\ T_2\ T_4$ ⇒ HP, LPN, HPN, LP

$T_4\ T_2\ T_1\ T_3$ ⇒ LP, HPN, LPN, HP

REALIZING BIQUAD SECTIONS (ACTIVE-RC)

3 MAIN CLASSES FOR REALIZING ACTIVE-RC BIQUADS.
(i.e. ACTIVE-RC 2nd ORDER TRANSFER-FUNCTIONS)

- SINGLE-AMPLIFIER BIQUADS (SABs)
  - ONE OP-AMP ← POPULAR IN 70's AND AS ANTI-ALIASING FILTERS ON SOME IC's
  - WORSE DYNAMIC RANGE
  - MORE SUSCEPTIBLE TO NON-IDEAL EFFECTS OF AMPLIFIER.

- TWO-INTEGRATOR-LOOP BIQUADS
  - KHN AND TOW-THOMAS ARE 2 POPULAR TYPES
  - TYPICALLY 3 OP-AMPS REQUIRED (SOMETIMES 4)
  - GOOD DYNAMIC RANGE + LESS SENSITIVE TO NON-IDEAL OP-AMPS THAN SABs

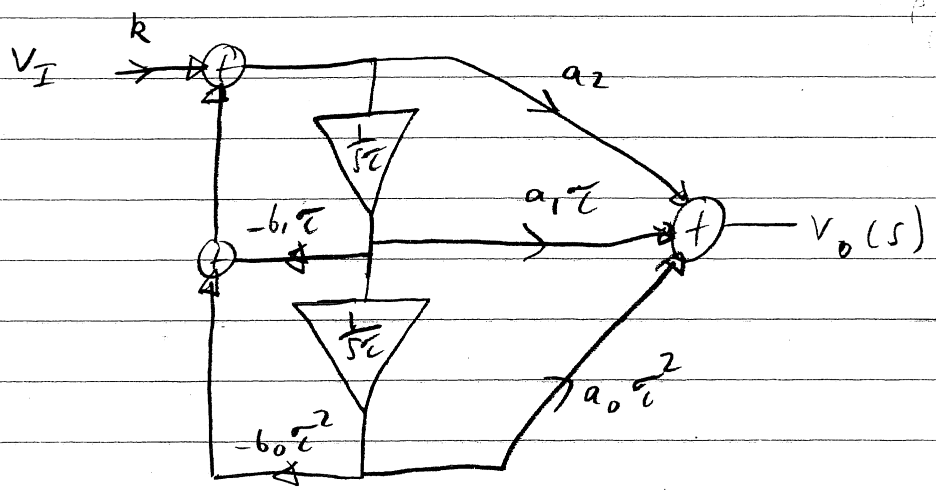
- GENERALIZED-IMPEDANCE-CONVERTER (GIC) BASED
  - ONLY 2 OP-AMPS REQUIRED (SOMETIMES 3)
  - DEVELOPED BY TWO CANADIAN EE PROFS (ANTONIOU + BRUTON)
  - GOOD DYNAMIC RANGE + LESS SENSITIVE TO NON-IDEAL OP-AMPS THAN (SABs)
  - COULD OSCILLATE IF LARGE STRAY CAPACITANCES (MIGHT REQUIRE COMPENSATION)

TWO-INTEGRATOR-LOOP BIQUADS

BASED ON A DIRECT-FORM REALIZATION FOR $N=2$

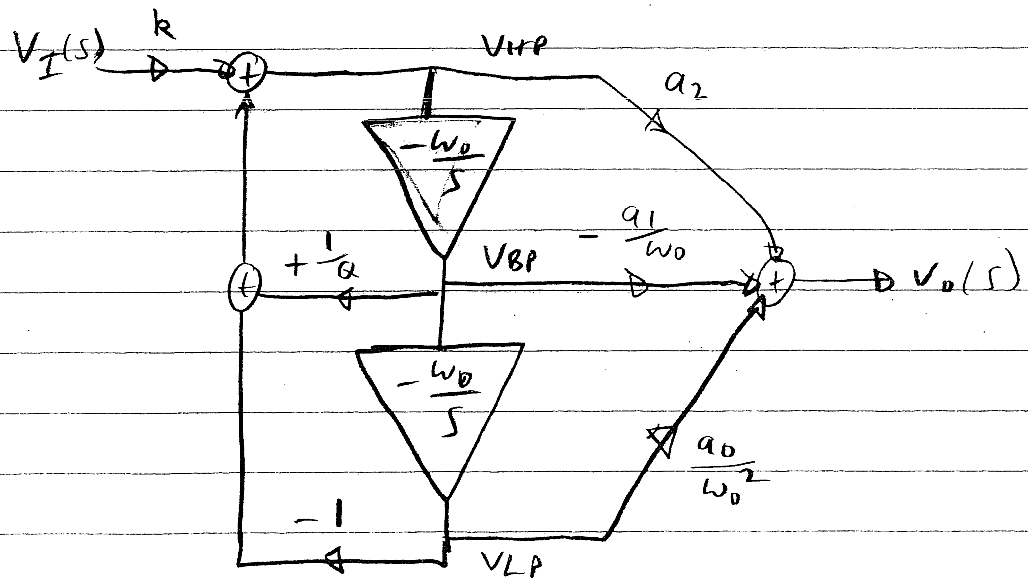
KHN (KERWIN-HUELSMAN-NEWCOMB) BIQUAD

(ALSO KNOWN AS THE STATE-VARIABLE BIQUAD)

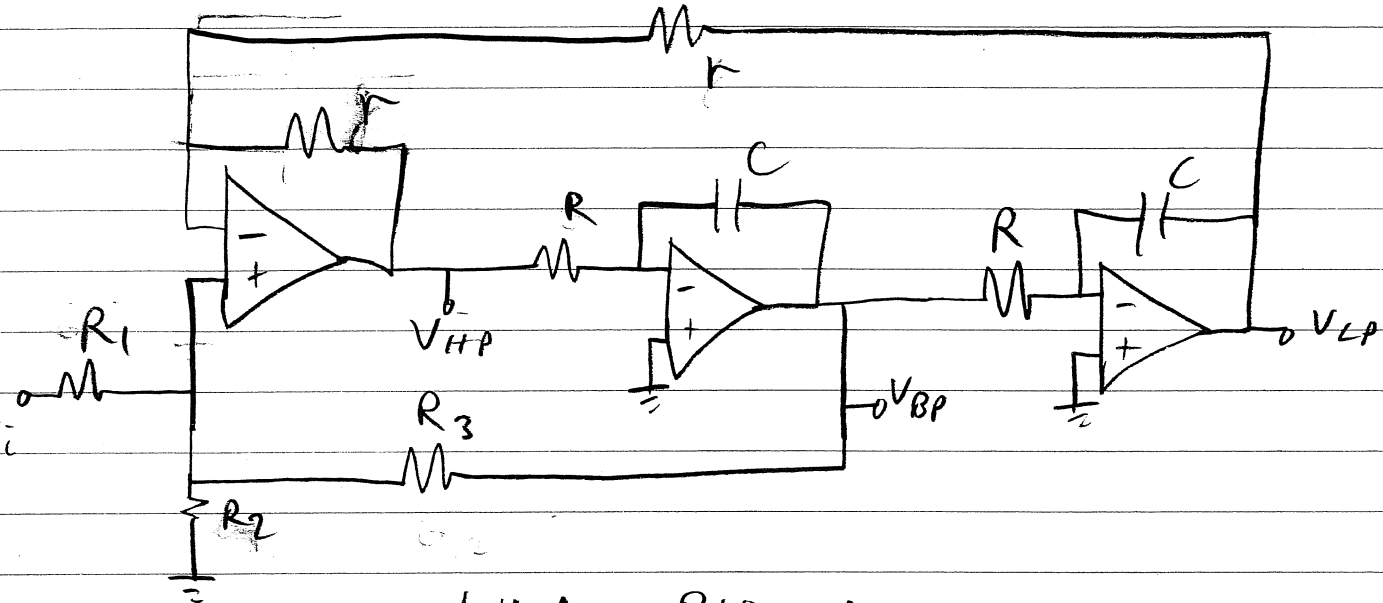


$$T(s) \triangleq \frac{V_o}{V_I} = k\,\frac{a_2 s^2 + a_1 s + a_0}{s^2 + b_1 s + b_0} \triangleq k\,\frac{a_2 s^2 + a_1 s + a_0}{s^2 + \frac{\omega_0}{Q} s + \omega_0^2}$$

AND USE NEGATIVE INTEGRATORS WITH $\tau = \frac{1}{\omega_0}$ RESULTS IN



WE CAN REALIZE THIS BLOCK DIAGRAM WITH THE FOLLOWING ACTIVE-RC CIRCUIT.



A KHN BIQUAD

IN ABOVE CIRCUIT, $RC = \tau = \frac{1}{\omega_0}$ $\boxed{RC = \frac{1}{\omega_0}}$

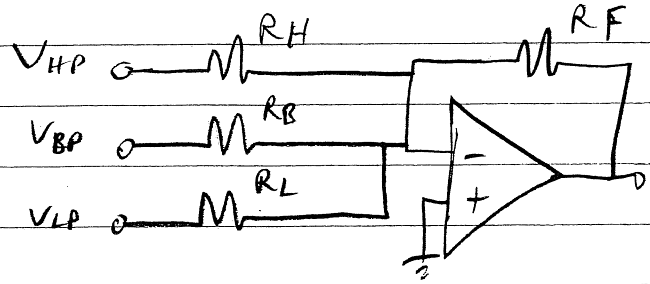
USING SUPERPOSITION FOR SUMMER

$$V_{HP} = -V_{LP} + \underbrace{\left(1+\frac{r}{r}\right)}_{2}\left(\frac{R_1 \| R_2}{R_1 \| R_2 + R_3}\right)V_{BP} + \underbrace{\left(1+\frac{r}{r}\right)}_{2}\left(\frac{R_2 \| R_3}{R_2 \| R_3 + R_1}\right)V_i$$

$$\therefore \boxed{2\frac{R_1 \| R_2}{R_1 \| R_2 + R_3} = \frac{1}{Q}} \quad \& \quad \boxed{2\frac{R_2 \| R_3}{R_2 \| R_3 + R_1} = k}$$

2 EQUATIONS ⇒ 3 UNKNOWNS ⇒ CAN CHOOSE SIZE OF RESISTORS.
(ie 1k, 100k OR 1M)

THEN OUTPUT IS



$$V_o = -\left(\frac{R_F}{R_H} V_{HP} + \frac{R_F}{R_B} V_{BP} + \frac{R_F}{R_L} V_{LP}\right)$$

$\therefore$ $\boxed{\frac{R_F}{R_H} = a_2}$ $\boxed{\frac{R_F}{R_B} = -\frac{a_1}{\omega_0}}$ & $\boxed{\frac{R_F}{R_L} = \frac{a_0}{\omega_0^2}}$

AND

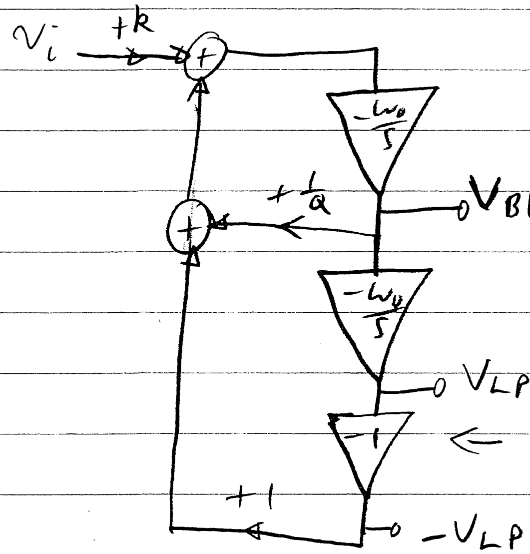
$$T(s) = -k \, \frac{a_2 s^2 + a_1 s + a_0}{s^2 + \frac{\omega_0}{Q} s + \omega_0^2}$$

IF $R_H = R_L = R_F$ & $R_B = R_F Q$ THEN AN ALL-PASS FUNCTION IS REALIZED.

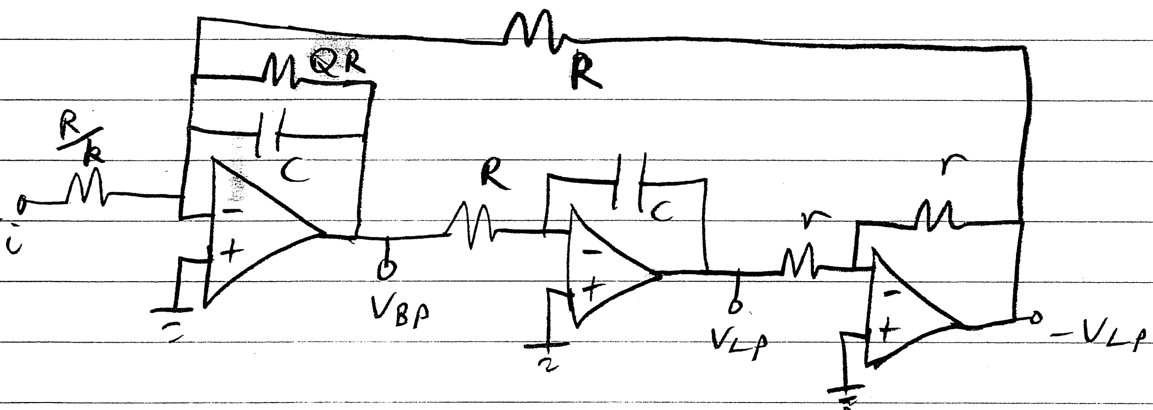
$$T(s) = -k \, \frac{s^2 - \frac{\omega_0}{Q} s + \omega_0^2}{s^2 + \frac{\omega_0}{Q} s + \omega_0^2}$$

TOW-THOMAS BIQUAD

ELIMINATE THE V_{HP} AS AN OP-AMP OUTPUT BY MAKING ALL COEFFICIENTS INTO IT '+'ve



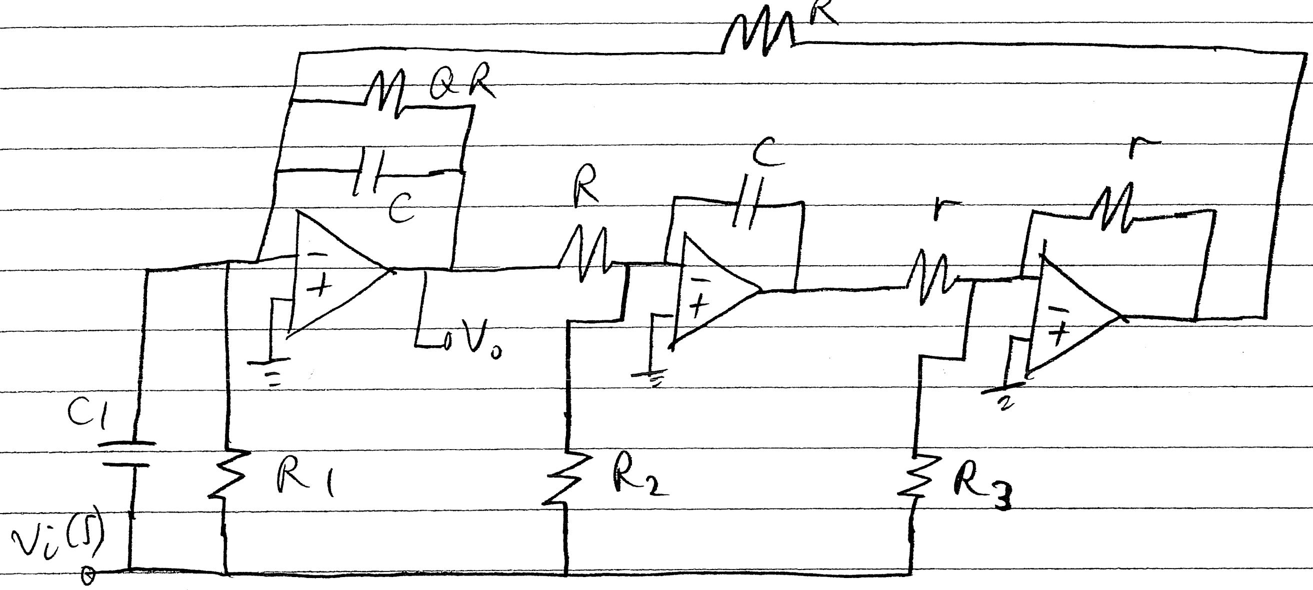
⇐ NEEDED TO MAKE "−1" ⇒ "+1"



$$RC = \frac{1}{\omega_0}$$

FOR FINITE- ZEROS ~~POLES (ZEROS)~~ ⇒ CAN USE FEED-IN ELEMENTS.

TOW – THOMAS BIQUAD USING INPUT ELEMENTS TO REALIZE FINITE ZEROS



$$H(s) = \frac{V_o}{V_i} = -\frac{s^2\left(\frac{C_1}{C}\right) + s\left(\frac{1}{C}\right)\left(\frac{1}{R_1} - \frac{r}{RR_3}\right) + \frac{1}{RR_2C^2}}{s^2 + \frac{1}{RCQ}s + \frac{1}{C^2R^2}}$$

$$\omega_0 = \frac{1}{CR}$$

LOWPASS $\Rightarrow$ $C_1 = 0$ $\quad R_1 = R_3 \to \infty$

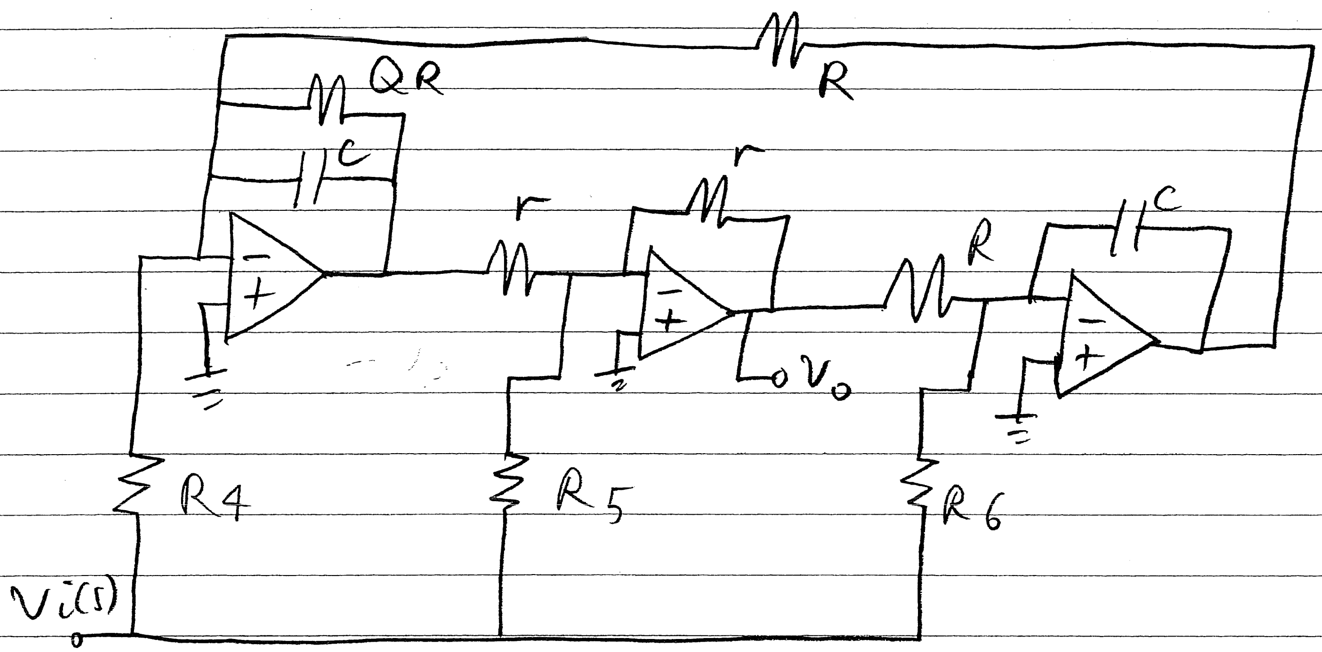
BANDPASS $\Rightarrow$ $C_1 = 0$ $\quad R_2 = R_3 \to \infty$ (NEG BP)

HIGHPASS $\Rightarrow$ $R_1 = R_2 = R_3 \to \infty$

LPN, HPN $\Rightarrow$ $R_1 = R_3 \to \infty$ $\quad C_1 = \frac{C}{G}$ $\quad R_2 = \frac{R\left(\frac{\omega_0}{\omega_N}\right)^2}{G}$

"G" = HIGH FREQ GAIN

ALTERNATE TOW-THOMAS WITH FINITE ZEROS



$$H(s) = -\frac{\left(\frac{r}{R_5}\right)s^2 + \frac{1}{RCQ}\left(\frac{r}{R_5} - \frac{QR}{R_4}\right)s + \frac{1}{RR_6C^2}}{s^2 + \frac{1}{RCQ}s + \frac{1}{C^2R^2}}$$

$$W_o = \frac{1}{CR}$$

LOWPASS $\Rightarrow$ $R_4 = R_5 \to \infty$

BANDPASS $\Rightarrow$ $R_5 = R_6 \to \infty$

HIGHPASS $\Rightarrow$ $R_6 \to \infty$ $\quad R_4 = \frac{QR}{G}$ $\quad R_5 = \frac{r}{G}$

"G" HIGHPASS GAIN