**Last Name:** ________________

# ECE512 Analog Signal Processing Equation Sheet

**Constants:** $k = 1.38 \times 10^{-23}\ \mathrm{JK}^{-1}$ ; $q = 1.602 \times 10^{-19}\ \mathrm{C}$ ; $\varepsilon_0 = 8.854 \times 10^{-12}\ \mathrm{F/m}$ ; $V_T = kT/q \approx 26\mathrm{mV}$ at 300 °K ;
$h(t)$ is impulse response of LTI system; $H(s)$ is the Laplace transform of $h(t)$ .

$H(j\omega) = |H(j\omega)|e^{j\phi(\omega)}$ ; $T_d(\omega) = -\dfrac{d\phi(\omega)}{d\omega}$ ; $|H(j\omega)|_{\mathrm{dB}} = 20\log|H(j\omega)|$ ; $H(s) = \dfrac{a_m s^m + \ldots + a_0}{s^N + b_{n-1}s + \ldots + b_0}$ ;

$|H(j\omega)|^2 = H(s)H(-s)|_{s=j\omega} = H(j\omega)H(-j\omega)$ ; General Lowpass $|H(j\omega)|^2 = A_0^2/(1 + F(\omega^2))$ ;

**Butterworth:** $F(\omega^2) = \varepsilon^2\left(\dfrac{\omega}{\omega_p}\right)^{2N}$ ; $A_{\max} = 20\log\sqrt{1+\varepsilon^2}$ ; $A_{\min} \le 10\log\left[1 + \varepsilon^2\left(\dfrac{\omega}{\omega_p}\right)^{2N}\right]$ ;

Poles lie on circle of radius $\omega_p(1/\varepsilon)^{1/N}$ spaced apart by $\pi/N$ with first half angle from $j\omega$ axis

**Chebyshev:** (for $\omega_p = 1$ ) $F(\omega^2) = \varepsilon^2 C_N^2(\omega)$ ; $C_N(\omega) = \cos(N\cos^{-1}(\omega))\ |\omega| \le 1$ $C_N(\omega) = \cosh(N\cosh^{-1}(\omega))\ |\omega| \ge 1$ ;

$C_{N+1}(\omega) = 2\omega C_N(\omega) - C_{N-1}(\omega)$ ; $A_{\max} = 20\log\sqrt{1+\varepsilon^2}$ ; $A_{\min} \le 10\log[1 + \varepsilon^2\cosh^2(N\cosh^{-1}(\omega_s/\omega_p))]$

**Second-order polynomial:** $s^2 + (\omega_0/Q)s + \omega_0^2$ ; for $Q > 0.5$ ; poles complex at radius $\omega_0$ and real part is $-\omega_0/(2Q)$

**Lowpass and highpass:** peaking occurs if $Q > 1/\sqrt{2}$ and $\omega_{\max} = \omega_0\sqrt{1 - 1/(2Q^2)}$

**Bandpass:** for complex poles; peak occurs at $\omega_0$ and has 3dB bandwidth of $\omega_0/Q$

**LCR:** $\omega_0 = 1/\sqrt{LC}$ ; $Q = \omega_0 CR$

**KHN Biquad:** $RC = 1/\omega_0$ ; $2((R_1 \| R_2)/((R_1 \| R_2) + R_3)) = 1/Q$ ; $2((R_2 \| R_3)/((R_2 \| R_3) + R_1)) = k$

**Tow-Thomas Biquad:** $RC = 1/\omega_0$ ; damping resistor is $QR$ ; numerator is $-s^2\left(\dfrac{C_1}{C}\right) - s\left(\dfrac{1}{C}\right)\left(\dfrac{1}{R_1} - \dfrac{r}{RR_3}\right) - \dfrac{1}{RR_2C^2}$

**Noise:** Noise equivalent bandwidth = $(\pi/2)f_{3\mathrm{dB}}$ ; $V_R^2(f) = 4kTR$ ; $I_d^2(f) = 2qI_D$ ; $r_d = (kT)/(qI_D)$ ; $V_C^2 = (kT)/C$

**Discrete-Time:** $X_s(s) = \sum x_c(nT)e^{-snT}$ ; $X(z) = \sum x_c(nT)z^{-n}$ ; $p = (z-1)/(z+1)$ ; $z = (1+p)/(1-p)$ ; $\Omega = \tan(\omega/2)$

**Switched-Cap:** $R_{eq} = T/C$ ; $Q_{\mathrm{CH}} = -WLC_{\mathrm{ox}}(V_{\mathrm{GS}} - V_t)$

**Data Converters:** $B_{\mathrm{in}} = b_1 2^{-1} + \ldots + b_N 2^{-N}$ ; $V_{\mathrm{LSB}} = V_{\mathrm{ref}}/2^N$ ; $V_{\mathrm{out}} = V_{\mathrm{ref}}B_{\mathrm{in}}$ ; $V_{\mathrm{ref}}B_{\mathrm{out}} = V_{\mathrm{in}} + V_Q$ ; $|V_{\mathrm{Q}}| \le 0.5V_{\mathrm{LSB}}$ ;

$V_{\mathrm{Q(rms)}} = V_{\mathrm{LSB}}/\sqrt{12}$ ; $\mathrm{SNR} = 6.02N + 1.76$ ; $E_{\mathrm{off(D/A)}} = V_{\mathrm{out}}/V_{\mathrm{LSB}}|_{0\ldots0}$ ; $E_{\mathrm{off(A/D)}} = V_{0\ldots01}/V_{\mathrm{LSB}} - 0.5\mathrm{LSB}$ ; $\Delta t < 1/(2^N\pi f_{\mathrm{in}})$

$E_{\mathrm{gain(D/A)}} = (V_{\mathrm{out}}/V_{\mathrm{LSB}}|_{1\ldots1} - V_{\mathrm{out}}/V_{\mathrm{LSB}}|_{0\ldots0}) - (2^N - 1)$ ; $E_{\mathrm{gain(A/D)}} = (V_{1\ldots1}/V_{\mathrm{LSB}} - V_{0\ldots01}/V_{\mathrm{LSB}}) - (2^N - 2)$ ;

**Oversampling:** $\mathrm{OSR} = f_s/(2f_0)$ ; 0 order $\mathrm{SNR}_{\max} = 6.02N + 1.76 + 10\log(\mathrm{OSR})$ ; 1 order $\mathrm{SNR}_{\max} = 6.02N + 1.76 - 5.17 + 30\log(\mathrm{OSR})$ ;

2 order $\mathrm{SNR}_{\max} = 6.02N + 1.76 - 12.9 + 50\log(\mathrm{OSR})$ ; $S_{\mathrm{TF}}(z) = H(z)/(1 + H(z))$ ; $N_{\mathrm{TF}}(z) = 1/(1 + H(z))$ ;

$T_{\mathrm{avg}}(z) = \dfrac{1}{M}\sum_{i=0}^{M-1} z^{-i} = \dfrac{1}{M}\left(\dfrac{1 - z^{-M}}{1 - z^{-1}}\right)$ ; $T_{\mathrm{avg}}(e^{j\omega}) = \dfrac{\mathrm{sinc}((\omega M)/2)}{\mathrm{sinc}(\omega/2)}$ ; $|N_{\mathrm{TF}}(e^{j\omega})| \le 1.5$ for 1-bit quantizer stability