

Switched-Capacitor Circuits



Basic Building Blocks

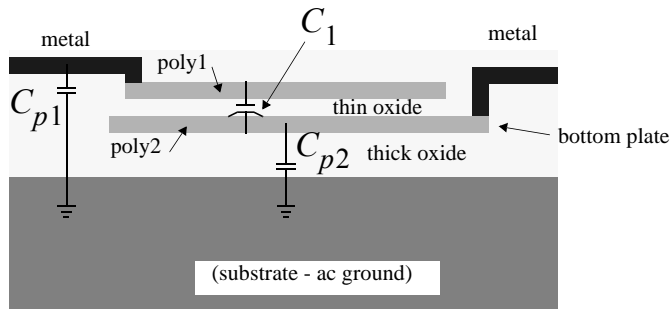
Opamps

- Ideal opamps usually assumed.
- Important non-idealities
 - dc gain: sets the accuracy of charge transfer, hence, transfer-function accuracy.
 - unity-gain freq, phase margin & slew-rate: sets the max clocking frequency. A general rule is that unity-gain freq should be 5 times (or more) higher than the clock-freq.
 - dc offset: Can create dc offset at output. Circuit techniques to combat this which also reduce $1/f$ noise.

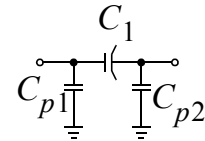


Basic Building Blocks

Double-Poly Capacitors



cross-section view



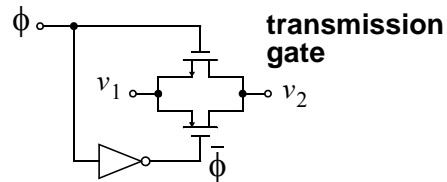
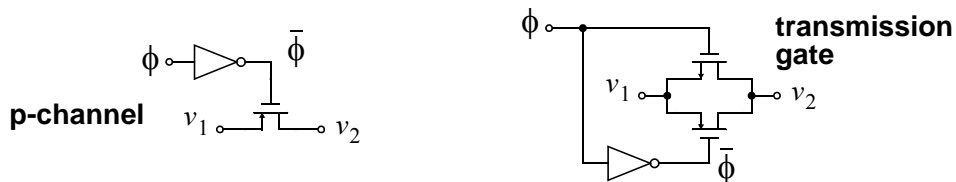
equivalent circuit

- Substantial parasitics with large bottom plate capacitance (20 percent of C_1)
- Also, metal-metal capacitors are used but have even larger parasitic capacitances.



Basic Building Blocks

Switches

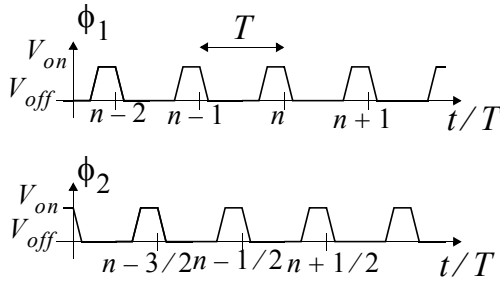


- Mosfet switches are good switches.
 - off-resistance near $G\Omega$ range
 - on-resistance in 100Ω to $5k\Omega$ range (depends on transistor sizing)
- However, have non-linear parasitic capacitances.

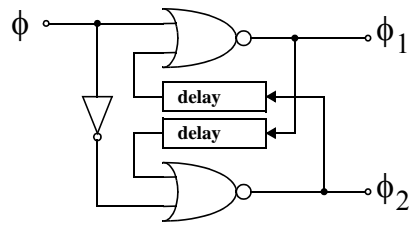


Basic Building Blocks

Non-Overlapping Clocks



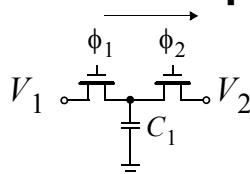
$$f_s \equiv \frac{1}{T}$$



- Non-overlapping clocks — both clocks are never on at same time
- Needed to ensure charge is not inadvertently lost.
- Integer values occur at end of ϕ_1 .
- End of ϕ_2 is 1/2 off integer value.



Switched-Capacitor Resistor Equivalent



$$V_1 \overset{R_{eq}}{\text{---}} V_2$$

$$\Delta Q = C_1(V_1 - V_2) \text{ every clock period}$$

$$R_{eq} = \frac{T}{C_1}$$

$$Q_x = C_x V_x \tag{1}$$

- C_1 charged to V_1 and then V_2 during each clk period.

$$\Delta Q_1 = C_1(V_1 - V_2) \tag{2}$$

- Find equivalent average current

$$I_{avg} = \frac{C_1(V_1 - V_2)}{T} \tag{3}$$

where T is the clk period.



Switched-Capacitor Resistor Equivalent

- For equivalent resistor circuit

$$I_{eq} = \frac{V_1 - V_2}{R_{eq}} \quad (4)$$

- Equating two, we have

$$R_{eq} = \frac{T}{C_1} = \frac{1}{C_1 f_s} \quad (5)$$

- This equivalence is useful when looking at low-freq portion of a SC-circuit.
- For higher frequencies, discrete-time analysis is used.



Resistor Equivalence Example

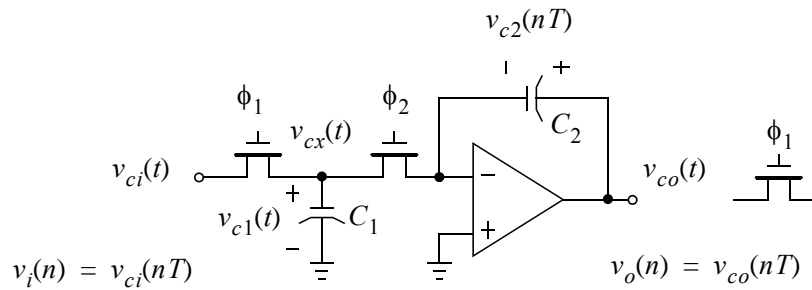
- What is the equivalent resistance of a $5pF$ capacitance sampled at a clock frequency of $100kHz$.
- Using (5), we have

$$R_{eq} = \frac{1}{(5 \times 10^{-12})(100 \times 10^3)} = 2M\Omega$$

- Note that a very large equivalent resistance of $2M\Omega$ can be realized.
- Requires only 2 transistors, a clock and a relatively small capacitance.
- In a typical CMOS process, such a large resistor would normally require a huge amount of silicon area.



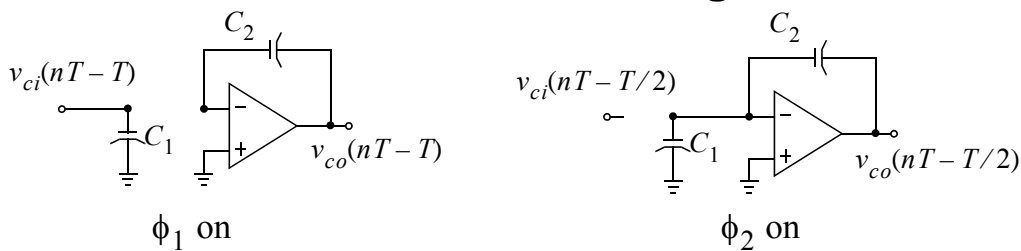
Parasitic-Sensitive Integrator



- Start by looking at an integrator which IS affected by parasitic capacitances
- Want to find output voltage at end of ϕ_1 in relation to input sampled at end of ϕ_1 .



Parasitic-Sensitive Integrator



- At end of ϕ_2

$$C_2 v_{co}(nT - T/2) = C_2 v_{co}(nT - T) - C_1 v_{ci}(nT - T) \quad (6)$$

- But would like to know the output at end of ϕ_1

$$C_2 v_{co}(nT) = C_2 v_{co}(nT - T/2) \quad (7)$$

- leading to

$$C_2 v_{co}(nT) = C_2 v_{co}(nT - T) - C_1 v_{ci}(nT - T) \quad (8)$$



Parasitic-Sensitive Integrator

- Modify above to write

$$v_o(n) = v_o(n-1) - \frac{C_1}{C_2} v_i(n-1) \quad (9)$$

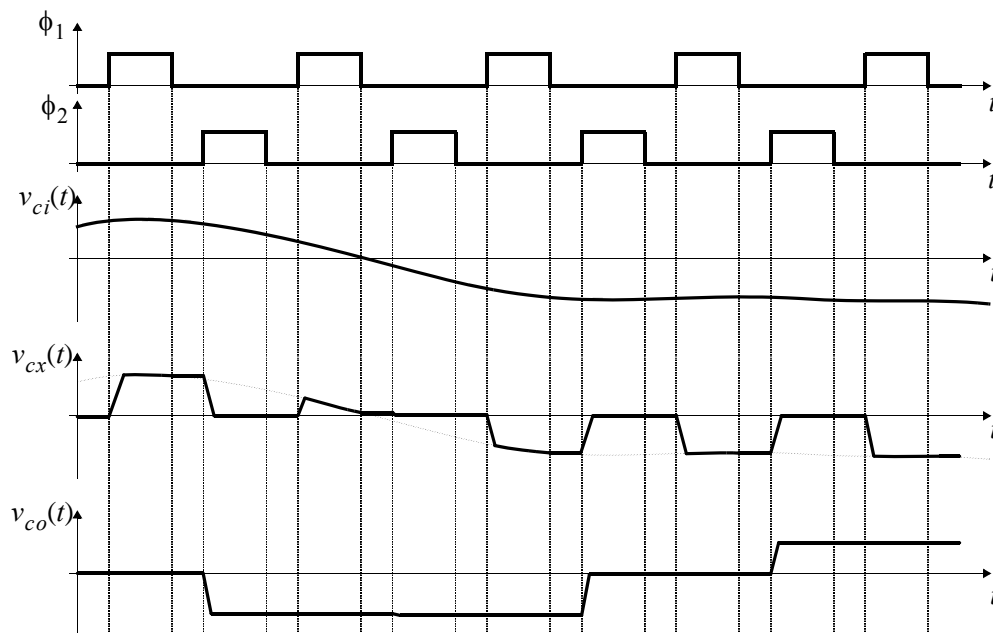
and taking z-transform and re-arranging, leads to

$$H(z) \equiv \frac{V_o(z)}{V_i(z)} = -\left(\frac{C_1}{C_2}\right) \frac{1}{z-1} \quad (10)$$

- Note that gain-coefficient is determined by a ratio of two capacitance values.
- Ratios of capacitors can be set VERY accurately on an integrated circuit (within 0.1 percent)
- Leads to very accurate transfer-functions.



Typical Waveforms



Low Frequency Behavior

- Equation (10) can be re-written as

$$H(z) = -\left(\frac{C_1}{C_2}\right) \frac{z^{-1/2}}{z^{1/2} - z^{-1/2}} \quad (11)$$

- To find freq response, recall

$$z = e^{j\omega T} = \cos(\omega T) + j \sin(\omega T) \quad (12)$$

$$z^{1/2} = \cos\left(\frac{\omega T}{2}\right) + j \sin\left(\frac{\omega T}{2}\right) \quad (13)$$

$$z^{-1/2} = \cos\left(\frac{\omega T}{2}\right) - j \sin\left(\frac{\omega T}{2}\right) \quad (14)$$

$$H(e^{j\omega T}) = -\left(\frac{C_1}{C_2}\right) \frac{\cos\left(\frac{\omega T}{2}\right) - j \sin\left(\frac{\omega T}{2}\right)}{j 2 \sin\left(\frac{\omega T}{2}\right)} \quad (15)$$



Low Frequency Behavior

- Above is exact but when $\omega T \ll 1$ (i.e., at low freq)

$$H(e^{j\omega T}) \cong -\left(\frac{C_1}{C_2}\right) \frac{1}{j\omega T} \quad (16)$$

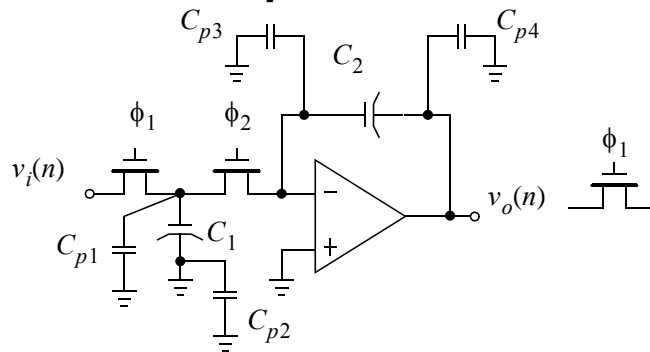
- Thus, the transfer function is same as a continuous-time integrator having a gain constant of

$$K_I \cong \frac{C_1}{C_2 T} \quad (17)$$

which is a function of the integrator capacitor ratio and clock frequency only.



Parasitic Capacitance Effects



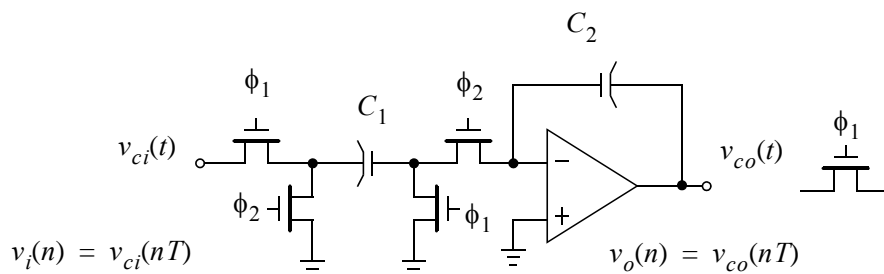
- Accounting for parasitic capacitances, we have

$$H(z) = -\left(\frac{C_1 + C_{p1}}{C_2}\right) \frac{1}{z-1} \quad (18)$$

- Thus, gain coefficient is not well controlled and partially non-linear (due to C_{p1} being non-linear).



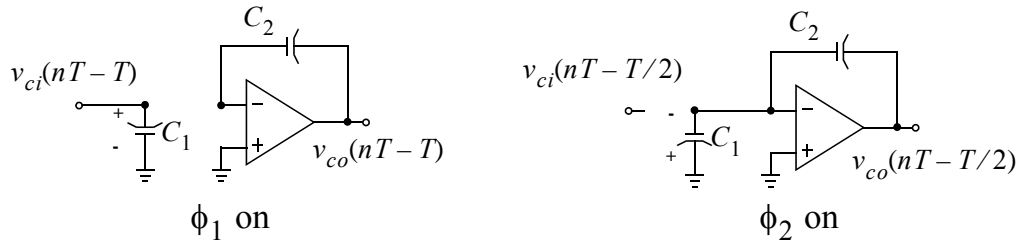
Parasitic-Insensitive Integrators



- By using 2 extra switches, integrator can be made insensitive to parasitic capacitances
 - more accurate transfer-functions
 - better linearity (since non-linear capacitances unimportant)



Parasitic-Insensitive Integrators



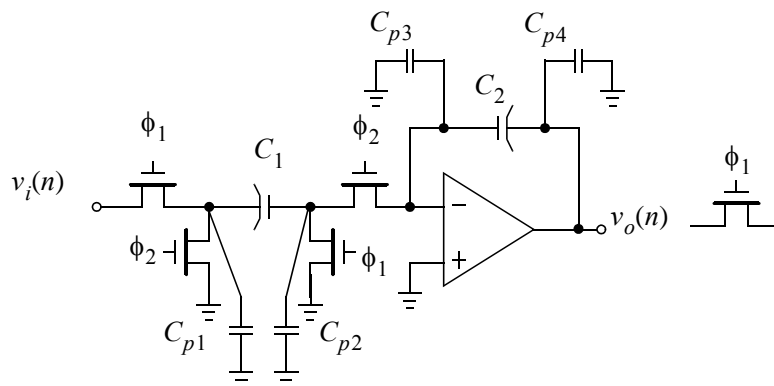
- Same analysis as before except that C_1 is switched in polarity before discharging into C_2 .

$$H(z) \equiv \frac{V_o(z)}{V_i(z)} = \left(\frac{C_1}{C_2} \right) \frac{1}{z-1} \quad (19)$$

- A positive integrator (rather than negative as before)



Parasitic-Insensitive Integrators



- C_{p3} has little effect since it is connected to virtual gnd
- C_{p4} has little effect since it is driven by output
- C_{p2} has little effect since it is either connected to virtual gnd or physical gnd.

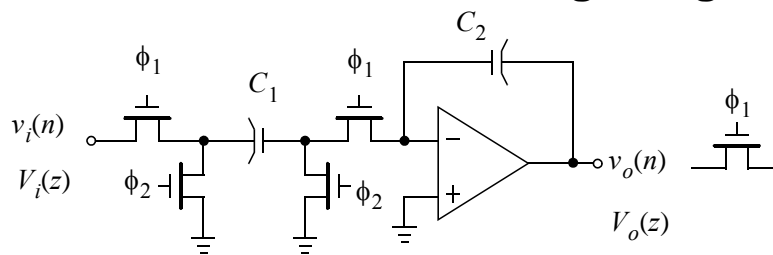


Parasitic-Insensitive Integrators

- C_{p1} is continuously being charged to $v_i(n)$ and discharged to ground.
- ϕ_1 on — the fact that C_{p1} is also charged to $v_i(n-1)$ does not affect C_1 charge.
- ϕ_2 on — C_{p1} is discharged through the ϕ_2 switch attached to its node and does not affect the charge accumulating on C_2 .
- While the parasitic capacitances may slow down settling time behavior, they do not affect the discrete-time difference equation



Parasitic-Insensitive Inverting Integrator



$$C_2 v_{co}(nT - T/2) = C_2 v_{co}(nT - T) \quad (20)$$

$$C_2 v_{co}(nT) = C_2 v_{co}(nT - T/2) - C_1 v_{ci}(nT) \quad (21)$$

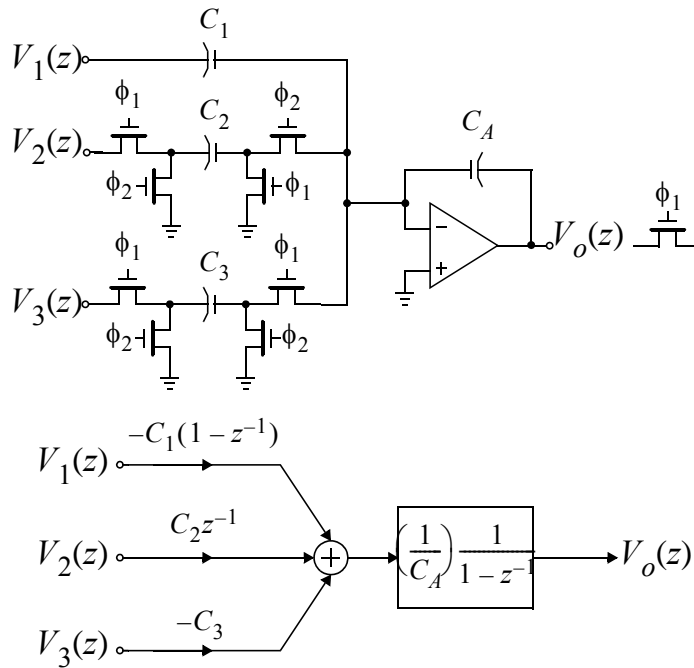
- Present output depends on present input (**delay-free**)

$$H(z) \equiv \frac{V_o(z)}{V_i(z)} = -\left(\frac{C_1}{C_2}\right) \frac{z}{z-1} \quad (22)$$

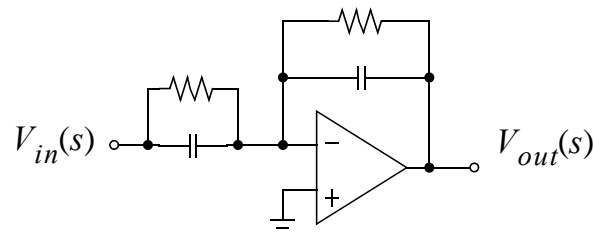
- Delay-free integrator has negative gain while delaying integrator has positive gain.



Signal-Flow-Graph Analysis



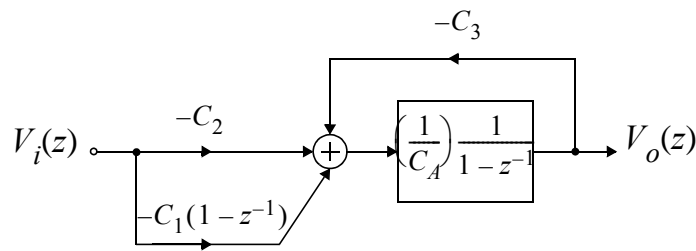
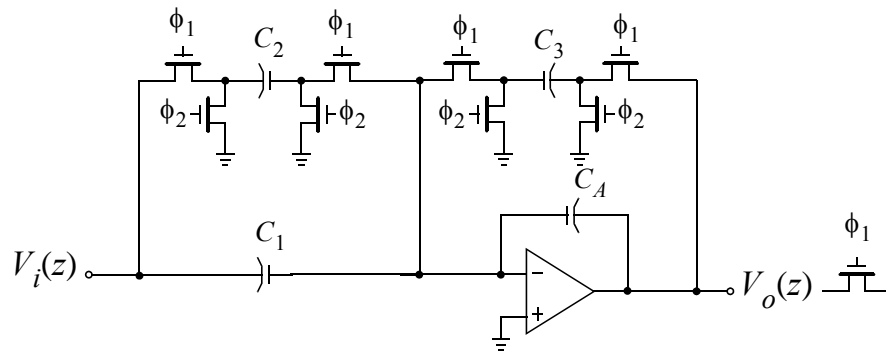
First-Order Filter



- Start with an active-RC structure and replace resistors with SC equivalents.
- Analyze using discrete-time analysis.



First-Order Filter



First-Order Filter

$$C_A(1-z^{-1})V_o(z) = -C_3V_o(z) - C_2V_i(z) - C_1(1-z^{-1})V_i(z) \quad (23)$$

$$H(z) \equiv \frac{V_o(z)}{V_i(z)} = \frac{\left(\frac{C_1}{C_A}\right)(1-z^{-1}) + \left(\frac{C_2}{C_A}\right)}{1-z^{-1} + \frac{C_3}{C_A}} \quad (24)$$

$$= \frac{\left(\frac{C_1 + C_2}{C_A}\right)z - \frac{C_1}{C_A}}{\left(1 + \frac{C_3}{C_A}\right)z - 1}$$



First-Order Filter

- The pole of (24) is found by equating the denominator to zero

$$z_p = \frac{C_A}{C_A + C_3} \quad (25)$$

- For positive capacitance values, this pole is restricted to the real axis between 0 and 1 — circuit is always stable.
- The zero of (24) is found to be given by

$$z_z = \frac{C_1}{C_1 + C_2} \quad (26)$$

- Also restricted to real axis between 0 and 1.



First-Order Filter

The dc gain is found by setting $z = 1$ which results in

$$H(1) = \frac{-C_2}{C_3} \quad (27)$$

- Note that in a fully-differential implementation, effective negative capacitances for C_1 , C_2 and C_3 can be achieved by simply interchanging the input wires.
- In this way, a zero at $z = -1$ could be realized by setting

$$C_1 = -0.5C_2 \quad (28)$$



First-Order Example

- Find the capacitance values needed for a first-order SC-circuit such that its 3dB point is at $10kHz$ when a clock frequency of $100kHz$ is used.
- It is also desired that the filter have zero gain at $50kHz$ (i.e. $z = -1$) and the dc gain be unity.
- Assume $C_A = 10pF$.

Solution

- Making use of the bilinear transform
 $p = (z - 1)/(z + 1)$ the zero at -1 is mapped to $\Omega = \infty$.
- The frequency warping maps the $-3dB$ frequency of $10kHz$ (or 0.2π rad/sample) to



First-Order Example

$$\Omega = \tan\left(\frac{0.2\pi}{2}\right) = 0.3249 \quad (29)$$

- in the continuous-time domain leading to the continuous-time pole, p_p , required being

$$p_p = -0.3249 \quad (30)$$

- This pole is mapped back to z_p given by

$$z_p = \frac{1 + p_p}{1 - p_p} = 0.5095 \quad (31)$$

- Therefore, $H(z)$ is given by

$$H(z) = \frac{k(z + 1)}{z - 0.5095} \quad (32)$$



First-Order Example

- where k is determined by setting the dc gain to one (i.e. $H(1) = 1$) resulting

$$H(z) = \frac{0.24525(z+1)}{z-0.5095} \quad (33)$$

- or equivalently,

$$H(z) = \frac{0.4814z + 0.4814}{1.9627z - 1} \quad (34)$$

- Equating these coefficients with those of (24) (and assuming $C_A = 10pF$) results in

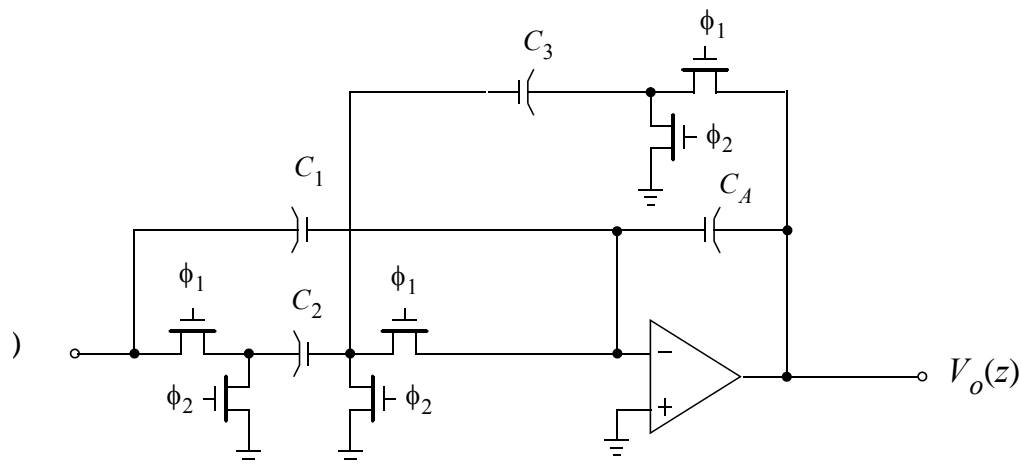
$$C_1 = 4.814pF \quad (35)$$

$$C_2 = -9.628pF \quad (36)$$

$$C_3 = 9.628pF \quad (37)$$



Switch Sharing

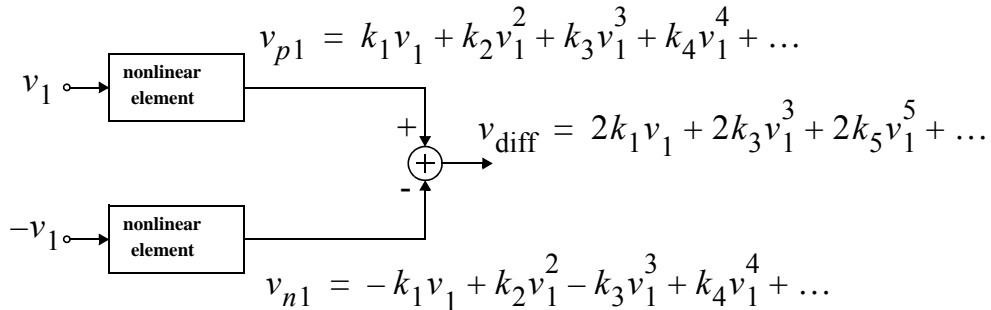


- Share switches that are always connected to the same potentials.

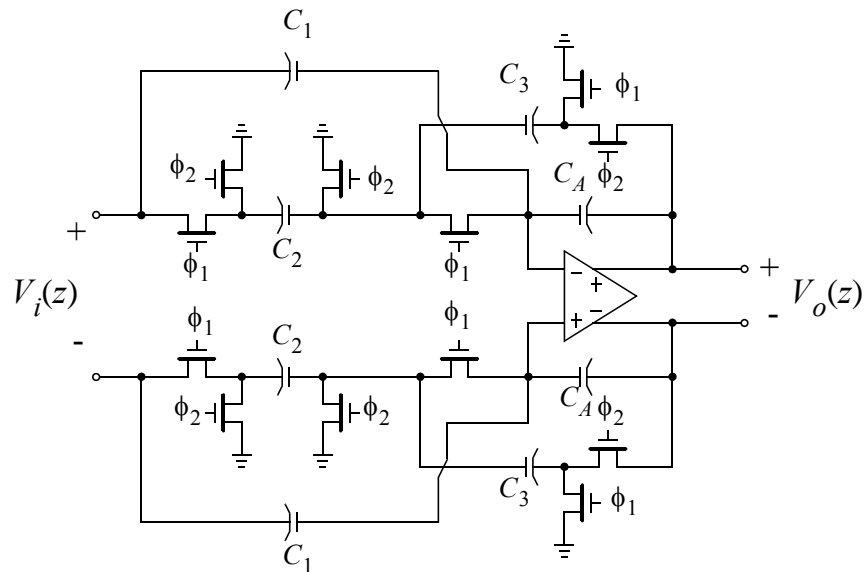


Fully-Differential Filters

- Most modern SC filters are fully-differential
- Difference between two voltages represents signal (also balanced around a common-mode voltage).
- Common-mode noise is rejected.
- Even order distortion terms cancel

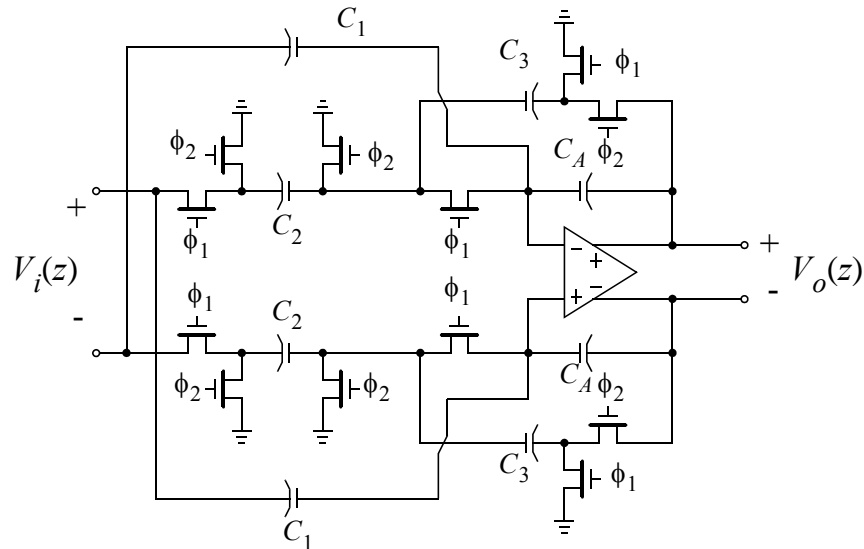


Fully-Differential Filters



Fully-Differential Filters

- Negative continuous-time input
— equivalent to a negative C_1



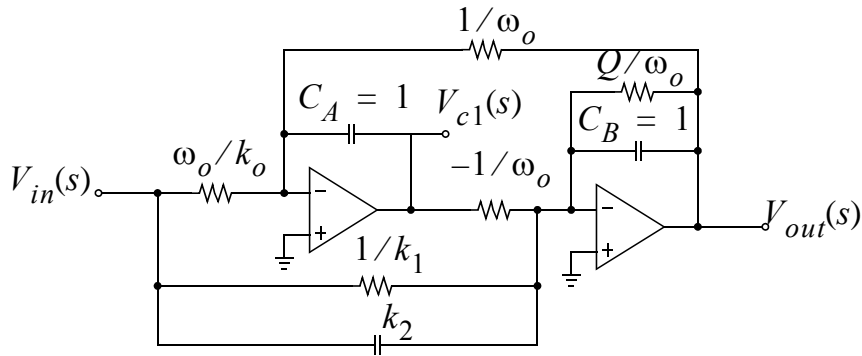
Fully-Differential Filters

- Note that fully-differential version is essentially two copies of single-ended version, however ... area penalty not twice.
- Only one opamp needed (though common-mode circuit also needed)
- Input and output signal swings have been doubled so that same dynamic range can be achieved with half capacitor sizes (from kT/C analysis)
- Switches can be reduced in size since small caps used.
- However, there is more wiring in fully-differ version but better noise and distortion performance.

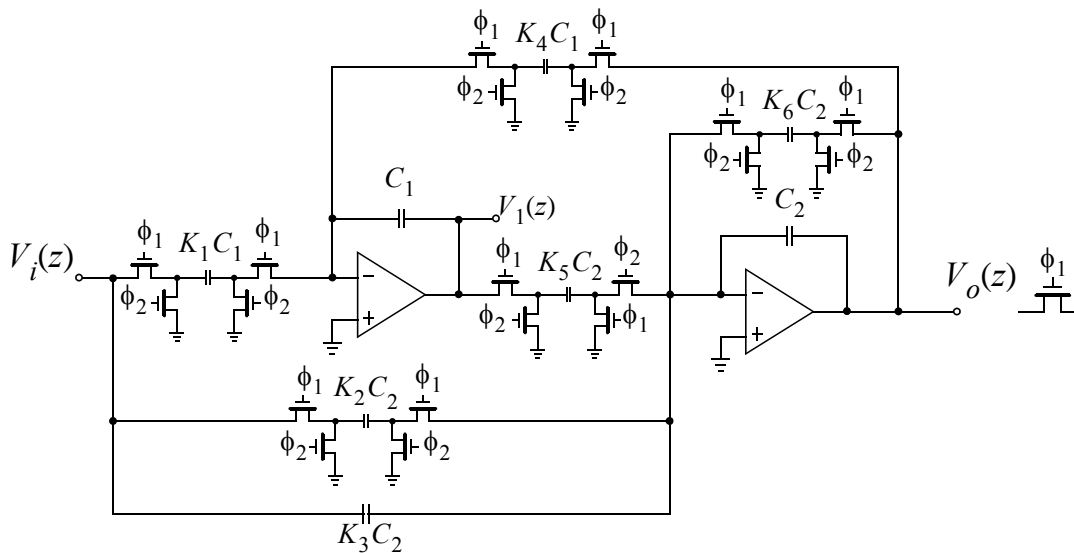


Low-Q Biquad Filter

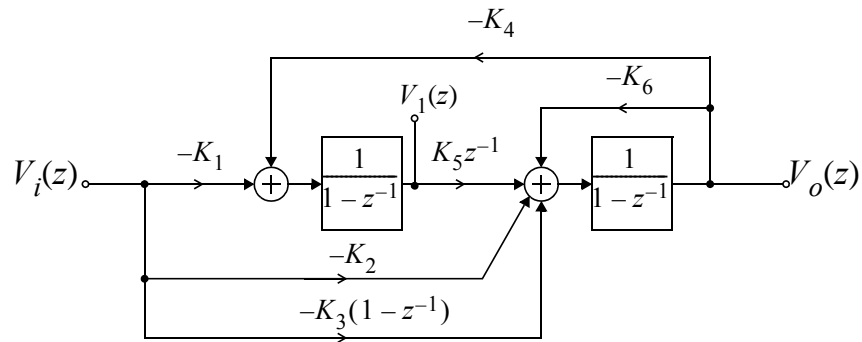
$$H_a(s) \equiv \frac{V_{out}(s)}{V_{in}(s)} = -\frac{k_2 s^2 + k_1 s + k_o}{s^2 + \left(\frac{\omega_o}{Q}\right)s + \omega_o^2} \quad (38)$$



Low-Q Biquad Filter



Low-Q Biquad Filter



$$H(z) \equiv \frac{V_o(z)}{V_i(z)} = \frac{(K_2 + K_3)z^2 + (K_1K_5 - K_2 - 2K_3)z + K_3}{(1 + K_6)z^2 + (K_4K_5 - K_6 - 2)z + 1} \quad (39)$$



Low-Q Biquad Filter Design

$$H(z) = \frac{a_2z^2 + a_1z + a_0}{b_2z^2 + b_1z + 1} \quad (40)$$

- we can equate the individual coefficients of “z” in (39) and (40), resulting in:

$$K_3 = a_0 \quad (41)$$

$$K_2 = a_2 - a_0 \quad (42)$$

$$K_1K_5 = a_0 + a_1 + a_2 \quad (43)$$

$$K_6 = b_2 - 1 \quad (44)$$

$$K_4K_5 = b_1 + b_2 + 1 \quad (45)$$

- A degree of freedom is available here in setting internal $V_1(z)$ output



Low-Q Biquad Filter Design

- Can do proper dynamic range scaling
- Or let the time-constants of 2 integrators be equal by

$$K_4 = K_5 = \sqrt{b_1 + b_2 + 1} \quad (46)$$

Low-Q Biquad Capacitance Ratio

- Comparing resistor circuit to SC circuit, we have

$$K_4 \approx K_5 \approx \omega_o T \quad (47)$$

$$K_6 \approx \frac{\omega_o T}{Q} \quad (48)$$

- However, the sampling-rate, $1/T$, is typically much larger than the approximated pole-frequency, ω_o ,

$$\omega_o T \ll 1 \quad (49)$$



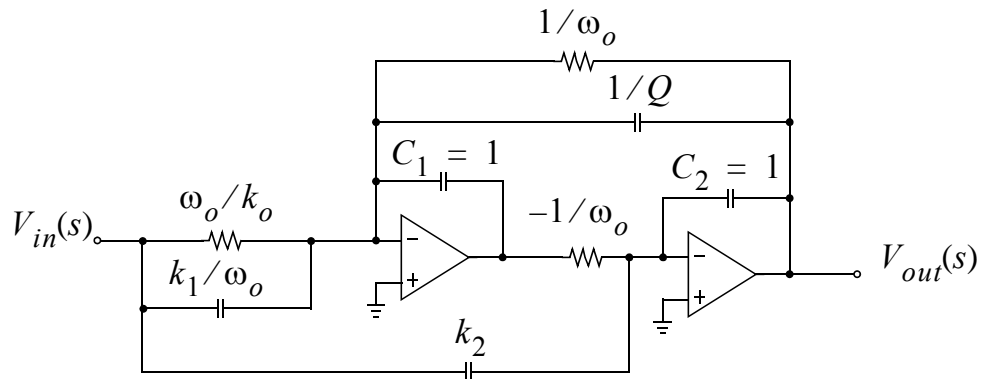
Low-Q Biquad Capacitance Ratio

- Thus, the largest capacitors determining pole positions are the integrating capacitors, C_1 and C_2 .
- If $Q < 1$, the smallest capacitors are $K_4 C_1$ and $K_5 C_2$ resulting in an approximate capacitance spread of $1/(\omega_o T)$.
- If $Q > 1$, then from (48) the smallest capacitor would be $K_6 C_2$ resulting in an approximate capacitance spread of $Q/(\omega_o T)$ — can be quite large for $Q \gg 1$

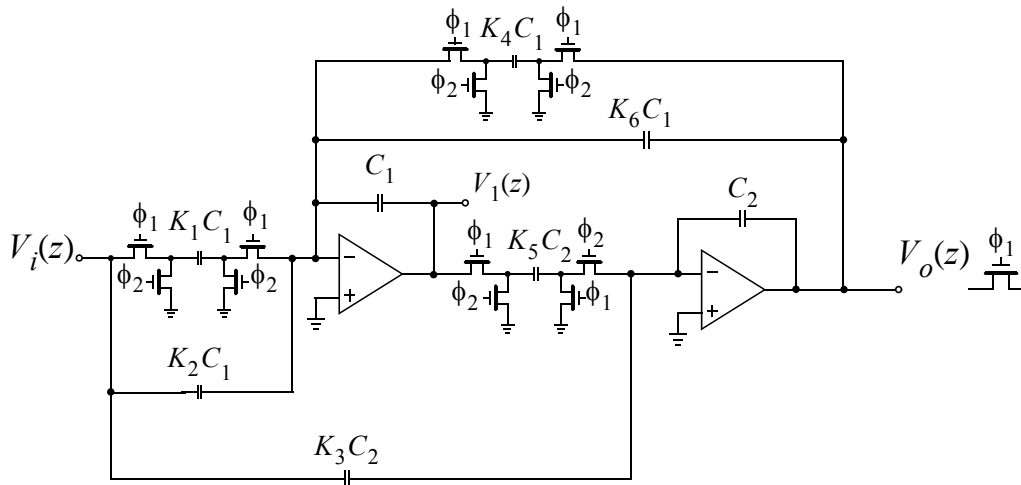


High-Q Biquad Filter

- Use a high-Q biquad filter circuit when $Q \gg 1$
- Q-damping done with a cap around both integrators
- Active-RC prototype filter



High-Q Biquad Filter



- Q-damping now performed by K_6C_1



High-Q Biquad Filter

- Input $K_1 C_1$: major path for lowpass
- Input $K_2 C_1$: major path for band-pass filters
- Input $K_3 C_2$: major path for high-pass filters
- General transfer-function is:

$$H(z) \equiv \frac{V_o(z)}{V_i(z)} = -\frac{K_3 z^2 + (K_1 K_5 + K_2 K_5 - 2K_3)z + (K_3 - K_2 K_5)}{z^2 + (K_4 K_5 + K_5 K_6 - 2)z + (1 - K_5 K_6)} \quad (50)$$

- If matched to the following general form

$$H(z) = -\frac{a_2 z^2 + a_1 z + a_0}{z^2 + b_1 z + b_0} \quad (51)$$



High-Q Biquad Filter

$$K_1 K_5 = a_0 + a_1 + a_2 \quad (52)$$

$$K_2 K_5 = a_2 - a_0 \quad (53)$$

$$K_3 = a_2 \quad (54)$$

$$K_4 K_5 = 1 + b_0 + b_1 \quad (55)$$

$$K_5 K_6 = 1 - b_0 \quad (56)$$

- And, as in lowpass case, can set

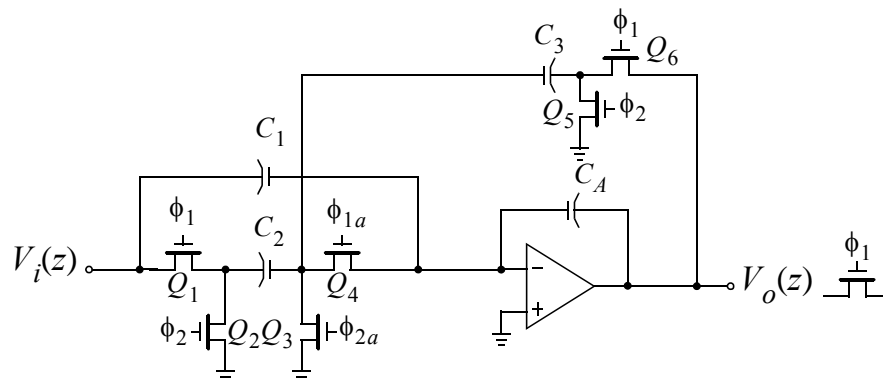
$$K_4 = K_5 = \sqrt{1 + b_0 + b_1} \quad (57)$$

- As before, K_4 and K_5 approx $\omega_o T \ll 1$ but $K_6 \cong 1/Q$



Charge Injection

- To reduce charge injection (thereby improving distortion) , turn off certain switches first.



- Advance ϕ_{1a} and ϕ_{2a} so that only their charge injection affect circuit (result is a dc offset)



Charge Injection

- Note: ϕ_{2a} connected to ground while ϕ_{1a} connected to virtual ground, therefore ...
 - can use single n-channel transistors
 - charge injection NOT signal dependent

$$Q_{CH} = -WLC_{ox}V_{eff} = -WLC_{ox}(V_{GS} - V_t) \quad (58)$$

- Charge related to V_{GS} and V_t and V_t related to substrate-source voltage.
- Source of Q_3 and Q_4 remains at 0 volts — **amount of charge injected by Q_3, Q_4 is not signal dependent and can be considered as a dc offset.**



Charge Injection Example

- Estimate dc offset due to channel-charge injection when $C_1 = 0$ and $C_2 = C_A = 10C_3 = 10pF$.
- Assume switches Q_3, Q_4 have $V_{tn} = 0.8V$, $W = 30\mu m$, $L = 0.8\mu m$, $C_{ox} = 1.9 \times 10^{-3} pF/\mu m^2$, and power supplies are $\pm 2.5V$.
- Channel-charge of Q_3, Q_4 (when on) is

$$\begin{aligned} Q_{CH3} = Q_{CH4} &= -(30)(0.8)(0.0019)(2.5 - 0.8) \quad (59) \\ &= -77.5 \times 10^{-3} pC \end{aligned}$$

- dc feedback keeps virtual opamp input at zero volts.



Charge Injection Example

- Charge transfer into C_3 given by

$$Q_{C_3} = -C_3 v_{out} \quad (60)$$

- We estimate half channel-charges of Q_3, Q_4 are injected to the virtual ground leading to

$$\frac{1}{2}(Q_{CH3} + Q_{CH4}) = Q_{C_3} \quad (61)$$

which leads to

$$v_{out} = \frac{77.5 \times 10^{-3} pC}{1pF} = 78 mV \quad (62)$$

- dc offset affected by the capacitor sizes, switch sizes and power supply voltage.

