$a_2 = 1$

$$T(s) = \frac{s^2 + (2\pi 60)^2}{s^2 + s\frac{2\pi 60}{1.005} + (2\pi 60)^2}$$

$$T(s) = \frac{s + 1.421\times10^5}{s^2 + 375.1s + 1.421\times10^5}$$

12.31

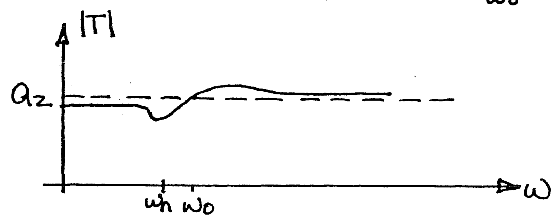
FOR ALL PASS:

$$T(s) = a_2 \frac{s^2 - s\,\omega_0/Q + \omega_0^2}{s^2 + s\,\omega_0/Q + \omega_0^2}$$

If zero frequency < pole frequency

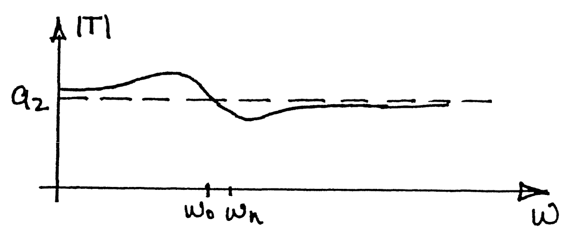
$$T(s) = a_2 \frac{s^2 - s\,\omega_n/Q + \omega_n^2}{s^2 + s\,\omega_0/Q + \omega_0^2} \qquad \omega_n < \omega_0$$

At DC: $|T| = a_2 \dfrac{\omega_n^2}{\omega_0^2}$ where $\dfrac{\omega_n^2}{\omega_0^2} < 1$



If zero frequency > pole frequency then $\omega_n > \omega_0$

At DC: $|T| = a_2\,\omega_n^2/\omega_0^2$ where $\dfrac{\omega_n^2}{\omega_0^2} > 1$



12.32

$$T(s) = \frac{s^2 - s\,\omega_0/Q' + \omega_0^2}{s^2 + s\,\omega_0/Q_0 + \omega_0^2}\, a_2$$

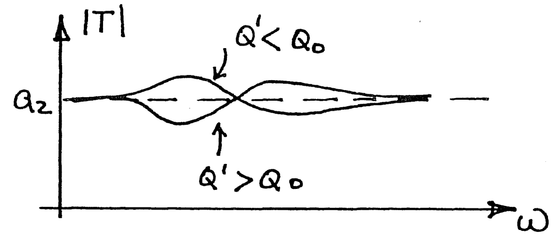
Zero Q < Pole Q $\Rightarrow Q' < Q_0$

At $\omega = \omega_0$:

$$|T| = \frac{a_2\,\omega_0^2/Q'}{\omega_0^2/Q_0} = \frac{a_2 Q_0}{Q'} > a_2$$

If $Q' > Q_0$

$$|T(j\omega_0)| = \frac{a_2 Q_0}{Q'} < a_2$$



12.33

$\omega_0 = 10^4$ rad/s, $Q = 2$, $R = 10\text{k}\Omega$

$$\omega_0 = \frac{1}{\sqrt{LC}} \Rightarrow \omega_0^2 = \frac{1}{LC} \Leftarrow L = \frac{R^2 C}{Q^2}$$

$$= \frac{Q^2}{R^2C^2}$$

$$C = \frac{Q}{R\omega_0} = \underline{\underline{20\text{nF}}}$$

$$L = \frac{1}{C\omega_0^2} = \underline{\underline{500\text{mH}}}$$

12.34

$\omega_o = 1/\sqrt{LC}$

If $L' = 1.01L$

$\omega_o' = (1.01LC)^{-1/2}$

$= 0.9950 \frac{1}{\sqrt{LC}}$

$= 0.9950\,\omega_o$.

$\therefore\ \Delta\omega_o = -0.5\%$

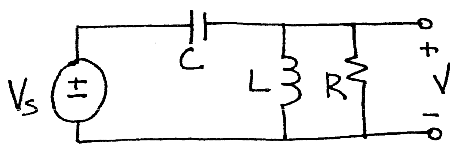
If $C' = 1.01C$

$\omega_o' = 0.9950\,\omega_o$

$\Delta\omega_o' = -0.5\%$

Changing R has no effect on ω_o

12.35



Use voltage divider rule:

$$V_o = \frac{Z_{R\|L}}{Z_{R\|L} + Z_C} V_s$$

$$\frac{V_o}{V_s} = \frac{\left(\frac{1}{R} + \frac{1}{sL}\right)^{-1}}{\left(\frac{1}{R} + \frac{1}{sL}\right)^{-1} + \frac{1}{sC}}$$

$$= \frac{sC}{\left(\frac{1}{sL} + \frac{1}{R}\right) + sC}$$

$$\therefore\ T(s) = \frac{V_o(s)}{V_s(s)} = \frac{s^2}{s^2 + s/RC + \frac{1}{LC}}$$

12.36

Low Pass: $\omega_o = 10^5$, $C = 0.1\,\mu F$
$Q = 1/\sqrt{2}$

$Q = \omega_o CR$

$R = \frac{Q}{\omega_o C}$

$= \frac{1}{\sqrt{2} \times 10^5 \times 0.1 \times 10^{-6}}$

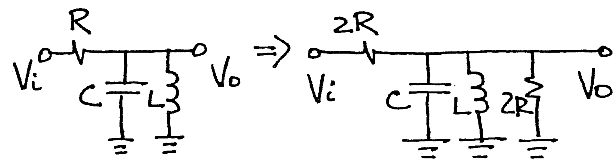
$= 70.7\,\Omega$

$\omega_o = 1/\sqrt{LC}$

$L = \frac{1}{\omega_o^2 C}$

$= 1\,mH$

12.37



$\omega_o = \frac{1}{\sqrt{LC}}$

$Q = \omega_o CR$

$A_{MID} = 1$

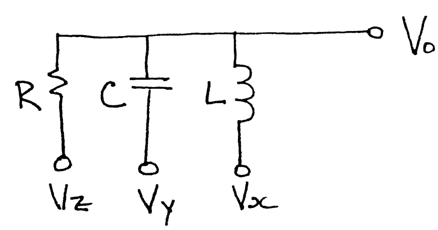
$\omega_o = 1/\sqrt{LC}$

$Q = \omega_o C\,(2R \| 2R)$

$= \omega_o CR$

$A_{MID} = \frac{2R}{2R + 2R} = 1/2$

12.38



$V_o/V_z \big|_{V_y = V_x = 0} = T_{BP}(s)$

$V_o/V_y \big|_{V_x = V_z = 0} = T_{HP}(s)$

CONT.

$$\left.\frac{V_o}{V_x}\right|_{V_y=V_z=0} = T_{LP}(s)$$

Using superposition

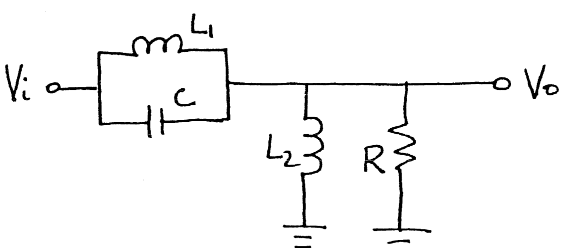
$$V_o = \frac{V_o}{V_x}V_x + \frac{V_o}{V_y}V_y + \frac{V_o}{V_z}V_z$$

$$= T_{LP}V_x + T_{HP}V_y + T_{BP}V_z$$

$$= \frac{\frac{1}{LC}V_x + s^2 V_y + \frac{s}{RC}V_z}{s^2 + s/RC + 1/LC}$$

$$\therefore V_o = V_x \frac{1/LC}{s^2 + s/RC + 1/LC} + V_y \frac{s^2}{s^2 + s/RC + 1/LC} + V_z \frac{s/RC}{s^2 + s/RC + 1/LC}$$

12.39



From Eq 12.46

$$T(s) = \frac{s^2 + 1/L_1C}{s^2 + s\,(1/CR) + \frac{1}{(L_1 \| L_2)C}}$$

Required notch $\omega_n^2 = \frac{1}{L_1C} = (0.9\omega_0)^2$

but:

$$\omega_0^2 = \frac{1}{(L_1\|L_2)C} \qquad \text{where } L_1\|L_2 = \frac{1}{L_1^{-1}+L_2^{-1}} = \frac{L_1L_2}{L_1+L_2}$$

$$= \frac{L_1+L_2}{L_1L_2C}$$

$$= \frac{L_1+L_2}{L_2}(.9\omega_0)^2$$

$$1 = \left(\frac{L_1}{L_2}+1\right)0.9^2$$

$$\therefore L_1/L_2 = \frac{1}{0.9^2} - 1 = \underline{\underline{0.2346}}$$

For $\omega \ll \omega_0$:-

$$|T| \cong \frac{1/L_1C}{1/(L_1\|L_2)C} = \frac{L_2}{L_1+L_2}$$

i.e. inductors dominate!

For $\omega \gg \omega_0$ L_1 & L_2 are "open" C is shorted

$$\underline{\underline{|T| \cong 1}}$$

12.40

$L = C_4 R_1 R_3 R_5 / R_2$

Choose $\underline{\underline{R_1 = R_2 = R_3 = R_5 = 10\,k\Omega}}$

$\therefore L = C_4 \times 10^8$

For:

$L = 10H = C_4 \times 10^8 \Rightarrow \underline{\underline{C_4 = 100nF}}$

$L = 1H \Rightarrow \underline{\underline{C_4 = 10nF}}$

$L = 0.1H \Rightarrow \underline{\underline{C_4 = 1nF}}$

$$T(S) = \frac{0.4508\,\omega_p\,(s^2 + 1.6996\,\omega_p^2)}{(s + 0.7294\,\omega_p)(s^2 + 0.2786\,\omega_p s + 1.0504\,\omega_p^2)}$$

SUB $\omega_p = 10^4$ rad/s

$$T(S) = \frac{4508\,(s^2 + 1.6996 \times 10^8)}{(s + 7294)(s^2 + 2786 s + 1.0504 \times 10^8)}$$

Part (b)

First decompose T(s) into 1st-and 2nd-order sections with unity DC gain!

$$T_1(s) = \frac{k_1}{s + 7294} \qquad T_1(0) = \frac{k_1}{7294} = 1$$

$$\Rightarrow k_1 = 7294$$

Now $k_1 k_2 = 4508 \Rightarrow k_2 = 0.6180$

$$\therefore T_2(s) = \frac{0.6180\,(s^2 + 1.6996 \times 10^8)}{s^2 + 2786 s + 1.0504 \times 10^8}$$

As a check:

$$T_2(0) = \frac{0.6180\,(1.6996 \times 10^8)}{1.0504 \times 10^8} = 1.000$$

As EXPECTED!

$$\therefore T(S) = T_1(s) \cdot T_2(s)$$

For first-order section use Fig 12.13(a)

$\omega_0 = 7294$ rad/s DC Gain $= 1$

Let $C = 10\,nF$

$$R_1 = R_2 = \frac{1}{\omega_0 C} \Rightarrow R_1 = R_2 = 13.71\,k\Omega$$

For second-order section

$\omega_n^2 = 1.6996 \times 10^8 \Rightarrow \omega_n = 13.037 \times 10^3$

$\omega_0^2 = 1.0504 \times 10^8 \Rightarrow \omega_0 = 10.249 \times 10^3$

$\frac{\omega_0}{Q} = 2786 \Rightarrow Q = 3.6787$

For LPN use table 12.1 and Fig 12.22 (e)

Make $R_1 = R_2 = R_3 = R_5 = R$ and $C = C_4 = C_6 = 10\,nF$

$$R = \frac{1}{\omega_0 C} = 9.757\,k\Omega = R_1 = R_2 = R_3 = R_5$$

$$\omega_n^2 = \frac{1}{C R^2 C_{61}} \Rightarrow C_{61} = 6.18\,nF$$

$$C_{62} = C - C_{61} = 3.82\,nF$$

$$Q = R_6 \sqrt{\frac{C_6}{C_4} \frac{R_2}{R_1 R_3 R_5}} = \frac{R_6}{\sqrt{R^2}} = \frac{R_6}{R}$$

$$\therefore R_6 = RQ = (9.757 \times 10^3)(3.6787) = 35.89\,k\Omega$$

12.49

$f_0 = 1\,kHz$

The 3dB bandwidth for a 2nd order filter is given by:

$$B = \omega_0 / Q \Rightarrow Q = \frac{2\pi 10^3}{2\pi 50} = 20$$

Choose $C = 10\,nF$

$$R = \frac{1}{\omega_0 C} = 15.92\,k\Omega$$

Use $R_1 = R_f = 10\,k\Omega$

CONT.

$$\frac{R_3}{R_2} = 2Q-1 = 39$$

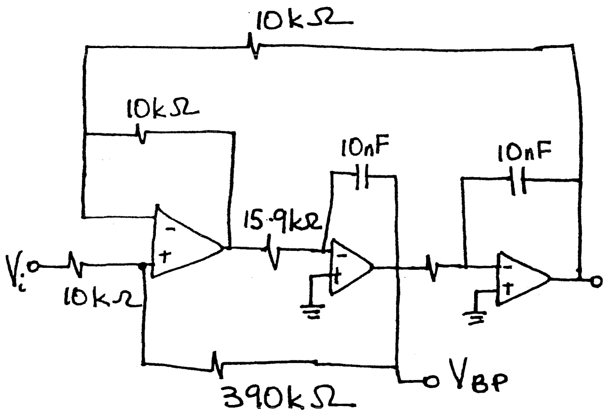
choose $\underline{\underline{R_2 = 10k\Omega}}$ $\underline{\underline{R_3 = 390k\Omega}}$

Now $T(s) = \dfrac{-k\omega_0 s}{s^2 + s\frac{\omega_0}{Q} + \omega_0^2}$

$$\Rightarrow |T(j\omega_0)| = \frac{k\omega_0^2}{\omega_0^2/Q} = kQ$$

but $k = 2 - 1/Q = 1.95$

$\therefore$ Centre-freq gain $= kQ = \underline{\underline{39}}$



12.50

$R_L = R_H = R_B/Q \Rightarrow R_B = QR_H$
$R_L = R_H$

Using Eq 12.66 :

$$\frac{V_o}{V_i} = -k\,\frac{\frac{R_F}{R_H}s^2 - s\left(\frac{R_F}{R_B}\right)\omega_0 + \left(\frac{R_F}{R_L}\right)\omega_0^2}{s^2 + s\frac{\omega_0}{Q} + \omega_0^2}$$

$$= -k\frac{R_F}{R_H}\;\frac{s^2 - \frac{\omega_0}{Q}s + \omega_0^2}{s^2 + \frac{\omega_0}{Q}s + \omega_0^2}$$

Flat Gain $= \underline{-k\,R_F/R_H}$

Part (b) − $\omega_0 = 10^4$ rad/s $Q = 2$ Flat Gain $= 10$

choose $\underline{\underline{C = 10nF}} \Rightarrow R = \dfrac{1}{\omega_0 C} = \underline{\underline{10k\Omega}}$

choose $\underline{\underline{R_F = R_1 = 10k\Omega}}$

$\dfrac{R_3}{R_2} = 2Q-1 = 3 \Rightarrow \underline{\underline{R_2 = 10k\Omega}}$
$\underline{\underline{R_3 = 30k\Omega}}$

Now $k = 2 - 1/Q = 1.5$

$\therefore$ Flat Gain $= 10 = (1.5)\dfrac{R_F}{R_H}$

$$\therefore \frac{R_H}{R_F} = 0.15$$

choose $\underline{\underline{R_F = 100k\Omega}}$

$\underline{\underline{R_H = R_L = 15k\Omega}}$

$R_B = QR_H = \underline{\underline{30k\Omega}}$

12.51

Note ω_n does not depend on R or C
From eq. 12.67 :

$$\frac{R_H}{R_L} = \left(\frac{\omega_n}{\omega_0}\right)^2$$

$\therefore \omega_n = \omega_0\sqrt{\dfrac{R_H}{R_L}}$ Nominally $R_H = R_L \pm 1\%$

Thus :

$\omega_n' = \omega_0\sqrt{\dfrac{1.01}{0.99}} = 1.01\omega_0$ $\omega_n'' = \omega_0\sqrt{\dfrac{0.99}{1.01}} = 0.99\omega_0$

$\therefore \omega_n$ can deviate from ω_0 by $\underline{\underline{\pm 1\%}}$

12.52

Use Tow Thomas to realize a LPN (Fig 12.26)

$\omega_0 = 10^4$ $\omega_n = 1.2\omega_0$ $Q = 10$ DC Gain $= 1$

$C = 10nF$ $r = 20k\Omega$

$$R = \frac{1}{\omega_0 C} = \underline{\underline{10k\Omega}}$$

from 12.16 (e):

$$\text{DC Gain} = a_2 \frac{\omega_n^2}{\omega_0^2} = 1$$

$$a_2 \frac{1.2^2 \omega_0^2}{\omega_0^2} = 1$$

$$a_2 = \frac{1}{1.2^2} = \text{HF. Gain}$$

$$C_1 = C a_2 = \frac{10 \times 10^{-9}}{1.2^2} = \underline{\underline{6.94nF}}$$

$$R_2 = \frac{R(\omega_0/\omega_n)^2}{\text{HF Gain}} = R\left(\frac{1}{1.2}\right)^2 \times (1.2)^2$$

$$= R = \underline{\underline{10k\Omega}}$$

$\underline{\underline{R_1 = R_3 = \infty}}$

$$Q_z = \frac{\sqrt{\frac{1}{C^2 R R_2} \frac{C}{C_1}}}{\frac{1}{C}\left(\frac{1}{R_1} - \frac{r}{R R_3}\right)\left(\frac{C}{C_1}\right)}$$

$$= \underline{\underline{\frac{1}{\sqrt{R R_2}\left(\frac{1}{R_1} - \frac{r}{R R_3}\right)\sqrt{\frac{C}{C_1}}}}}$$

For All Pass $R_1 \to \infty$

To adjust Q_z, trim r or R_2 (independent of ω_z!)

Now $\omega_0 = \frac{1}{CR}$ so do not trim R or C!

Note if we trim R_2 or C_1 to adjust ω_z, this will also affect Q_z. So the options are:

For ω_z: (a) trim R_2 AND (r or R_3) to maintain the value of Q_z

OR

(b) trim C_1, and r or R_3

Prefer not to trim a capacitor so use (a)!

12.53

For all pass:

$$T(s) = \frac{-s^2\left(\frac{C_1}{C}\right) + s\frac{1}{C}\left(\frac{1}{R_1} - \frac{r}{RR_3}\right) + \frac{1}{C^2 R R_2}}{s^2 + s\frac{\omega_0}{Q} + \omega_0^2}$$

$$\omega_z^2 = \frac{1}{C^2 R R_2} \cdot \frac{C}{C_1} \Rightarrow \underline{\underline{\omega_z = \frac{1}{C\sqrt{R R_2}} \cdot \sqrt{\frac{C}{C_1}}}}$$

$$Q_z = \frac{\omega_z}{\frac{1}{C}\left(\frac{1}{R_1} - \frac{r}{RR_3}\right)\frac{C}{C_1}}$$

12.54

$$T(s) = \frac{0.4508(s^2 + 1.6996)}{(s + 0.7294)(s^2 + 0.2786s + 1.0504)}$$

Part (a) Replace s with s/ω_p, $\omega_p = 10^4$ rad/s.

$$T(s) = \frac{0.4508\left(s^2/\omega_p^2 + 1.6996\right)}{\left(\frac{s}{\omega_p} + 0.7294\right)\left(\frac{s^2}{\omega_p^2} + \frac{0.2786s}{\omega_p} + 1.0504\right)}$$

CONT.

$$T(s) = \frac{0.4508\,\omega_p(s^2 + 1.6996\,\omega_p^2)}{(s + 0.7294\,\omega_p)(s^2 + 0.2786\,\omega_p s + 1.0504\,\omega_p^2)}$$

$$= \frac{4508(s^2 + 1.6996 \times 10^8)}{(s + 7294)(s^2 + 2786 s + 1.0504 \times 10^8)}$$

For FIRST ORDER SECTION use Fig 12.13(a)

$\omega_0 = 7294$ DC gain $= 1$

choose $C = 10\,nF$

$R_1 = R_2 = \frac{1}{\omega_0 C} \Rightarrow R_1 = R_2 = 13.71\,k\Omega$

For SECOND ORDER SECTION - use FIG 12.26

$\omega_n^2 = 1.6996 \times 10^8 \Rightarrow \omega_n = 13.037 \times 10^3$

$\omega_0^2 = 1.0504 \times 10^8 \Rightarrow \omega_0 = 10.249 \times 10^3$

$\frac{\omega_0}{Q} = 2786 \Rightarrow Q = 3.6787$

DC gain $= 1$

For Tow Thomas LPN use table 12.2

choose $C = 10\,nF$

$R = \frac{1}{\omega_0 C} = \frac{1}{10.249 \times 10^3 \; 10 \times 10^{-9}}$

$= 9.757\,k\Omega$

choose $r = 20\,k\Omega$

Now from Fig 12.16 (e):

$T(0) = a_2 \frac{\omega_n^2}{\omega_0^2} = 1 \Rightarrow a_2 = \frac{\omega_0^2}{\omega_n^2} = 0.618$

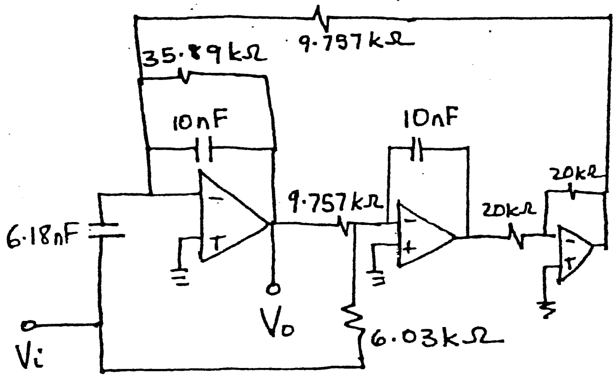
$\therefore$ HF gain $= a_2 = 0.618$

$C_1 = C \times$ HF gain $\Rightarrow C_1 = 6.18\,nF$

$R_2 = R(\omega_0/\omega_n)^2 / 0.618$

$R_2 = R \Rightarrow R_2 = 9.757K$

$R_1 = R_3 = \infty$ $QR = 35.89\,k\Omega$



12.55

Make $C_1 = C_2 = 1nF = C$

$\omega_0 = \frac{1}{\sqrt{C_1 C_2 R_3 R_4}}$ $Q = \left[\frac{\sqrt{C_1 C_2 R_3 R_4}}{R_3}\left(\frac{1}{C_1} + \frac{1}{C_2}\right)\right]^{-1}$

Let $R_3 = R$ $m = 4Q^2$

$R_4 = \frac{R}{m}$ $= 4/2 = 2$

$\therefore \omega_0 = \frac{1}{\sqrt{C^2 R^2/2}} = \frac{\sqrt{2}}{RC} = 10^4$

$\Rightarrow R = \frac{\sqrt{2}}{10^4 10^{-9}} = 141.42\,k\Omega$

$R_3 = 141.4\,k\Omega$

$R_4 = \frac{R_3}{2} \Rightarrow R_4 = 70.7\,k\Omega$