**Last Name:** ________________

# ECE512 Analog Signal Processing Equation Sheet

**Constants:** $k = 1.38 \times 10^{-23}\ \mathrm{JK}^{-1}$ ; $q = 1.602 \times 10^{-19}$ C ; $\varepsilon_0 = 8.854 \times 10^{-12}$ F/m ; $V_T = kT/q \approx 26$ mV at 300 °K ;

**General:** $h(t)$ is impulse response of LTI system; $H(s)$ is the Laplace transform of $h(t)$

$H(j\omega) = |H(j\omega)|e^{j\phi(\omega)}$; $T_d(\omega) = -\dfrac{d\phi(\omega)}{d\omega}$; $|H(j\omega)|_{\mathrm{dB}} = 20\log|H(j\omega)|$; $H(s) = \dfrac{a_m s^m + \ldots + a_0}{s^N + b_{n-1}s + \ldots + b_0}$; $|H(j\omega)|^2 = H(s)H(-s)|_{s=j\omega} = H(j\omega)H(-j\omega)$ ;

**General Lowpass:** $|H(j\omega)|^2 = A_0^2/(1 + F(\omega^2))$ ;

**Butterworth:** $F(\omega^2) = \varepsilon^2\left(\dfrac{\omega}{\omega_p}\right)^{2N}$ ; $A_{\max} = 20\log\sqrt{1 + \varepsilon^2}$ ; $A_{\min} \le 10\log\left[1 + \varepsilon^2\left(\dfrac{\omega}{\omega_p}\right)^{2N}\right]$ ; Poles lie on circle of radius $\omega_p(1/\varepsilon)^{1/N}$ spaced apart by $\pi/N$ with first angle being $\pi/(2N)$

**Chebyshev:** (for $\omega_p = 1$); $F(\omega^2) = \varepsilon^2 C_N^2(\omega)$; $C_N(\omega) = \cos(N\cos^{-1}(\omega))$ $|\omega| \le 1$; $C_N(\omega) = \cosh(N\cosh^{-1}(\omega))$ $|\omega| \ge 1$;

$C_{N+1}(\omega) = 2\omega C_N(\omega) - C_{N-1}(\omega)$; $A_{\max} = 20\log\sqrt{1 + \varepsilon^2}$ ; $A_{\min} \le 10\log[1 + \varepsilon^2\cosh^2(N\cosh^{-1}(\omega_s/\omega_p))]$ ;

**Second-order polynomial:** $s^2 + (\omega_0/Q)s + \omega_0^2$ ; for $Q > 0.5$; poles complex at radius $\omega_0$ and real part is $-\omega_0/(2Q)$;

**Lowpass and highpass:** peaking occurs if $Q > 1/\sqrt{2}$ and $\omega_{\max} = \omega_0\sqrt{1 - 1/2Q^2}$

**Bandpass:** for complex poles; peak occurs at $\omega_0$ and has 3 dB bandwidth of $\omega_0/Q$

**LCR:** $\omega_0 = 1/\sqrt{LC}$ ; $Q = \omega_0 CR$;

**KHN Biquad:** $RC = 1/\omega_0$ ; $2((R_1 \| R_2)/((R_1 \| R_2) + R_3)) = 1/Q$ ; $2((R_2 \| R_3)/((R_2 \| R_3) + R_1)) = k$ ;

**Tow-Thomas Biquad:** $RC = 1/\omega_0$ ; damping resistor is $QR$ ; numerator is $-s^2\left(\dfrac{C_1}{C}\right) - s\left(\dfrac{1}{C}\right)\left(\dfrac{1}{R_1} - \dfrac{r}{RR_3}\right) - \dfrac{1}{RR_2C^2}$ ;

**Noise:** noise equivalent bandwidth = $(\pi/2)f_{3\mathrm{dB}}$ ; $V_R^2(f) = 4kTR$; $I_d^2(f) = 2qI_D$; $r_d = (kT)/(qI_D)$ ; $V_C^2 = (kT)/C$ ;

**Discrete-Time:** $X_s(s) = \sum x_c(nT)e^{-snT}$; $X(z) = \sum x_c(nT)z^{-n}$; $p = (z-1)/(z+1)$ ; $z = (1+p)/(1-p)$ ; $\Omega = \tan(\omega/2)$ ;

**Switched-Cap:** $R_{eq} = T/C$ ; $Q_{\mathrm{CH}} = -WLC_{\mathrm{ox}}(V_{\mathrm{GS}} - V_t)$ ;

**Data Converters:** $B_{\mathrm{in}} = b_1 2^{-1} + \ldots + b_N 2^{-N}$; $V_{\mathrm{LSB}} = V_{\mathrm{ref}}/2^N$; $V_{\mathrm{out}} = V_{\mathrm{ref}}B_{\mathrm{in}}$; $V_{\mathrm{ref}}B_{\mathrm{out}} = V_{\mathrm{in}} + V_Q$; $|V_{\mathrm{Q}}| \le 0.5V_{\mathrm{LSB}}$;

$V_{\mathrm{Q(rms)}} = V_{\mathrm{LSB}}/\sqrt{12}$ ; SNR $= 6.02N + 1.76$ ; $E_{\mathrm{off(D/A)}} = V_{\mathrm{out}}/V_{\mathrm{LSB}}|_{0\ldots0}$ ; $E_{\mathrm{off(A/D)}} = V_{0\ldots01}/V_{\mathrm{LSB}} - 0.5\mathrm{LSB}$ ; $\Delta t < 1/(2^N\pi f_{\mathrm{in}})$ ;

$E_{\mathrm{gain(D/A)}} = (V_{\mathrm{out}}/V_{\mathrm{LSB}}|_{1\ldots1} - V_{\mathrm{out}}/V_{\mathrm{LSB}}|_{0\ldots0}) - (2^N - 1)$ ; $E_{\mathrm{gain(A/D)}} = (V_{1\ldots1}/V_{\mathrm{LSB}} - V_{0\ldots01}/V_{\mathrm{LSB}}) - (2^N - 2)$ ;

**Oversampling:** OSR $= f_s/(2f_0)$ ; $\mathrm{SNR}_0 = 6.02N + 1.76 + 10\log(\mathrm{OSR})$ ; $\mathrm{SNR}_1 = 6.02N + 1.76 - 5.17 + 30\log(\mathrm{OSR})$ ;

$\mathrm{SNR}_2 = 6.02N + 1.76 - 12.9 + 50\log(\mathrm{OSR})$ ; $S_{\mathrm{TF}}(z) = H(z)/(1 + H(z))$ ; $N_{\mathrm{TF}}(z) = 1/(1 + H(z))$

$T_{\mathrm{avg}}(z) = \dfrac{1}{M}\sum_{i=0}^{M-1} z^{-i} = \dfrac{1}{M}\left(\dfrac{1 - z^{-M}}{1 - z^{-1}}\right)$ ; $T_{\mathrm{avg}}(e^{j\omega}) = \dfrac{\mathrm{sinc}((\omega M)/2)}{\mathrm{sinc}(\omega/2)}$ ; $|N_{\mathrm{TF}}(e^{j\omega})| \le 1.5$ for 1-bit quantizer stability;

**Bipolar transistors:** $I_C = I_{\mathrm{CS}}e^{V_{\mathrm{BE}}/V_T}$ ; $g_m = I_C/V_T$ ; $r_e = \alpha/g_m = V_T/I_E$ ;

**CMOS transistors:** $K_n = 0.5\mu_n C_{\mathrm{ox}}(W/L)$ ; $I_D = 2K_n((V_{\mathrm{GS}} - V_{tn})V_{\mathrm{DS}} - (V_{\mathrm{DS}}^2/2))$ ; $r_{\mathrm{ds}} = (2K_n(V_{\mathrm{GS}} - V_{tn}))^{-1}$ ;

$I_D = K_n(V_{\mathrm{GS}} - V_{tn})^2$ ; $g_m = 2K_n(V_{\mathrm{GS}} - V_{tn}) = (2I_D)/(V_{\mathrm{GS}} - V_{tn})$ ; $r_s = 1/g_m$ ;

**Ideal Transcondcutor:** $i_o = G_m v_i$ ; **Bipolar Diff Pair:** $I_{C2} = I_1/(1 + e^{v_i/V_T})$ ;

**CMOS Pair:** $K_{\mathrm{eq}} = (K_nK_p)/(\sqrt{K_n} + \sqrt{K_p})^2$ ; $V_{\mathrm{t\text{-}eq}} = V_{\mathrm{tn}} - V_{\mathrm{tp}}$ ;

**Dynamic Range:** (all in dB or dBm) $ID_3 = I_{D3} - I_{D1}$ ; $\mathrm{OIP}_3 = I_{D1} - ID_3/2$ ; SFDR $= (2/3)(\mathrm{OIP}_3 - N_o)$ ;

THD $= 10\log((V_{h2}^2 + V_{h3}^2 + \ldots)/V_f^2)$ ;