12.10

Numerator is given by

$$a_7 (s-0)(s^2+(10^3)^2)(s^2+(3\times10^3)^2) \times (s^2+(6\times10^3)^2)$$

$$= a_7 s (s^2+10^6)(s^2+9\times10^6)(s^2+36\times10^6)$$

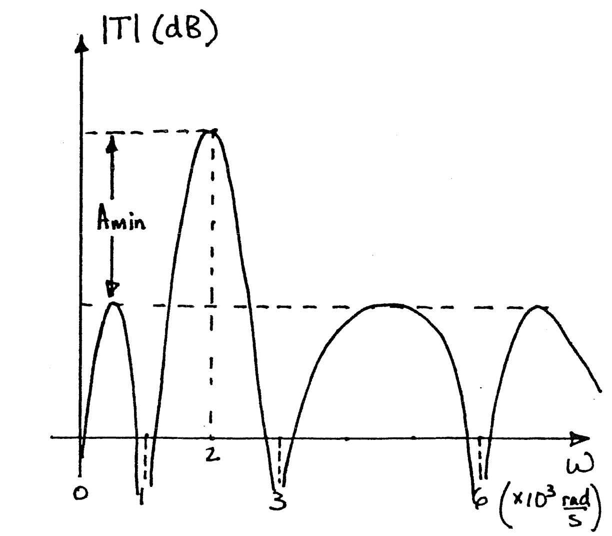
Degree of Numerator $\triangleq M = 7$

Degree of Denominator $\triangleq N$

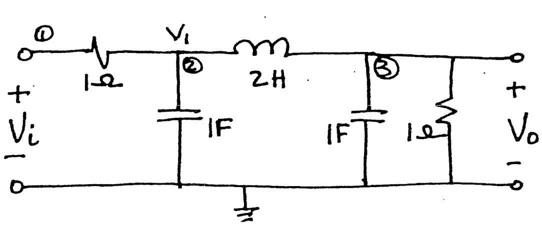
Given that there is one zero at ∞:

$$N - M = 1 \Rightarrow \underline{\underline{N = 8}}$$

$$\therefore T(s) = \frac{a_7 s (s^2+10^6)(s^2+9\times10^6)(s^2+36\times10^6)}{s^8 + b_7 s^7 + b_6 s^6 + \cdots + b_0}$$



12.11



The easiest way to solve the circuit is to use nodal analysis at nodes ①, ②, ③

At node ③ $\sum I = 0$

$$\frac{V_o}{1} + \frac{V_o}{1/s} + \frac{V_o - V_1}{2s} = 0$$

$$\therefore V_1 = V_o(2s^2+2s+1) \qquad \text{Eq.(a)}$$

At node ② $\sum I = 0$

$$\frac{V_1 - V_i}{1} + \frac{V_1}{1/s} + \frac{V_1 - V_o}{2s} = 0$$

$$\therefore V_1(2s^2+2s+1) = V_o + 2sV_i \qquad \text{Eq.(b)}$$

(a) $\rightarrow$ (b)

$$V_o(2s^2+2s+1)^2 = V_o + 2sV_i$$

$$V_o(4s^4 + s^3(4+4) + s^2(2+4+2) + s(2+2)+1) = V_o + 2sV_i$$

$$\frac{V_o(s)}{V_i(s)} \triangleq T(s) = \frac{2s}{4s^4+8s^3+8s^2+4s}$$

$$\underline{\underline{T(s) = \frac{0.5}{s^3+2s^2+2s+1}}}$$

Poles are given by:

$$s^3+2s^2+2s+1 = 0$$

$$(s+1)(s^2+s+1) = 0$$

$\therefore$ Poles are $\underline{\underline{s = -1}}$ and

$$\underline{\underline{s = -\tfrac{1}{2} \pm j\sqrt{3}/2}}$$

12.12

$A_{max} = 1\,dB$, $A_{min} = 20\,dB$, $\omega_s/\omega_p = 1.3$

Using:

$$A(\omega_s) = 10\log\left[1+\epsilon^2\left(\frac{\omega_s}{\omega_p}\right)^{2N}\right] \qquad \text{Eq (12.15)}$$

$$= A_{MIN}$$

CONT

$\epsilon = \left[10^{1/10} - 1\right]^{1/2} = 0.5088$

$A_{min} = 10 \log\left[1 + \epsilon^2 \left(\frac{\omega_s}{\omega_p}\right)^{2N}\right]$

$10^{A_{MIN}/10} - 1 = \epsilon^2 \left(\frac{\omega_s}{\omega_p}\right)^{2N}$

$\log\left(10^{A_{MIN}/10} - 1\right) = \log\left(\epsilon^2 \left(\frac{\omega_s}{\omega_p}\right)^{2N}\right)$

$$N = \frac{\log\left[\left(10^{A_{MIN}/10} - 1\right)/\epsilon^2\right]}{2 \log\left(\omega_s/\omega_p\right)}$$

$= 11.3 \Rightarrow$ choose $\underline{\underline{N = 12}}$

The actual value of stopband attenuation can be calculated using the integer value of N:

$A(\omega_s) = 10 \log\left[1 + \epsilon^2 \left(\frac{\omega_s}{\omega_p}\right)^{2N}\right]$; $N = 12$

$= \underline{\underline{27.35\,dB}}$ actual attenuation

If the stopband specs are to be met exactly we need to find Amax.

Eq. 12.15 can be rearranged to give

$$\epsilon^2 = \frac{10^{A_{min}/10} - 1}{\left(\omega_s/\omega_p\right)^{2N}} \qquad \begin{array}{l} A_{min} = 20 \\ N = 12 \end{array}$$

$= 0.1824$

$\therefore A_{max} = 10 \log\left(1 + \epsilon^2\right)$

$= \underline{\underline{0.73\,dB}}$

12.13

$N = 7$, $A_{max} = 3\,dB$

We want attenuation at $\omega = 1.6\,\omega_p$ or $\frac{\omega}{\omega_p} = 1.6$

$\epsilon = \sqrt{10^{A_{max}/10} - 1} = 0.998$

$A(\omega) = 10 \log\left[1 + \epsilon^2 \left(\frac{\omega}{\omega_p}\right)^{2N}\right]$

$= 10 \log\left[1 + 0.998^2 (1.6)^{14}\right]$

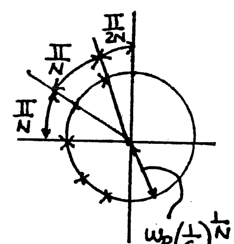
$= \underline{\underline{28.56\,dB}}$

12.14

$\omega_p = 10^3$ rad/s , $N = 5$

$A_{max} = 1\,dB \Rightarrow \epsilon = 0.5088$

Find solution graphically



$P_1 = \omega_p \left(\frac{1}{\epsilon}\right)^{1/N} \angle \left(\frac{\pi}{2} + \frac{\pi}{2N}\right)$

$= 873.59 \angle (6\pi/10)$

$= 873.59 \left[\cos(6\pi/10) \pm j \sin\left(\frac{6\pi}{10}\right)\right]$

$= \underline{\underline{-269.96 \pm j\,830.84}}$

$P_2 = 873.59 \angle \left[\frac{\pi}{2} + \frac{\pi}{2N} + \frac{\pi}{N}\right]$

$= \underline{\underline{-706.75 \pm j\,513.49}}$

$P_3 = 873.59 \angle \pi = \underline{\underline{-873.59}}$

12.15

$f_p = 10\,kHz$ $\quad \frac{\omega_s}{\omega_p} = 1.5 \quad A_{MIN} = 15\,dB$

$f_s = 15\,kHz \quad\quad\quad\quad\quad A_{max} = 2\,dB$

$\epsilon^2 = 10^{A_{max}/10} - 1 \Rightarrow \epsilon = 0.76478$

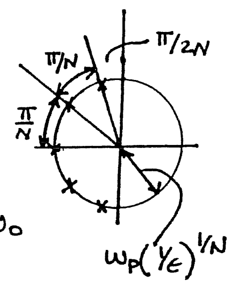
CONT.

Manipulation Eq (12·15) we get:

$$N = \frac{\log\left[\left(10^{A_{min}/10} - 1\right)/\epsilon^2\right]}{2\log(\omega_s/\omega_p)} = 4.88$$

$\therefore$ Use $\underline{\underline{N = 5}}$

Finding naturalmodes graphically :-



$$\text{radius} = \omega_p\left(\frac{1}{\epsilon}\right)^{1/N} \triangleq \omega_0$$

$$\omega_0 = 6.629 \times 10^4$$

$$P_1 = \omega_0 \measuredangle \left(\pi/2 + \pi/2N\right) = \omega_0 \measuredangle \left(6\pi/10\right)$$

$$= \omega_0\left(\cos\left(\tfrac{6\pi}{10}\right) \pm j\sin\left(\tfrac{6\pi}{10}\right)\right)$$

$$= \underline{\omega_0\left(-0.309 \pm j0.951\right)}$$

$$P_2 = \omega_0\left(\cos\tfrac{8\pi}{10} \pm j\sin\tfrac{8\pi}{10}\right)$$

$$= \underline{\underline{\omega_0\left(-0.809 \pm j0.588\right)}}$$

$$P_3 = \omega_0\left(\cos\pi \pm j\sin\pi\right) = \underline{\underline{-\omega_0}}$$

Given a natural mode $-\alpha \pm j\beta$, the following term results

$$(s + \alpha + j\beta)(s + \alpha - j\beta)$$

$$= s^2 + 2\alpha s + \alpha^2 + \beta^2$$

$$= \underline{s^2 + 2\,\mathrm{Re}[P]\,s + |P|^2}$$

Also, note that for a Butterworth, all natural modes have a magnitude of ω_0.

P_1 yields: $s^2 + 0.618\omega_0 s + \omega_0^2$

P_2 yields: $s^2 + 1.618\omega_0 s + \omega_0^2$

P_3 yields: $s + \omega_0$

$$\therefore T(s) = \frac{k}{(s + \omega_0)(s^2 + .618\omega_0 s + \omega_0^2)} \times \frac{1}{s^2 + 1.618\omega_0 s + \omega_0^2}$$

For unity dc gain

$$|T(j0)| = \frac{k}{\omega_0^5} = 1 \Rightarrow k = \omega_0^5$$

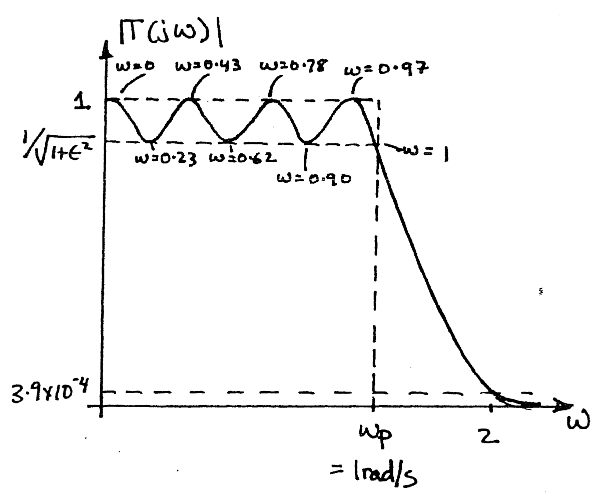
$$\therefore T(s) = \underline{\underline{\frac{\omega_0^5}{(s + \omega_0)(s^2 + .618\omega_0 s + \omega_0^2)}}} \times \underline{\underline{\frac{1}{(s^2 + 1.618\omega_0 s + \omega_0^2)}}}$$

For attenuation at 20kHz use Eq (12.15) with $\frac{\omega_s}{\omega_p} = \frac{20}{10} = 2$

$$A(\omega_s) = 10\log\left[1 + \epsilon^2\left(\frac{\omega_s}{\omega_p}\right)^{2N}\right]$$

$$= \underline{27.8\,\mathrm{dB}}$$

12·16



Given $A_{max} = 1\,\mathrm{dB} \Rightarrow \epsilon = 0.5088$

CONT.

Using Eq 12.18

$$|T(j\omega)| = \left[1+\epsilon^2 \cos^2\left(N \cos^{-1}\left(\frac{\omega}{\omega_p}\right)\right)\right]^{-1/2}$$

for $\omega \le \omega_p$

If $|T(j\omega)| = 1$

$$1 = 1+\epsilon^2 \cos^2\left(N \cos^{-1}(\omega/\omega_p)\right) \qquad \omega_p = 1$$

$$N \cos^{-1}(\omega/1) = \cos^{-1}(0)$$

$$\cos^{-1}(\omega) = \frac{2i+1}{2N}\pi$$

ω's repeat after this value

$$\therefore \omega_i = \cos\left[\frac{2i+1}{2N}\pi\right] \qquad i = 0, 1, \ldots, \frac{N-1}{2}$$

$$\boxed{\begin{aligned} \omega_0 &= 0.9749 \\ \omega_1 &= 0.7818 \\ \omega_2 &= 0.4339 \\ \omega_3 &= 0 \end{aligned}}$$

ω values at which $|T| = 1$

note $\omega_4 = -0.4339 = -\omega_2$!

If $|T| = 1/\sqrt{1+\epsilon^2}$, then

$$1/\sqrt{1+\epsilon^2} = \left[1+\epsilon^2 \cos^2\left(N \cos^{-1}(\omega/\omega_p)\right)\right]^{-\frac{1}{2}}$$

$$1 = \cos\left(N \cos^{-1}\left(\frac{\omega}{\omega_p}\right)\right)$$

$$N \cos^{-1}(\omega) = \cos^{-1}(0)$$

$$= i\pi \qquad i = 0, 1, 2 \ldots$$

$$\omega_i = \cos\left[\frac{i\pi}{N}\right] \qquad i = 0, 1, 2, \ldots \frac{N}{2}$$

$$\boxed{\begin{aligned} \omega_0 &= 1.0 \\ \omega_1 &= 0.9010 \\ \omega_2 &= 0.6235 \\ \omega_3 &= 0.2252 \end{aligned}}$$

ω values at which $|T| = (1+\epsilon^2)^{-1/2}$

Note $\omega_4 = -0.2252 = -\omega_3$!

To find $|T(j2)|$ use Eq (12.19) since $\omega > \omega_p$

$$|T(j\omega)| = \left[1+\epsilon^2 \cosh^2\left(N \cosh^{-1}\left(\frac{\omega}{\omega_p}\right)\right)\right]^{-\frac{1}{2}}$$

$$= \left[1+0.5088^2 \cosh^2(7 \cosh^{-1} 2)\right]^{-\frac{1}{2}}$$

$$= \underline{\underline{3.898 \times 10^{-4}\ \text{V/V}}}$$

$$|T|_{dB} = \underline{\underline{-68.2\ dB}}$$

For roll-off consider

$$T(s) = \frac{k}{s^7 + b_6 s^6 + \ldots + b_0}$$

for $\omega \gg \omega_p$ $\quad T(j\omega) \cong \dfrac{k}{\omega^7}$

$\therefore$ Rolloff is $\frac{1}{2^7}$ or $20\log(1/2^7)$ per octave $= \underline{\underline{-42\ dB/octave}}$.

12.17

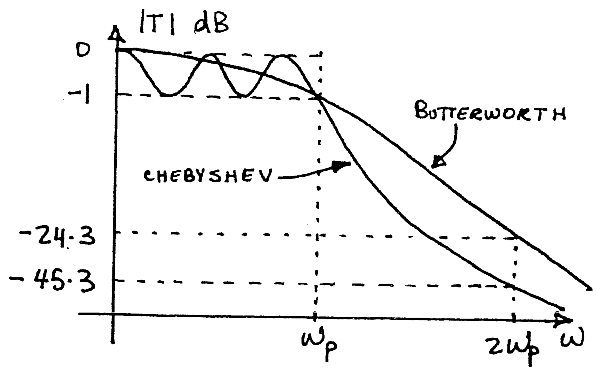
$\omega_s/\omega_p = 2 \qquad A_{max} = 1dB \Rightarrow \epsilon = \sqrt{10^{\frac{A_{max}}{10}} - 1} = 0.5088$

$$|T_B| = \left[1+\epsilon^2 (\omega_s/\omega_p)^{2N}\right]^{-1/2}$$

$$|T_c| = \left[1+\epsilon^2 \cosh^2\left(N \cosh^{-1}\left[\frac{\omega_s}{\omega_p}\right]\right)\right]^{-1/2}$$

$|T_B| = 6.13 \times 10^{-2} \Rightarrow \underline{\underline{-24.3\ dB}}$

$|T_c| = 5.43 \times 10^{-3} \Rightarrow \underline{\underline{-45.3\ dB}}$



12.18

$f_p = 3.4\,kHz$ $\qquad A_{max} = 1\,dB \Rightarrow \epsilon = 0.5088$

$f_s = 4\,Khz$ $\qquad A_{min} = 35\,dB$

$\omega_s/\omega_p = 1.176$

Using Eq (12.22) :

$$A(\omega_s) = 10\log\left[1+\epsilon^2\cosh^2\left(N\cosh^{-1}\left(\frac{\omega_s}{\omega_p}\right)\right)\right]$$

& trying different values for N

N	$A(\omega_s)$
8	28.8 dB
9	33.9 dB
10	38.98 dB

$\therefore$ Use $N = 10$

Excess attenuation $= 39 - 35 = 4\,dB$

Poles are given by:

$$P_k = -\omega_p \sin\left(\frac{2k-1}{N}\cdot\frac{\pi}{2}\right)\sinh\left(\frac{1}{N}\sinh^{-1}\left(\frac{1}{\epsilon}\right)\right)$$
$$+ j\,\omega_p \cos\left(\frac{2k-1}{N}\cdot\frac{\pi}{2}\right)\cosh\left(\frac{1}{N}\sinh^{-1}\left(\frac{1}{\epsilon}\right)\right)$$

for $k = 1, 2, \ldots, N$.

Since $\epsilon = 0.5088$ and $N = 10$

$\sinh\left(1/N \sinh^{-1}(1/\epsilon)\right) = 0.1433$

$\cosh\left(1/N \sinh^{-1}(1/\epsilon)\right) = 1.010$

$$\therefore P_1 = \omega_p\left[-0.1433\sin\left(\tfrac{\pi}{20}\right) + j1.010\cos\left(\tfrac{\pi}{20}\right)\right]$$
$$= \omega_p(-0.0224 + j0.9978)$$

$P_2 = \omega_p(-0.0650 + j0.900)$

$P_3 = \omega_p(-0.1013 + j0.7143)$

$P_4 = \omega_p(-0.1277 + j0.4586)$

$P_5 = \omega_p(-0.1415 + j0.1580)$

Now it should be realized that the remaining poles are complex conjugates of the above.

Pole-pair P_1 & P_1^* give a factor:

$$s^2 + 2(0.0224)\omega_p s + \omega_p^2(0.0224^2 + 0.9978^2)$$
$$= s^2 + 0.0448\omega_p s + 1.023\omega_p^2$$

i.e. this factor is from $(s-P_1)(s-P_1^*)$

P_2 yields: $s^2 + 0.130\omega_p s + 0.902\omega_p^2$

P_3 yields: $s^2 + 0.203\omega_p s + 0.721\omega_p^2$

P_4 yields: $s^2 + 0.255\omega_p s + 0.476\omega_p^2$

P_5 yields: $s^2 + 0.283\omega_p s + 0.212\omega_p^2$

Now T(s) is given by

$$T(s) = \frac{k\,\omega_p^{10}}{\epsilon\, 2^9 (s-P_1)(s-P_1^*)\cdots(s-P_5)(s-P_5^*)}$$

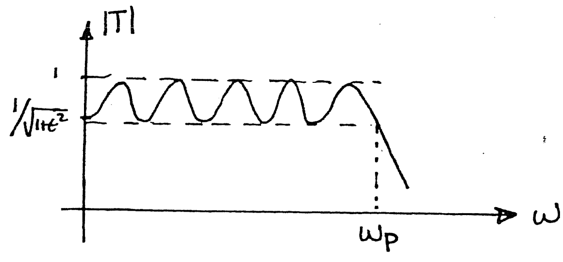
where the second order terms of the denominator are given above.

k is the dc gain

$\therefore$ we want the dc gain to be

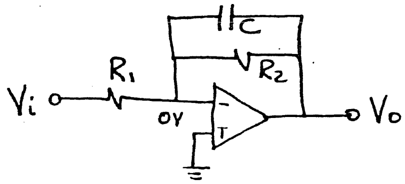
$$k = \frac{1}{1+\epsilon^2} = 0.8913$$

$\omega_p = 2\pi \times 3400$



12.19

$f_o = 10\text{kHz}$ DC gain $= 10$ $R_{in} = 10\text{k}\Omega$



$R_{in} = R_1 = \underline{\underline{10\text{k}\Omega}}$

$$DC\ gain = -R_2/R_1 = -10$$

$$R_2 = 10R_1 = \underline{\underline{100\text{k}\Omega}}$$

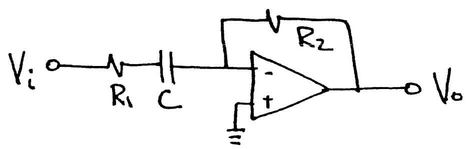
$$R_2 C = 1/\omega_o$$

$$C = \frac{1}{\omega_o R_2} = \frac{1}{2\pi\, 10^4 \times 100\times10^3}$$

$$= \underline{\underline{0.159\text{nF}}}$$

12.20

$f_o = 100\text{kHz}$ $R_i(\infty) = 100\text{k}\Omega$ $|T(\infty)| = 1$



$R_i(\infty) = R_1 = \underline{\underline{100\text{k}\Omega}}$

$$|T(\infty)| = R_2/R_1 = 1$$

$$R_2 = R_1 = \underline{\underline{100\text{k}\Omega}}$$

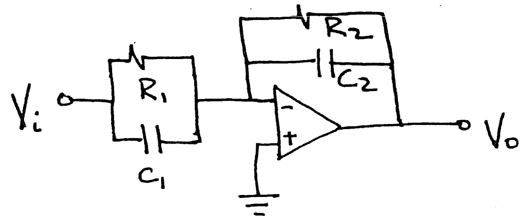
$$CR_1 = 1/\omega_o$$

$$C = \frac{1}{\omega_o R_1} = \frac{1}{2\pi\, 100\times10^3 \times 100\times10^3}$$

$$= \underline{\underline{15.9\text{nF}}}$$

12.21

Use general first-order circuit:



- Zero at 1kHz ; Pole at 100kHz
- $|T(0)| = 1$; $R_i(0) = 1\text{k}\Omega$

Thus: $R_i(DC) = R_1 = \underline{\underline{1\text{k}\Omega}}$

$$T(DC) = -R_2/R_1 = -1$$

$$R_2 = R_1 = \underline{\underline{1\text{k}\Omega}}$$

For a pole at 100kHz

$$C_2 R_2 = \frac{1}{\omega_o} \Rightarrow C_2 = \frac{1}{2\pi f_o R_2}$$

$$= \underline{\underline{1.59\text{nF}}}$$

For the circuit $T(s) = \dfrac{a_1 s + a_0}{s + \omega_o}$

Thus the zero at $-a_0/a_1 = -2\pi\, 10^3$

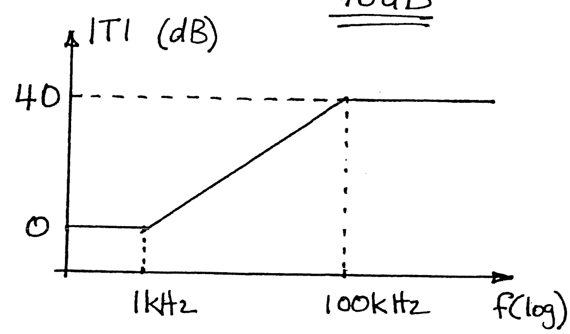
$$C_1 R_1 = a_1/a_0$$

$$C_1 = \frac{1}{2\pi\, 10^3 R_1}$$

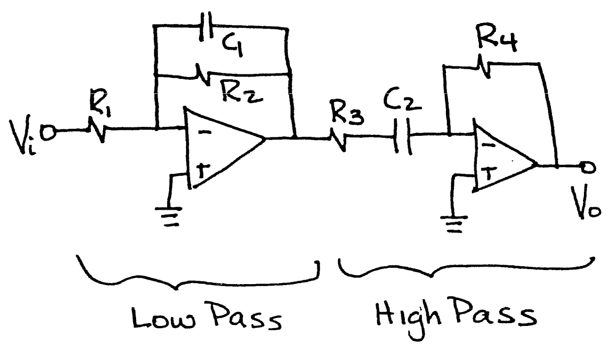
$$= \underline{\underline{159\text{nF}}}$$

High freq gain $= \dfrac{-C_1}{C_2} = \underline{\underline{-100}}$

$$= \underline{\underline{40\text{dB}}}$$



12.22



gain $= 10^{12/20} = 3.98 \approx \underline{4}$

want $R_i = R_1$ large

$\therefore \; R_1 = \underline{\underline{100k\Omega}}$

Total gain $= A_{LP} A_{HP} = 4$

$A_{LP} = -R_2/R_1 \Rightarrow R_2 = -A_{LP} R_1$ and $R_2 \leq 100k\Omega$

$\therefore$ make $A_{LP} = -1$ $\quad A_{HP} = -4$

$R_2 = \underline{\underline{100k\Omega}}$

$$R_2 C_1 = \frac{1}{\omega_{0,LP}}$$

$$C_1 = \frac{1}{2\pi f_{o,LP} R_2} = \frac{1}{2\pi (10\times10^3)\, 100\times10^3}$$

$= \underline{\underline{0.159nF}}$

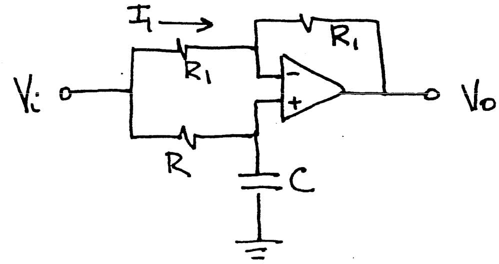
$A_{HP} = -R_4/R_3 = -4$, $R_4 = 4R_3$ $\Big\}$ make $R_4 = \underline{\underline{100k\Omega}}$, $R_3 = \underline{\underline{25k\Omega}}$

Now $R_3 C_2 = 1/\omega_{0,HP}$

$$C_2 = \frac{1}{2\pi f_{o,HP} R_3} = \frac{1}{2\pi (100\times10^3)\, 25\times10^3}$$

$= \underline{\underline{63.7nF}}$

12.23



At +ve terminal

$$V_+ = \frac{1/sC}{1/sC + R_1} V_i = \frac{1}{1+s\tau} V_i \qquad \underline{\tau = RC}$$

$V_- = V_+$ due to virtual short between terminals.

$$\therefore I_1 = \left(V_i - \frac{1}{1+s\tau} V_i\right)\frac{1}{R_1}$$

$$V_o = V_- - I_1 R_1 = \frac{V_i}{1+s\tau} - \left(V_i - \frac{V_i}{1+s\tau}\right)\frac{R_1}{R_1}$$

$$\frac{V_o}{V_i} = \frac{1 - (1+s\tau) + 1}{1+s\tau} = \frac{1 - s\tau}{1+s\tau}$$

$$= \frac{\omega_0 - s}{\omega_0 + s} \qquad \omega_0 = \frac{1}{\tau}$$

$$= \underline{\underline{-\frac{s-\omega_0}{s+\omega_0} = T(s)}}$$

$$T(j\omega) = -\frac{j\omega - \omega_0}{j\omega + \omega_0}$$

$$\phi(\omega) = 180° + \mathrm{Tan}^{-1}\left(\frac{\omega}{-\omega_0}\right) - \mathrm{Tan}^{-1}\left(\frac{\omega}{\omega_0}\right)$$

$$= 360° - 2\tan^{-1}\left(\frac{\omega}{\omega_0}\right)$$

$$= -2\tan^{-1}(\omega/\omega_0)$$

$\therefore \tan^{-1}\left(\frac{\omega}{-\omega_0}\right) = 180 - \tan^{-1}\left(\frac{\omega}{\omega_0}\right)$

Now this equation can be rearranged:

$$\frac{\omega}{\omega_0} = \tan(-\phi/2) \Leftarrow \omega_0 = \frac{1}{\tau} = \frac{1}{RC}$$

$$RC\omega = \tan(-\phi/2)$$

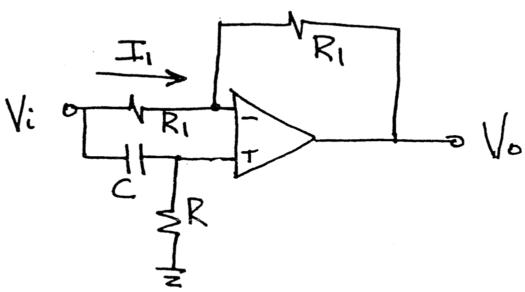
CONT.

$\therefore R = \dfrac{\tan(-\phi/2)}{C\omega} = 10^4 \tan(-\phi/2)$

$\phi = -30^\circ, -60^\circ, -90^\circ, -120^\circ, -150^\circ$

$R = 2.68\,k\Omega, 5.77\,k\Omega, 10\,k\Omega, 17.32\,k\Omega, 37.32\,k\Omega$

12.24



$V_+ = \dfrac{R}{R + 1/sC} V_i = \dfrac{s}{s + \omega_0} V_i$

where $\omega_0 = \dfrac{1}{RC}$

$I_1 = \dfrac{V_i - (s/s+\omega_0)V_i}{R_1}$

$V_o = \dfrac{s}{s+\omega_0} V_i - I_1 R_1$

$= \dfrac{s}{s+\omega_0} V_i - V_i\left(1 - \dfrac{s}{s+\omega_0}\right)$

$\dfrac{V_o}{V_i} = \dfrac{2s - s - \omega_0}{s + \omega_0} = \dfrac{s - \omega_0}{s + \omega_0}$

Now:

NOTE $\tan^{-1}\dfrac{\omega}{-\omega_0} = 180^\circ - \tan^{-1}\dfrac{\omega}{\omega_0}$

$\phi(\omega) = \tan^{-1}\left(\dfrac{\omega}{-\omega_0}\right) - \tan^{-1}\left(\dfrac{\omega}{\omega_0}\right)$

$= 180 - \tan^{-1}\left(\dfrac{\omega}{\omega_0}\right) - \tan^{-1}\left(\dfrac{\omega}{\omega_0}\right)$

$= 180 - 2\tan^{-1}\left(\dfrac{\omega}{\omega_0}\right)$

Clearly $\phi(0) = 180^\circ$ & $\phi(\omega \to \infty) = 0^\circ$

12.25

Low Pass $\omega_0 = 10^3$ rad/s

$Q = 1$

DC gain $= 1$

$T(s) = \dfrac{a_0}{s^2 + s\frac{\omega_0}{Q} + \omega_0^2}$

$T(0) = a_0/\omega_0^2 = 1$

$a_0 = \omega_0^2 = 10^6$

$\therefore T(s) = \dfrac{10^6}{s^2 + 10^3 s + 10^6}$

$\omega_{max} = \omega_0 \sqrt{1 - 1/2Q^2}$

$= \dfrac{\omega_0}{\sqrt{2}}$

$= 0.707$ rad/s

$|T_{max}| = \dfrac{|a_0|\,Q}{\omega_0^2\sqrt{1 - \frac{1}{4Q^2}}} \Leftarrow a_0 = \omega_0^2$

$= \dfrac{|\omega_0^2|}{\omega_0^2\sqrt{3/4}}$

$= 2/\sqrt{3}$

$= 1.15$ V/V

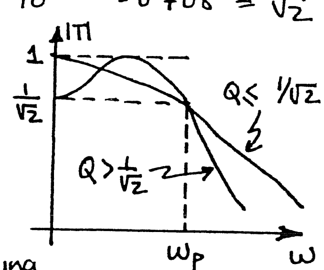
$= 1.21$ dB

12.26

$\omega_p = 1$ rad/s

$A_{max} = 3$ dB

$10^{-3/20} = 0.708 \cong \dfrac{1}{\sqrt{2}}$



There are many Q-values which may be used

$Q \leq 1/\sqrt{2}$ - no peaking

$Q > 1/\sqrt{2}$ - peaking

CONT.

Solution 1 $\quad Q \leq 1/\sqrt{2}$

For $Q = 1/\sqrt{2}$ the response is maximally flat. Because this is desirable, use : $\underline{\underline{Q = \frac{1}{\sqrt{2}}}}$

$$T(s) = \frac{a_0}{s^2 + s\omega_0\sqrt{2} + \omega_0^2}$$

$$|T(0)| = \frac{a_0}{\omega_0^2} = 1$$

$$\underline{\underline{a_0 = \omega_0^2}}$$

$$|T(j1)|^2 = \frac{\omega_0^2}{(\omega_0^2-1) + 2\omega_0^2} = \left(\frac{1}{\sqrt{2}}\right)^2$$

$$\omega_0 = 1 \text{ rad/s}$$

$$\therefore \underline{\underline{T_1(s) = \frac{1}{s^2 + \sqrt{2}s + 1}}}$$

Solution 2 $\quad Q > 1/\sqrt{2}$

From the figure : $|T(0)| = 1/\sqrt{2} = \frac{a_0}{\omega_0^2}$

$$\therefore \underline{\underline{a_0 = \omega_0^2/\sqrt{2}}}$$

Now $|T|_{max} = \dfrac{|a_0|\,Q}{\omega_0^2\sqrt{1 - 1/4Q^2}} = 1$

$$\frac{Q}{\sqrt{2}\sqrt{1 - 1/4Q^2}} = 1$$

$$Q = \sqrt{2}\sqrt{1 - 1/4Q^2}$$

$$\therefore Q^2 = 2\left(1 - \frac{1}{4Q^2}\right)$$

$$= 2 - \frac{1}{2Q^2}$$

$$Q^4 - 2Q^2 + \tfrac{1}{2} = 0$$

Solving for Q^2 gives:-

$$Q^2 = 1 \pm \sqrt{2}$$

ASIDE:

$\because Q > 1/\sqrt{2}$

$Q^2 > 1/2$

$4Q^2 > 2$

$\frac{1}{4Q^2} < \frac{1}{2}$

$\therefore 1 - \frac{1}{4Q^2} > 1/2$

$\therefore \left|1 - \frac{1}{4Q^2}\right| = 1 - \frac{1}{4Q^2}$

$\Rightarrow Q = 0.5412$ or 1.3066

$\therefore Q > \frac{1}{\sqrt{2}}$ $\quad$ use $\underline{\underline{Q = 1.3066}}$

Now at the passband edge

$$|T(j1)| = 1/\sqrt{2}$$

$$|T(j1)|^2 = \frac{(\omega_0^2/\sqrt{2})^2}{(\omega_0^2-1)^2 + \frac{\omega_0^2}{Q^2}} = \frac{1}{2}$$

$$\frac{\cancel{\omega_0^4}}{\cancel{2}} = \frac{1}{\cancel{2}}\left[\cancel{\omega_0^4} - 2\omega_0^2 + 1 + \frac{\omega_0^2}{Q^2}\right]$$

$$\omega_0^2(2 - 1/Q^2) = 1$$

$$\underline{\underline{\omega_0 = 0.841}}$$

$$\therefore T_2(s) = \frac{\omega_0^2/\sqrt{2}}{s^2 + \omega_0/Q\, s + \omega_0^2}$$

$$= \underline{\underline{\frac{0.5}{s^2 + 0.644s + 0.707}}}$$

If $\omega_s = 2$

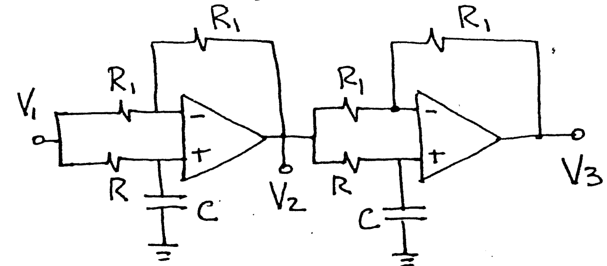
$|T_1(j2)| = 0.242 \qquad |T_2(j2)| = 0.1414$

$\therefore \underline{\underline{A_{min,1} = -12.3\text{dB}}} \qquad \underline{\underline{A_{MIN,2} = -17\text{dB}}}$

12.27

V_2 lags V_1 by $120°$

V_3 lags V_2 by $120°$



$\omega = 2\pi 60$ & $C = 1\,\mu F$

$T(s) = \dfrac{s - \omega_0}{s + \omega_0} \qquad \omega_0 = \dfrac{1}{RC}$

CONT.

$$\phi(\omega) = 180^\circ + \tan^{-1}\left(\frac{\omega}{-\omega_0}\right) - \tan^{-1}\left(\frac{\omega}{\omega_0}\right)$$

Sub: $\tan\left(\frac{\omega}{-\omega_0}\right) = 180 - \tan^{-1}\left(\frac{\omega}{\omega_0}\right)$

$$\Rightarrow \phi(\omega) = -2\tan(\omega/\omega_0)$$

Now $\phi = -120^\circ$ at $\omega = 2\pi 60$

$$-120 = -2\tan(\omega RC)$$

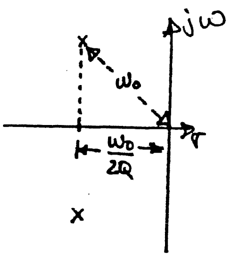
$$-60 = -\tan^{-1}(2\pi 60 \times R \times 10^{-6})$$

$$\underline{R = 4.59\,k\Omega}$$

R_1 can be arbitrarily chosen

use $\underline{R_1 = 10k\Omega}$

12.28



Natural Modes:

$$-\frac{1}{2} \pm j\frac{\sqrt{3}}{2}$$

$$\omega_0 = \sqrt{\left(\tfrac{1}{2}\right)^2 + \left(\tfrac{\sqrt{3}}{2}\right)^2} = \underline{1.0}$$

$$\frac{\omega_0}{2Q} = \frac{1}{2} \Rightarrow \frac{\omega_0}{Q} = 1$$

$$T(s) = \frac{a_2 s^2}{s^2 + s\frac{\omega_0}{Q} + \omega_0^2} = \frac{a_2 s^2}{s^2 + s + 1}$$

$$|T(j\infty)| = a_2 = 1$$

$$\therefore \quad T(s) = \frac{s^2}{s^2 + s + 1}$$

12.29

For a 2nd-order bandpass

$$T(s) = \frac{a_1 s}{s^2 + s\frac{\omega_0}{Q} + \omega_0^2}$$

$$T(j\omega) = \frac{j\omega a_1}{(\omega_0^2 - \omega^2) + \frac{j\omega\omega_0}{Q}}$$

$$|T(j\omega)| = \frac{a_1\omega}{\left[(\omega_0^2 - \omega^2)^2 + \frac{\omega^2\omega_0^2}{Q^2}\right]^{1/2}}$$

Part (a):

$$|T(j\omega_1)| = |T(j\omega_2)|$$

$$\frac{a_1\omega_1}{\sqrt{(\omega_0^2 - \omega_1^2)^2 + \left(\frac{\omega_1\omega_0}{Q}\right)^2}} = \frac{a_1\omega_2}{\sqrt{(\omega_0^2 - \omega_2^2)^2 + \left(\frac{\omega_2\omega_0}{Q}\right)^2}}$$

$$\omega_1^2\left[(\omega_0^2 - \omega_2^2)^2 + \cancel{\left(\frac{\omega_2\omega_0}{Q}\right)^2}\right] = \omega_2^2\left[(\omega_0^2 - \omega_1^2)^2 + \cancel{\left(\frac{\omega_1\omega_0}{Q}\right)^2}\right]$$

$$\omega_1^2(\omega_0^4 - 2\cancel{\omega_0^2}\omega_2^2 + \omega_2^4) = \omega_2^2(\omega_0^4 - 2\cancel{\omega_0^2}\omega_1^2 + \omega_1^4)$$

$$\omega_1^2\omega_0^4 + \omega_1^2\omega_2^4 = \omega_2^2\omega_0^4 + \omega_2^2\omega_1^4$$

$$\omega_0^4(\omega_1^2 - \omega_2^2) = \omega_2^2\omega_1^4 - \omega_1^2\omega_2^4$$

$$\omega_0^4(\omega_1^2 - \omega_2^2) = \omega_2^2\omega_1^2(\omega_1^2 - \omega_2^2)$$

$$\omega_0^4 = \omega_1^2\omega_2^2$$

$$\underline{\omega_0^2 = \omega_1\omega_2} \quad \text{Q.E.D.}$$

CONT.

For Fig. 12.4: $\omega_{p1} = 8100$ rad/s
$\omega_{p2} = 10\,000$ rad/s
$A_{max} = 1$ dB

$$\omega_0^2 = (8100)(10\,000)$$

$$\omega_0 = \underline{\underline{9000 \text{ rad/s}}}$$

$$|T(j\omega_{p1})| = |T(j\omega_{p2})| = 10^{-1/20} = \underline{0.8913}$$

$$|T(j\omega_0)| = \frac{\omega_0 a_1}{\sqrt{(\omega_0^2-\omega_0^2)^2 + \left(\frac{\omega_0^2}{Q}\right)^2}} = 1$$

$$\Rightarrow \frac{\omega_0 a_1}{\omega_0^2/Q} = 1$$

$$\therefore \frac{Q a_1}{\omega_0} = 1 \Rightarrow a_1 = \frac{\omega_0}{Q}$$

$$|T(j\omega_{p1})|^2 = |T(j0.9\omega_0)|^2 = 0.8913^2$$

$$\frac{(\omega_0/Q)^2(0.9\omega_0)^2}{\left(\omega_0^2-(0.9\omega_0)^2\right)^2 + \left(\frac{0.9\omega_0}{Q}\right)^2} = 0.8913^2$$

$$\left(\frac{\omega_0}{Q}(.9\omega_0)\right)^2 = 0.8913^2\left[\left(\omega_0^2-(0.9\omega_0)^2\right)^2 + \left(\frac{0.9\omega_0}{Q}\right)^2\right]$$

$$\frac{0.81\omega_0^4}{Q^2} = 0.8913^2\left[\omega_0^4(1-0.81)^2 + \frac{0.81\omega_0^4}{Q^2}\right]$$

$$\frac{0.81\omega_0^4}{Q^2}(1-0.8913^2) = 0.8913^2\omega_0^4 \times (1-0.81)^2$$

SUB $\omega_0 = 9000$ gives

$$\underline{\underline{Q = 2.41}}$$

Now $a_1 = \frac{\omega_0}{Q} = 0.415\omega_0$

$$\therefore T(s) = \underline{\underline{\frac{0.415\omega_0 s}{s^2 + 0.415\omega_0 s + \omega_0^2}}}$$

If $\omega_{s1} = 3000$ rad/s

$$|T(j3000)| = \frac{0.415\omega_0(3000)}{\sqrt{(\omega_0^2-3000^2)^2 + (\omega_0 3000 \times .415)^2}}$$

$$= 0.1537$$

$$\therefore A_{MIN} = -20\log(0.1537)$$

$$= \underline{\underline{16.3\text{dB}}}$$

Now ω_{s1} and ω_{s2} are geometrically symmetrical about ω_0:

$$\omega_{s1}\omega_{s2} = \omega_0^2$$

$$\omega_{s2} = \frac{9000^2}{3000}$$

$$= \underline{\underline{27\,000 \text{ rad/s}}}$$

$\boxed{12.30}$

From exercise 12.15

$$Q = \frac{\omega_0}{BW\sqrt{10^{A/10}-1}} \qquad \begin{cases} \omega_0 = 2\pi(60) \\ BW = 2\pi 6 \\ A = 20\text{dB} \end{cases}$$

$$= \underline{\underline{1.005}}$$

$$T(s) = a_2\frac{s^2+\omega_0^2}{s^2 + s\frac{\omega_0}{Q} + \omega_0^2}$$

$$|T(0)| = \frac{a_2\omega_0^2}{\omega_0^2} = 1 \leftarrow \text{DC Gain}$$

CONT.