

# FREQUENCY SCALING

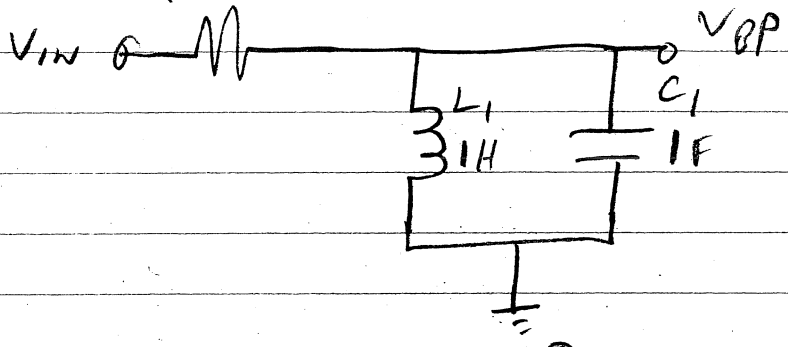
TO SCALE

$$H(s) \rightarrow H(s/k_f)$$

DIVIDE ALL CAPACITORS & INDUCTORS BY  $k_f$

EX

$$R_1 = 1 \Omega$$



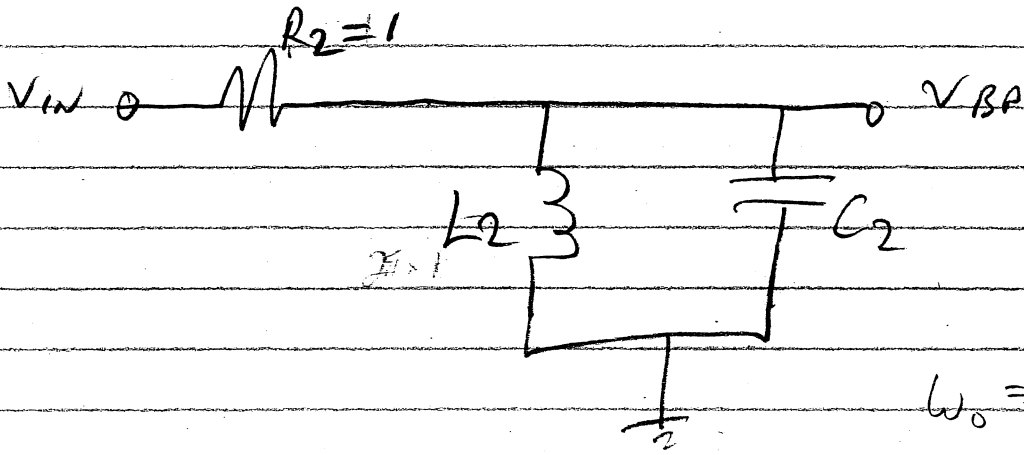
$$\omega_0 = \frac{1}{\sqrt{LC}} = 1$$

$$Q = \omega_0 CR = 1$$

$$H_1(s) \equiv \frac{V_{OP}}{V_{IN}} = \frac{s}{s^2 + s + 1}$$

LET  $k_f = 2\pi \times 1 \text{ GHz}$

$$H_2(s) = \frac{\frac{s}{(2\pi \times 10^9)}}{\frac{s^2}{(2\pi \times 10^9)^2} + \frac{s}{2\pi \times 10^9} + 1} = \frac{(2\pi \times 10^9)s}{s^2 + (2\pi \times 10^9)s + (2\pi \times 10^9)^2}$$



$\omega_0 = 2\pi \times 10^9$

$Q = 1$

$L_2 = \frac{L_1}{2\pi \times 10^9} = 15.9 \text{ pH}$

$C_2 = \frac{C_1}{2\pi \times 10^9} = 15.9 \text{ pF}$

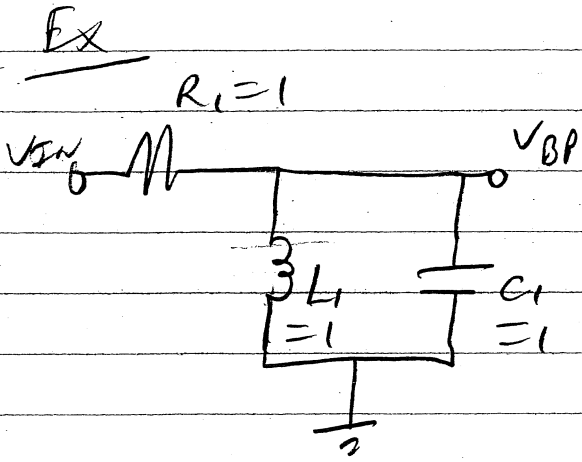
IMPEDANCE SCALING

TO KEEP  $H(s)$  UNCHANGED  
 IF  $H(s)$  IS DIMENSIONLESS (SUCH AS VOLTAGE GAIN OR CURRENT GAIN)

BUT SCALE IMPEDANCE BY  $k_z$

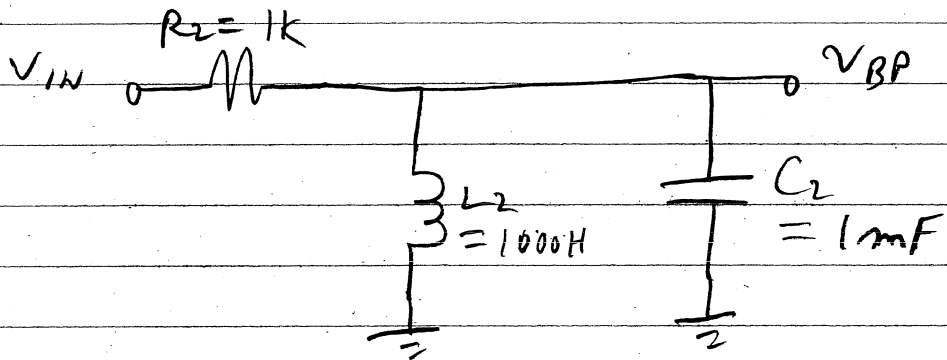
$\Rightarrow$  MULTIPLY ALL ELEMENTS HAVING DIMENSIONS OF RESISTANCE OR INDUCTANCE BY  $k_z$

DIVIDE ALL ELEMENTS HAVING DIMENSIONS OF CONDUCTANCE OR CAPACITANCE BY  $k_z$



$H_1(s)$

$K_2 = 1000$



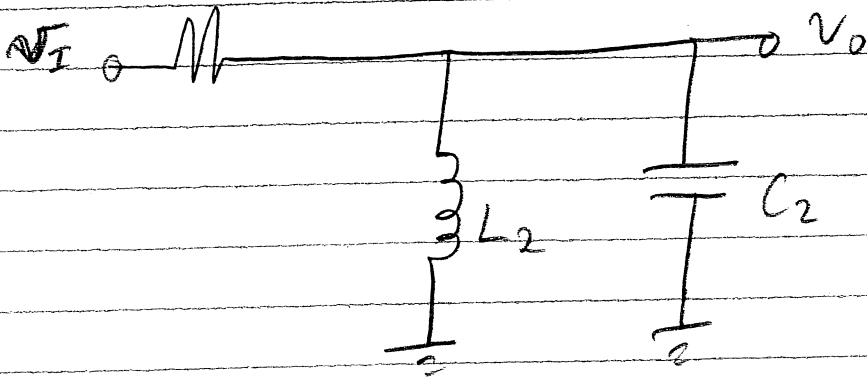
$H_2(s) = H_1(s)$

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SO BP FILTER WITH  $Q=1$ ,  $\omega_0 = 2\pi \times 10^9$

$$\text{or } R_{IN} = 75 \Omega$$

$$R = 75 \Omega$$



$$L_2 = 15.9 \text{ pF} \times 75 = 11.9 \text{ mH}$$

$$C_2 = 15.9 \text{ pF} / 75 = 2.12 \text{ pF}$$

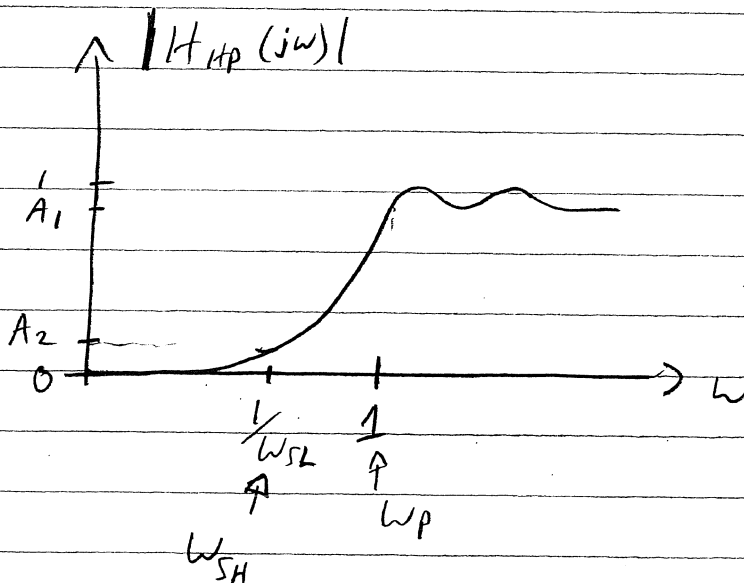
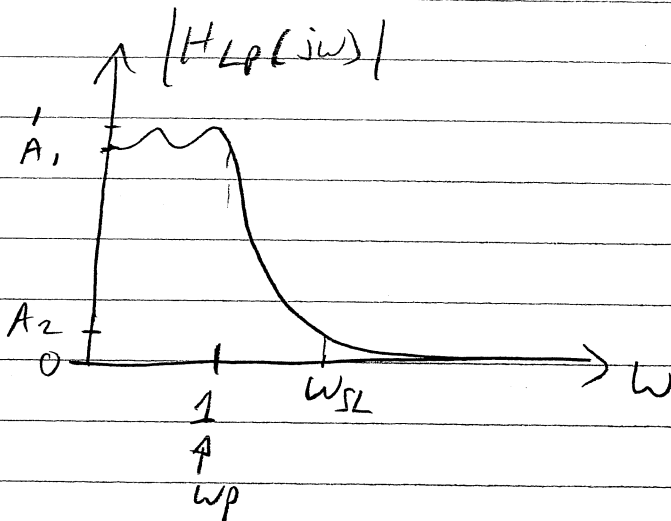
# FREQUENCY TRANSFORMATIONS

LOWPASS PROTOTYPE HAS  $\omega_p = 1$ ,  $H_{LP}(s)$

## LOWPASS TO HIGHPASS

MAKE SUBSTITUTION  $s \rightarrow \frac{1}{s}$

$$H_{HP}(s) = H_{LP}\left(\frac{1}{s}\right)$$



STOP BAND FREQ

$$\omega_{SH} = \frac{1}{\omega_{SL}}$$

BUTTERWORTH  
HIGHPASS EXAMPLE

$$\begin{aligned} \text{DESIRE PASSBAND EDGE} &= 1 \text{ MHz} \\ \text{STOPBAND EDGE} &= 200 \text{ kHz} \end{aligned}$$

$$\begin{aligned} \text{PASSBAND RIPPLE} &= 3 \text{ dB} \\ \text{STOPBAND ATTEN} &= 40 \text{ dB} \end{aligned}$$

FIRST FREQ TRANSFORM TO  $\omega_p = 1$

$$\omega_p = 1$$

$$\omega_{SH} = \frac{200 \text{ kHz}}{1 \text{ MHz}} = 0.2$$

NOW TRANSFORM TO LOWPASS

$$\omega_p = 1 \quad \omega_{SL} = \frac{1}{0.2} = 5$$

$$A_{MAX} = 3 \text{ dB} \quad A_{MIN} = 40 \text{ dB}$$

$$\epsilon = \sqrt{10^{\frac{A_{MAX}}{10}} - 1} = 0.9976$$

AND USE

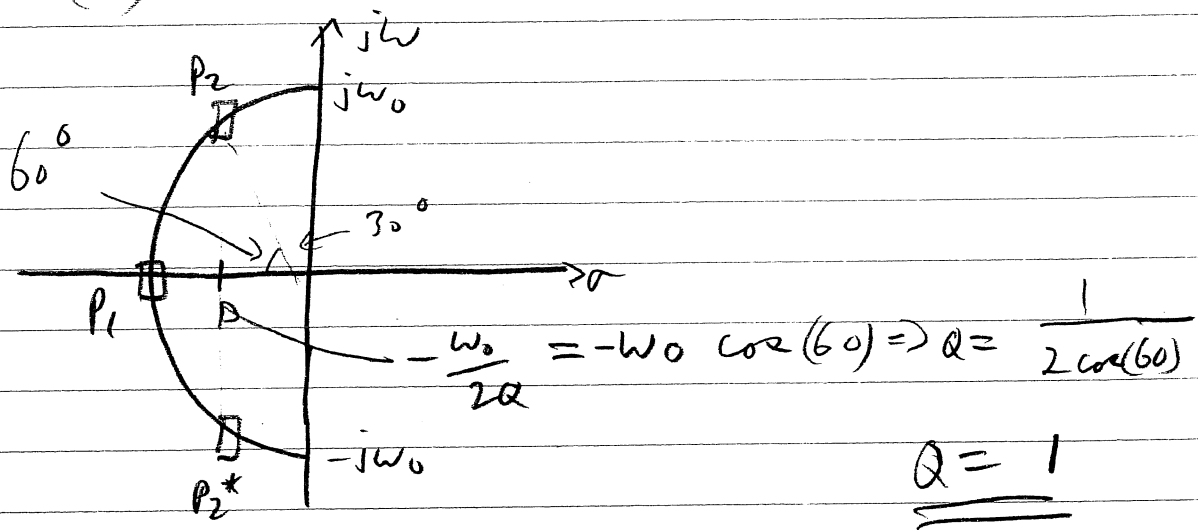
$$A_{MIN} \leq 10 \log \left[ 1 + \epsilon^2 \left( \frac{\omega_{SL}}{\omega_p} \right)^{2N} \right] \quad \text{TO FIND}$$

N

$$40 \leq 10 \log [1 + (0.9976)^2 (5)^{2N}]$$

$$\Rightarrow N \geq 2.86 \Rightarrow N = 3$$

$$\omega_0 = \left(\frac{1}{\epsilon}\right)^{\frac{1}{3}} = 1.0008$$



$$p_1 = -\omega_{01}, \quad p_2 \Rightarrow \omega_{02} \neq Q_2 = 1$$

$$\omega_{01} = \omega_0, \quad \omega_{02} = \omega_0$$

$$H_{LP}(s) = \frac{\omega_0}{(s + \omega_0)(s^2 + \frac{\omega_0}{Q}s + \omega_0^2)}$$

$$H_{HP}(s) = H_{LP}\left(\frac{1}{s}\right)$$

$$H_{HP}(s) = \frac{\omega_0^3}{\left(\frac{1}{s} + \omega_0\right) \left(\frac{1}{s^2} + \left(\frac{\omega_0}{Q}\right) \frac{1}{s} + \omega_0^2\right)}$$

$$= \frac{s^3 \omega_0^3}{(1 + s\omega_0) \left(1 + \frac{\omega_0}{Q} s + s^2 \omega_0^2\right)}$$

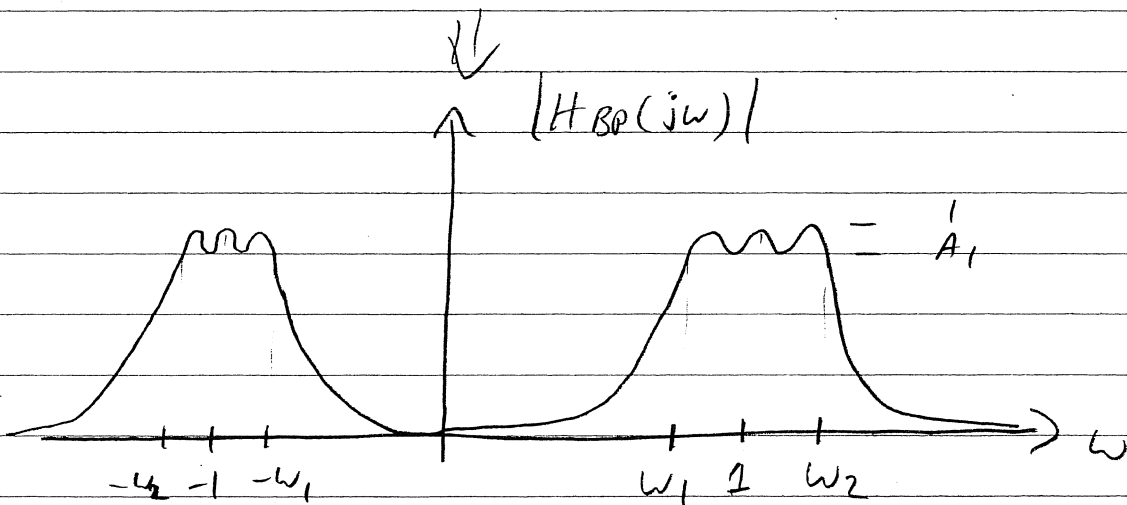
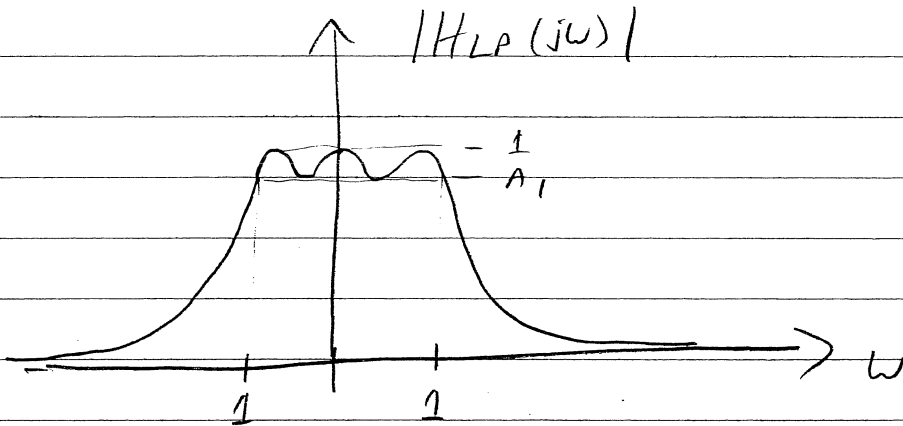
$$H_{HP}(s) = \frac{s^3}{\left(s + \frac{1}{\omega_0}\right) \left(s^2 + \frac{1}{\omega_0 Q} s + \frac{1}{\omega_0^2}\right)}$$

FINALLY FREQ SCALE BY  $s \rightarrow \frac{s}{(2\pi \times 10^6)}$

LOW PASS TO BANDPASS

LET  $S \rightarrow \frac{S^2 + 1}{BS}$

$H_{BP}(S) = H_{LP}\left(\frac{S^2 + 1}{BS}\right)$



$w_2 = \frac{1}{w_1}$        $w_2 - w_1 = B$

CAN SHOW       $w_1 = -\frac{B}{2} + \sqrt{\frac{B^2}{4} + 1}$

$w_2 = \frac{B}{2} + \sqrt{\frac{B^2}{4} + 1}$