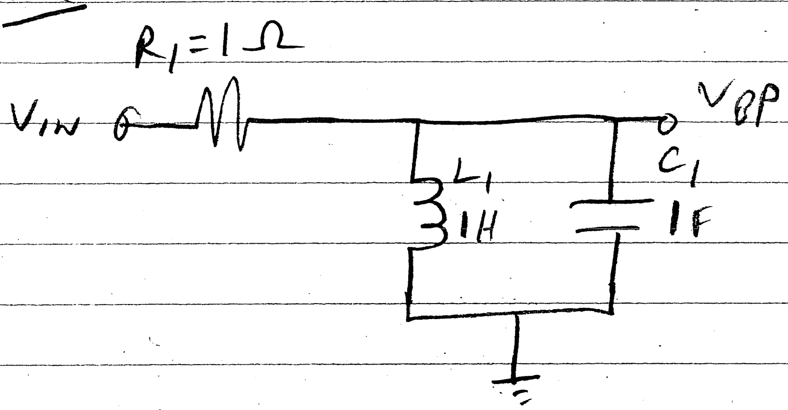
FREQUENCY SCALING

TO SCALE
$H(s) \rightarrow H(s/k_f)$

DIVIDE **ALL** CAPACITORS & INDUCTORS BY k_f

Ex



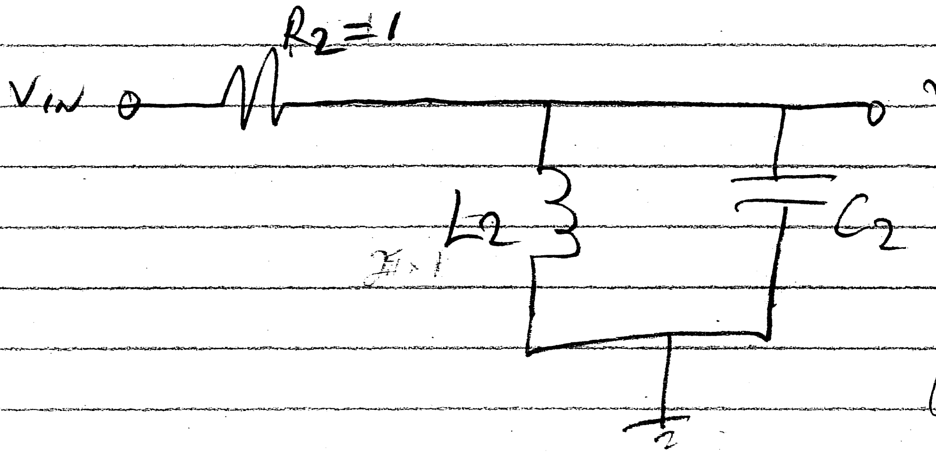
$\omega_0 = \frac{1}{\sqrt{LC}} = 1$

$Q = \omega_0 CR = 1$

$$H_1(s) \equiv \frac{V_{BP}}{V_{IN}} = \frac{s}{s^2 + s + 1}$$

LET $k_f = 2\pi \times 1\,GHz$

$$H_2(s) = \frac{\frac{s}{2\pi \times 1e9}}{\frac{s^2}{(2\pi \times 1e9)^2} + \frac{s}{2\pi \times 1e9} + 1} = \frac{(2\pi \times 1e9)s}{s^2 + (2\pi \times 1e9)s + (2\pi \times 1e9)^2}$$



$\omega_0 = 2\pi \times 1e9$

$Q = 1$

$L_2 = \dfrac{L_1}{2\pi \times 1e9} = 159\,pH$

$C_2 = \dfrac{C_1}{2\pi \times 1e9} = 159\,pF$

IMPEDANCE SCALING

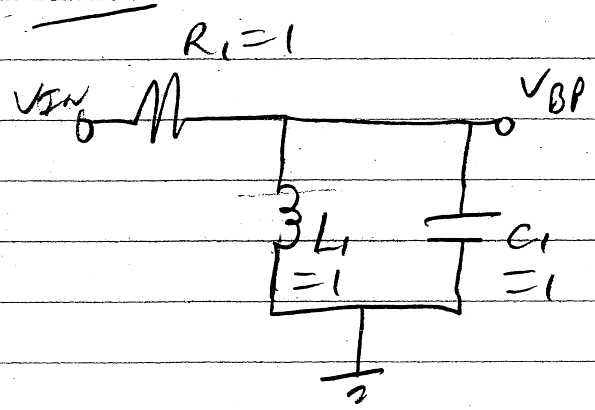
TO KEEP H(S) UNCHANGED
IF H(S) IS DIMENSIONLESS (SUCH AS VOLTAGE GAIN OR CURRENT GAIN)

BUT SCALE IMPEDANCE BY K_Z

$\Rightarrow$ MULTIPLY ALL ELEMENTS HAVING DIMENSIONS OF RESISTANCE OR INDUCTANCE BY K_Z

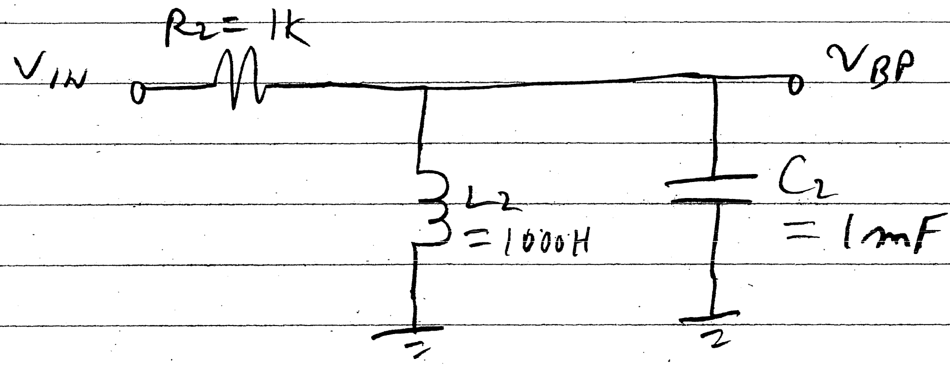
DIVIDE ALL ELEMENTS HAVING DIMENSIONS OF CONDUCTANCE OR CAPACITANCE BY K_Z

Ex



$H_1(s)$

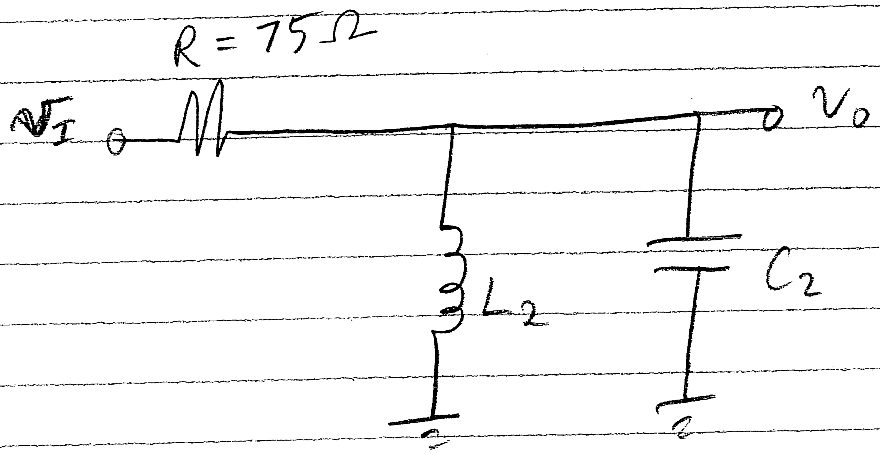
$K_Z = 1000$



$H_2(s) = H_1(s)$

SO BP FILTER WITH $Q = 1$, $\omega_0 = 2\pi \times 10^9$

& $R_{IN} = 75\ \Omega$



$L_2 = 159\ pH \times 75 = 11.9\ nH$

$C_2 = 159\ pF / 75 = 2.12\ pF$

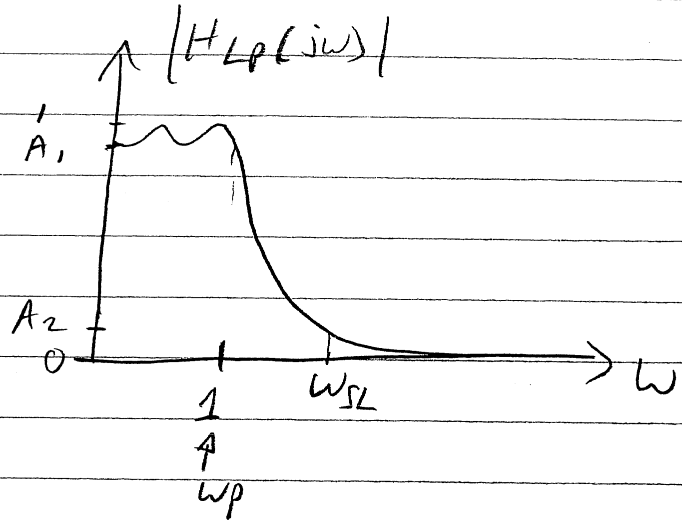
FREQUENCY TRANSFORMATIONS

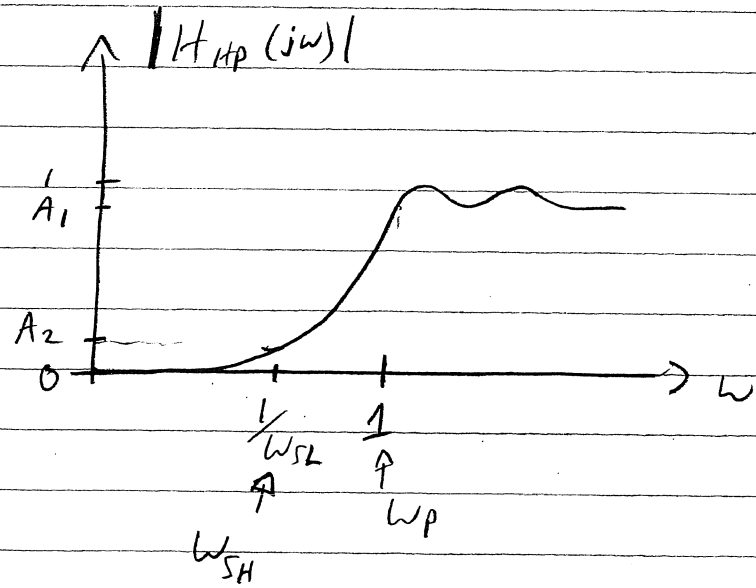
LOWPASS PROTOTYPE HAS $\omega_p = 1$, $H_{LP}(s)$

LOWPASS TO HIGHPASS

MAKE SUBSTITUTION $s \rightarrow \frac{1}{s}$

$$H_{HP}(s) = H_{LP}\left(\frac{1}{s}\right)$$





STOPBAND FREQ

$$\omega_{SH} = \frac{1}{\omega_{SL}}$$

BUTTERWORTH HIGHPASS EXAMPLE

DESIRE PASSBAND EDGE = 1 MHZ
STOPBAND EDGE = 200 KHZ

PASSBAND RIPPLE = 3 dB
STOPBAND ATTEN = 40 dB

FIRST FREQ TRANSFORM TO $W_p = 1$

$$W_p = 1$$

$$W_{SH} = \frac{200k}{1MHZ} = 0.2$$

NOW TRANSFORM TO LOWPASS

$$W_p = 1 \qquad W_{SL} = \frac{1}{0.2} = 5$$

$$A_{MAX} = 3\,dB \qquad A_{MIN} = 40\,dB$$

$$\epsilon = \sqrt{10^{\frac{A_{MAX}}{10}} - 1} = 0.9976$$

AND USE

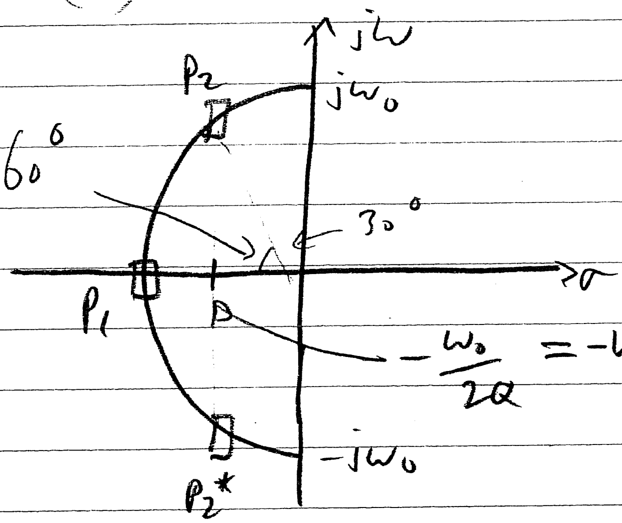
$$A_{MIN} \le 10 \log \left[1 + \epsilon^2 \left(\frac{W_{SL}}{W_p} \right)^{2N} \right]$$

TO FIND **N**

$$40 \le 10 \log\left[1 + (0.9976)^2 (5)^{2N}\right]$$

$$\Rightarrow \quad N \ge 2.86 \quad \Rightarrow \quad N = 3$$

$$w_0 = \left(\frac{1}{\epsilon}\right)^{\frac{1}{3}} = 1.0008$$



$$-\frac{w_0}{2Q} = -w_0 \cos(60) \Rightarrow Q = \frac{1}{2\cos(60)}$$

$$\underline{\underline{Q = 1}}$$

$P_1 = -w_{01}$, $\quad P_2 \Rightarrow w_{02}$ & $Q_2 = 1$

$w_{01} = w_0 \qquad w_{02} = w_0$

$$H_{LP}(s) = \frac{w_0^3}{(s + w_0)\left(s^2 + \frac{w_0}{Q} s + w_0^2\right)}$$

$$H_{HP}(s) = H_{LP}\left(\frac{1}{s}\right)$$

$$H_{HP}(s) = \frac{\omega_0^3}{\left(\frac{1}{s}+\omega_0\right)\left(\frac{1}{s^2}+\left(\frac{\omega_0}{Q}\right)\left(\frac{1}{s}\right)+\omega_0^2\right)}$$

$$= \frac{s^3\omega_0^3}{(1+s\omega_0)\left(1+\frac{\omega_0}{Q}s+s^2\omega_0^2\right)}$$

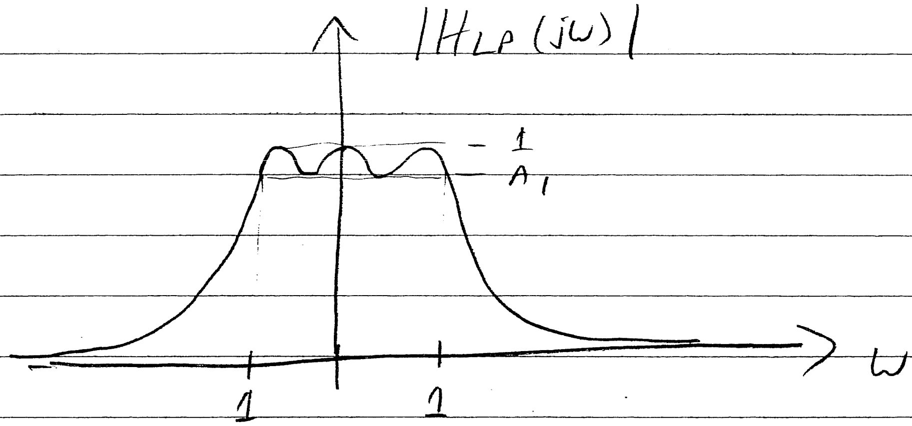
$$H_{HP}(s) = \frac{s^3}{\left(s+\frac{1}{\omega_0}\right)\left(s^2+\frac{1}{\omega_0 Q}s+\frac{1}{\omega_0^2}\right)}$$

FINALLY FREQ SCALE BY $s \to s/(2\pi \times 1e6)$

LOW PASS TO BANDPASS

LET $s \rightarrow \dfrac{s^2+1}{Bs}$

$$H_{BP}(s) = H_{LP}\left(\frac{s^2+1}{Bs}\right)$$



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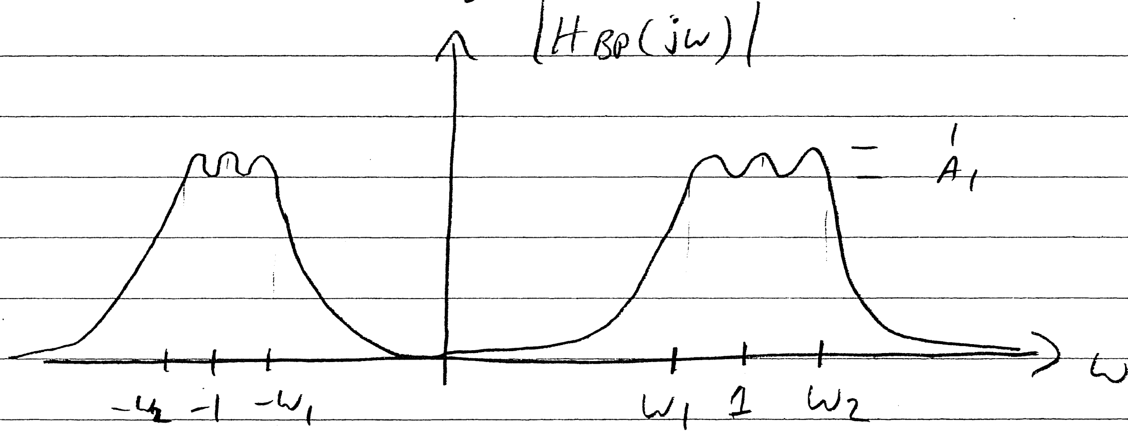

$\omega_2 = \dfrac{1}{\omega_1}$ $\qquad \omega_2 - \omega_1 = B$

CAN SHOW $\omega_1 = -\dfrac{B}{2} + \sqrt{\dfrac{B^2}{4}+1}$

$$\omega_2 = \frac{B}{2} + \sqrt{\frac{B^2}{4}+1}$$