University of Toronto

Final Exam

Date - Dec. 15, 2009

Duration: 2.5 hrs

ECE512 — Analog Signal Processing
Lecturer - D. Johns

ANSWER QUESTIONS ON THESE SHEETS **USING BACKS IF NECESSARY**

1. Both programmable and non-programmable calculators allowed.
2. Equation sheet on last page of this test.
3. Grading indicated by [ ]. Attempt all questions since a blank answer will certainly get 0.

Question	Mark
1	
2	
3	
4	
5	
6	
Total	

(max grade = 36)

Last Name: SOLUTIONS

First Name: ____________

Student #: ____________

[6] Question 1: Consider the following transfer-function, $H(s)$.

$$H(s) = \frac{2(s^2+9)}{s^4 + 1.3s^3 + s^2 + 0.75s + 1}$$

a) Determine the gain in dB at dc.

$$H(0) = \frac{2(9)}{1} = 18$$

$$H_{dB}(0) = 20\log(18) = \underline{\underline{25.1\ dB}}$$

b) Calculate the rate of attenuation increase in dB per decade at high frequencies.

$$H(j\infty) \propto \frac{1}{s^2}$$

SO FALLS OFF AT 40 dB/DECADE

c) At which frequencies is the attenuation infinite?

ATTEN INFINITE WHERE $|H(j\omega)| = 0$

OCCURS AT $s^2 + 9 = 0$

$$(j\omega)^2 + 9 = 0 \Rightarrow -\omega^2 + 9 = 0$$

$$\omega = \underline{\underline{3\ \text{RAD/S}}}$$

AND AS $\underline{\underline{\omega \rightarrow \infty}}$

[6] Question 2:

a) For the following lowpass filter specifications:

$f_p = 1\text{MHz}$ passband ripple $= 1\text{dB}$ $\leftarrow$ $A_{MAX} = 1dB$

$f_s = 4\text{MHz}$ Min stopband atten $= 60\text{dB}$ $A_{MIN} = 60dB$

Find the required filter order, N, to realize a Butterworth filter that meets this spec.

$1\,dB = 20\log\sqrt{1+\epsilon^2} \Rightarrow \epsilon^2 = 0.2589$

$60 \leq 10\log\left[1+\epsilon^2\left(\frac{4}{1}\right)^{2N}\right]$

$\epsilon^2\, 4^{2N} \geq (10^6 - 1) \gtrapprox 10^6 \Rightarrow 4^{2N} \gtrapprox \frac{10^6}{\epsilon^2} = 3.863e6$

$2N\log(4) \gtrapprox \log(3.863e6)$

$N \gtrapprox 5.47 \quad \therefore \quad \underline{\underline{N = 6}}$

b) Repeat part a) but assume a Chebyshev filter and find N.

$\epsilon^2 = 0.2589$

$60 \leq 10\log\left[1+\epsilon^2\cosh^2\left(N\cosh^{-1}\left(\frac{\omega_s}{\omega_p}\right)\right)\right]$

$3.863e6 \lessapprox \cosh^2(N\cosh^{-1}(4))$

$1.965e3 \lessapprox \cosh(2.063\,N)$

$2.063N \gtrapprox 8.2766$

$N \gtrapprox 4.012 \quad \therefore \quad \underline{\underline{N = 5}}$

c) Show that for frequencies much greater than the passband frequency of lowpass filters, an Nth -order Chebyshev filter has $6(N-1)$ dB more attenuation than an Nth-order Butterworth filter.

$$C_0 = \cos(0) = 1$$

$$C_1 = \cos(\cos^{-1}(\omega)) = \omega$$

$$C_2 = 2\omega C_1 - C_0 = 2\omega^2 - 1$$

$$C_3 = 4\omega^3 - 3\omega$$

$$C_4 = 8\omega^4 - 8\omega^2 + 1$$

$$C_N = 2^{N-1}\omega^N + \dots$$

BUTTERWORTH FOR $\omega_p = 1$ $F(\omega^2) = \epsilon^2 \omega^{2N}$

FOR $\omega \gg 1$ $|H(j\omega)|_B^2 \approx \frac{1}{\epsilon^2 \omega^{2N}}$

CHEBYSHEV FOR $\omega_p = 1$ $F(\omega^2) = \epsilon^2 C_N^2(\omega)$

FOR $\omega \gg 1$ $|H(j\omega)|_C^2 \approx \frac{1}{\epsilon^2 2^{2(N-1)} \omega^{2N}}$

$$\frac{|H(j\omega)|_C^2}{|H(j\omega)|_B^2} = \frac{1}{2^{2(N-1)}}$$

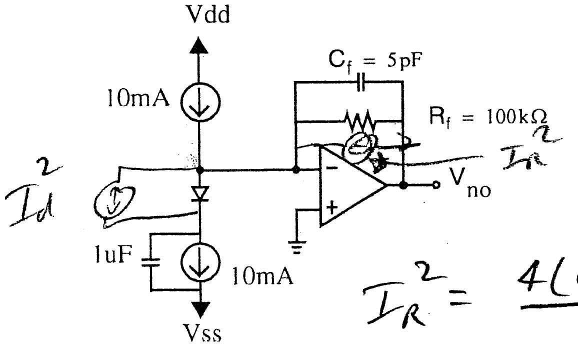
$$10 \log\left[2^{2(N-1)}\right] = 20(N-1)\log(2)$$

$$= 6(N-1) \text{ dB}$$

MORE ATTENUATION.

[6] Question 3: (in V_{RMS})

a) Find the total output noise for the circuit shown below where the opamp and current sources are ideal and noiseless.



$I_d^2 = 2qI_D \qquad I_D = 10mA$

$I_R^2 = \frac{4KT}{R_f}$

$I_d^2 = (2)(1.6\times10^{-19})(10mA) = 3.2e-21 \; [A^2/Hz]$

$I_R^2 = \frac{4(1.38e-23)(300)}{100e3} = 1.66e-25 \; [A^2/Hz]$

$$V_{no}^2 = (I_d^2 + I_R^2) R_f^2 \left[\frac{\pi}{2}\right]\left[\frac{1}{2\pi R_f C_f}\right]$$

$$= (3.2e-11)\left(\frac{1}{4(100e3)(5e-12)}\right) = 1.6e-5$$

$$V_{no} = \underline{4 \; mV_{RMS}}$$

$v_1^2 \quad v_2^2$

b) Find the total output noise (in dBm) if 2 noise sources of sizes -60 dBm and -66 dBm are added together?

$-60dBm = 10\log\frac{v_1^2}{(50)(1e-3)} \Rightarrow v_1^2 = 5e-8$

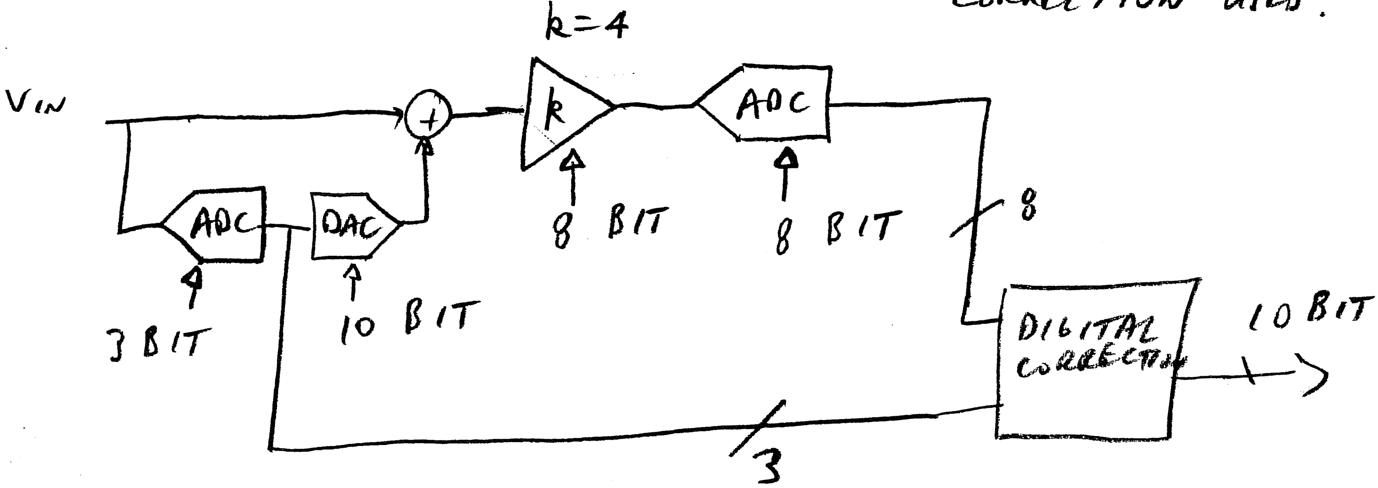
$-66dBm = 10\log\frac{v_2^2}{(50)(1e-3)} \Rightarrow v_2^2 = 1.256e-8$

$v_1^2 + v_2^2 = 6.256e-8$

$10\log\frac{6.256e-8}{(50)(1e-3)} = \underline{-59.0 \; dBm}$

[6] Question 4:

a) Draw a block diagram for a 10-bit two-step A/D converter where the first stage determines 3 bits. Indicate the accuracy needed in all blocks. ASSUME DIGITAL ERROR CORRECTION USED.



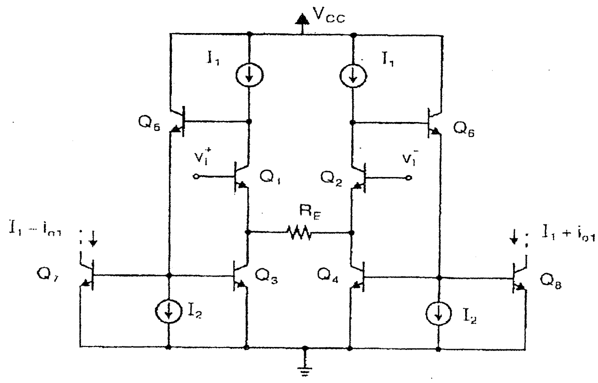
b) Why is there a speed benefit in using an interpolating A/D converter over a flash A/D converter?

AN INTERPOLATING A/D HAS LESS INPUT CAPACITANCE ON INPUT SIGNAL SINCE FEWER PRE-AMPS ARE USED.

c) Why is a thermometer code often used for the top portion of a D/A converter while the bottom portion of the D/A is a binary code?

TO REDUCE GLITCH ENERGY BY USING THERMOMETER PORTION. BINARY PORTION USED TO REDUCE COMPONENTS AND BINARY TO THERMOMETER DECODER SIZE

[6] Question 5: Consider the bipolar transconductor shown below where it is desired to have $G_m = 2$ mA/V and all transistors sizes are the same size except for Q_7 and Q_8 which are 4 times larger in BE area.



a) Find the value of R_E.

$$G_m = \frac{4}{R_E} \qquad R_E = \frac{4}{2(\text{mA/V})} = \underline{\underline{2\,k\Omega}}$$

b) Find the value I_1 such that all transistors do not fall below 30 percent of their nominal bias current when the peak value of $v_i = v_i^+ - v_i^- = 500\text{mV}$.

$$I_{RE} = \frac{v_i}{R_E} = \frac{0.5}{2k} = 0.25\,\text{mA}$$

$$I_{C3\_MIN} = I_1 - I_{RE} \qquad I_{C3\ \text{NOMINAL}} = I_1$$

$$\frac{I_{C3\_MIN}}{I_{C3\_NOMINAL}} = 0.7 \Rightarrow \frac{I_1 - 0.25\,\text{mA}}{I_1} = 0.7$$

$$0.3\, I_1 = 0.25\,\text{mA} \Rightarrow \underline{\underline{I_1 = 0.833\,\text{mA}}}$$

[6] Question 6:

a) If a circuit is measured to have $IIP_3 = 5$ dBm and has a gain of 4 dB, what output-signal level should be used such that the third-order intermodulation products are 60 dB below the fundamental?

$IIP_3 = 5\,dBm$ & $GAIN = 4\,dB$ $\quad OIP_3 = IIP_3 + GAIN$

$\Rightarrow OIP_3 = 9\,dBm$

$OIP_3 = I_{D1} - ID_3/2$

$9\,dBm = I_{D1} + \left(\frac{60}{2}\right)$

$\Rightarrow I_{D1} = 9 - 30 = \underline{\underline{-21\,dBm}}$

b) If the noise of the circuit in a) is measured to be -60 dBm, what is the SFDR?

$SFDR = \left(\frac{2}{3}\right)(OIP_3 - N_o)$

$= \left(\frac{2}{3}\right)(9 + 60) = \underline{\underline{46\,dB}}$

(page intentionally left blank for rough work)

ECE512 Analog Signal Processing Equation Sheet

Constants: $k = 1.38 \times 10^{-23}\ \mathrm{JK}^{-1}$; $q = 1.602 \times 10^{-19}$ C ; $\varepsilon_0 = 8.854 \times 10^{-12}$ F/m ; $V_T = kT/q \approx 26\mathrm{mV}$ at 300 °K ;

General: $h(t)$ is impulse response of LTI system; $H(s)$ is the Laplace transform of $h(t)$

$H(j\omega) = |H(j\omega)|e^{j\phi(\omega)}$; $T_d(\omega) = -\dfrac{d\phi(\omega)}{d\omega}$; $|H(j\omega)|_{\mathrm{dB}} = 20\log|H(j\omega)|$; $H(s) = \dfrac{a_m s^m + .. + a_0}{s^N + b_{n-1}s + .. + b_0}$; $|H(j\omega)|^2 = H(s)H(-s)|_{s=j\omega} = H(j\omega)H(-j\omega)$;

General Lowpass: $|H(j\omega)|^2 = A_0^2/(1 + F(\omega^2))$;

Butterworth: $F(\omega^2) = \varepsilon^2\left(\dfrac{\omega}{\omega_p}\right)^{2N}$; $A_{\max} = 20\log\sqrt{1+\varepsilon^2}$; $A_{\min} \le 10\log\left[1 + \varepsilon^2\left(\dfrac{\omega}{\omega_p}\right)^{2N}\right]$; Poles lie on circle of radius $\omega_p(1/\varepsilon)^{1/N}$ spaced apart by π/N with first angle being $\pi/(2N)$

Chebyshev: (for $\omega_p = 1$); $F(\omega^2) = \varepsilon^2 C_N^2(\omega)$; $C_N(\omega) = \cos(N\cos^{-1}(\omega))$ $|\omega| \le 1$; $C_N(\omega) = \cosh(N\cosh^{-1}(\omega))$ $|\omega| \ge 1$;

$C_{N+1}(\omega) = 2\omega C_N(\omega) - C_{N-1}(\omega)$; $A_{\max} = 20\log\sqrt{1+\varepsilon^2}$; $A_{\min} \le 10\log[1 + \varepsilon^2\cosh^2(N\cosh^{-1}(\omega_s/\omega_p))]$;

Second-order polynomial: $s^2 + (\omega_0/Q)s + \omega_0^2$; for $Q > 0.5$; poles complex at radius ω_0 and real part is $-\omega_0/(2Q)$;

Lowpass and highpass: peaking occurs if $Q > 1/\sqrt{2}$ and $\omega_{\max} = \omega_0\sqrt{1 - 1/2Q^2}$

Bandpass: for complex poles; peak occurs at ω_0 and has 3 dB bandwidth of ω_0/Q

LCR: $\omega_0 = 1/\sqrt{LC}$; $Q = \omega_0 CR$;

KHN Biquad: $RC = 1/\omega_0$; $2((R_1 || R_2)/((R_1 || R_2) + R_3)) = 1/Q$; $2((R_2 || R_3)/((R_2 || R_3) + R_1)) = k$;

Tow-Thomas Biquad: $RC = 1/\omega_0$; damping resistor is QR ; numerator is $-s^2\left(\dfrac{C_1}{C}\right) - s\left(\dfrac{1}{C}\right)\left(\dfrac{1}{R_1} - \dfrac{r}{RR_3}\right) - \dfrac{1}{RR_2C^2}$;

Noise: noise equivalent bandwidth = $(\pi/2)f_{3\mathrm{dB}}$; $V_R^2(f) = 4kTR$; $I_d^2(f) = 2qI_D$; $r_d = (kT)/(qI_D)$; $V_C^2 = (kT)/C$;

Discrete-Time: $X_s(s) = \sum x_c(nT)e^{-snT}$; $X(z) = \sum x_c(nT)z^{-n}$; $p = (z-1)/(z+1)$; $z = (1+p)/(1-p)$; $\Omega = \tan(\omega/2)$;

Switched-Cap: $R_{eq} = T/C$; $Q_{\mathrm{CH}} = -WLC_{\mathrm{ox}}(V_{\mathrm{GS}} - V_t)$;

Data Converters: $B_{\mathrm{in}} = b_1 2^{-1} + .. + b_N 2^{-N}$; $V_{\mathrm{LSB}} = V_{\mathrm{ref}}/2^N$; $V_{\mathrm{out}} = V_{\mathrm{ref}}B_{\mathrm{in}}$; $V_{\mathrm{ref}}B_{\mathrm{out}} = V_{\mathrm{in}} + V_Q$; $|V_Q| \le 0.5V_{\mathrm{LSB}}$;

$V_{Q(\mathrm{rms})} = V_{\mathrm{LSB}}/\sqrt{12}$; SNR $= 6.02N + 1.76$; $E_{\mathrm{off(D/A)}} = V_{\mathrm{out}}/V_{\mathrm{LSB}}|_{0...0}$; $E_{\mathrm{off(A/D)}} = V_{0...01}/V_{\mathrm{LSB}} - 0.5\mathrm{LSB}$; $\Delta t < 1/(2^N\pi f_{\mathrm{in}})$;

$E_{\mathrm{gain(D/A)}} = \left(V_{\mathrm{out}}/V_{\mathrm{LSB}}|_{1...1} - V_{\mathrm{out}}/V_{\mathrm{LSB}}|_{0...0}\right) - (2^N - 1)$; $E_{\mathrm{gain(A/D)}} = (V_{1...1}/V_{\mathrm{LSB}} - V_{0...01}/V_{\mathrm{LSB}}) - (2^N - 2)$;

Oversampling: OSR $= f_s/(2f_0)$; $\mathrm{SNR}_0 = 6.02N + 1.76 + 10\log(\mathrm{OSR})$; $\mathrm{SNR}_1 = 6.02N + 1.76 - 5.17 + 30\log(\mathrm{OSR})$;

$\mathrm{SNR}_2 = 6.02N + 1.76 - 12.9 + 50\log(\mathrm{OSR})$; $S_{\mathrm{TF}}(z) = H(z)/(1 + H(z))$; $N_{\mathrm{TF}}(z) = 1/(1 + H(z))$

$T_{\mathrm{avg}}(z) = \dfrac{1}{M}\sum_{i=0}^{M-1} z^{-i} = \dfrac{1}{M}\left(\dfrac{1 - z^{-M}}{1 - z^{-1}}\right)$; $T_{\mathrm{avg}}(e^{j\omega}) = \dfrac{\mathrm{sinc}((\omega M)/2)}{\mathrm{sinc}(\omega/2)}$; $|N_{\mathrm{TF}}(e^{j\omega})| \le 1.5$ for 1-bit quantizer stability;

Bipolar transistors: $I_C = I_{CS}e^{V_{\mathrm{BE}}/V_T}$; $g_m = I_C/V_T$; $r_e = \alpha/g_m = V_T/I_E$;

CMOS transistors: $K_n = 0.5\mu_n C_{\mathrm{ox}}(W/L)$; $I_D = 2K_n((V_{\mathrm{GS}} - V_{tn})V_{\mathrm{DS}} - (V_{\mathrm{DS}}^2/2))$; $r_{\mathrm{ds}} = (2K_n(V_{\mathrm{GS}} - V_{tn}))^{-1}$;

$I_D = K_n(V_{\mathrm{GS}} - V_{tn})^2$; $g_m = 2K_n(V_{\mathrm{GS}} - V_{tn}) = (2I_D)/(V_{\mathrm{GS}} - V_{tn})$; $r_s = 1/g_m$;

Ideal Transcondcutor: $i_o = G_m v_i$; **Bipolar Diff Pair:** $I_{C2} = I_1/(1 + e^{v_i/V_T})$;

CMOS Pair: $K_{\mathrm{eq}} = (K_n K_p)/(\sqrt{K_n} + \sqrt{K_p})^2$; $V_{\mathrm{t\text{-}eq}} = V_{\mathrm{tn}} - V_{\mathrm{tp}}$;

Dynamic Range: (all in dB or dBm) $ID_3 = I_{D3} - I_{D1}$; $\mathrm{OIP}_3 = I_{D1} - ID_3/2$; SFDR $= (2/3)(\mathrm{OIP}_3 - N_o)$;

THD $= 10\log((V_{h2}^2 + V_{h3}^2 + ..)/V_f^2)$;