ECE512 Analog Signal Processing Equation Sheet

$h(t)$ is impulse response of LTI system; $H(s)$ is Laplace transform of $h(t)$.

$H(j\omega) = |H(j\omega)|e^{j\phi(\omega)}$; $T_d(\omega) = -\dfrac{d\phi(\omega)}{d\omega}$; $|H(j\omega)|_{\mathrm{dB}} = 20\log|H(j\omega)|$; $H(s) = \dfrac{a_m s^m + \ldots + a_0}{s^N + b_{\mathrm{n-1}}s + \ldots + b_0}$

$|H(j\omega)|^2 = H(s)H(-s)|_{s=j\omega} = H(j\omega)H(-j\omega)$; General Lowpass $|H(j\omega)|^2 = \dfrac{A_0^2}{1 + F(\omega^2)}$;

Butterworth: $F(\omega^2) = \varepsilon^2\left(\dfrac{\omega}{\omega_p}\right)^{2\mathrm{N}}$; $A_{\max} = 20\log\sqrt{1+\varepsilon^2}$; $A_{\min} \le 10\log\left[1 + \varepsilon^2\left(\dfrac{\omega}{\omega_p}\right)^{2\mathrm{N}}\right]$;

Poles lie on circle of radius $\omega_p\left(\dfrac{1}{\varepsilon}\right)^{\frac{1}{N}}$ spaced apart by $\dfrac{\pi}{N}$ with first half angle from $j\omega$ axis

Chebyshev: (for $\omega_p = 1$) $F(\omega^2) = \varepsilon^2 C_N^2(\omega)$; $C_N(\omega) = \cos(N\cos^{-1}(\omega))$ $|\omega| \le 1$

$C_N(\omega) = \cosh(N\cosh^{-1}(\omega))$ $|\omega| \ge 1$; $C_{\mathrm{N+1}}(\omega) = 2\omega C_{\mathrm{N}}(\omega) - C_{\mathrm{N-1}}(\omega)$;

$A_{\max} = 20\log\sqrt{1+\varepsilon^2}$; $A_{\min} \le 10\log[1 + \varepsilon^2\cosh^2(N\cosh^{-1}(\omega_s/\omega_p))]$

Second-order polynomial: $s^2 + \left(\dfrac{\omega_0}{Q}\right)s + \omega_0^2$; for $Q > 0.5$, poles complex at radius ω_0 and real part is $-\dfrac{\omega_0}{2Q}$

Lowpass and highpass: peaking occurs if $Q > 1/\sqrt{2}$ and $\omega_{\max} = \omega_0\sqrt{1 - 1/(2Q^2)}$

Bandpass: for complex poles, peak occurs at ω_0 and has 3dB bandwidth of ω_0/Q

LCR: $\omega_0 = 1/\sqrt{LC}$; $Q = \omega_0 CR$

KHN Biquad: $RC = \dfrac{1}{\omega_0}$; $2\left(\dfrac{R_1 \parallel R_2}{(R_1 \parallel R_2) + R_3}\right) = \dfrac{1}{Q}$; $2\left(\dfrac{R_2 \parallel R_3}{(R_2 \parallel R_3) + R_1}\right) = k$

Tow-Thomas Biquad: $RC = \dfrac{1}{\omega_0}$; damping resistor is QR; numerator is $-s^2\left(\dfrac{C_1}{C}\right) + s\left(\dfrac{1}{C}\right)\left(\dfrac{1}{R_1} - \dfrac{r}{RR_3}\right) + \dfrac{1}{RR_2C^2}$